Engineering Thermodynmaics (ETD)

Semester : 4TH

Branch : Mechanical Engineering

Module - I

CONTENTS

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4. First Law of Thermodynamics

Some Important Notes

- $đQ$ is an inexact differential, and we write

$$\int_1^2 đQ = Q_{1-2} \quad or \quad {}_1Q_2 \neq Q_2 - Q_1$$

- $đW$ is an inexact differential, and we write

$$W_{1-2} = \int_1^2 đW = \int_1^2 pdV \neq W_2 - W_1$$

- $(\Sigma Q)_{cycle} = (\Sigma W)_{cycle}$ or $\oint \delta Q = \oint \delta W$

  The summations being over the entire cycle.

- **$\delta Q - \delta W = dE$**

  E consists of $\rightarrow$ E=U+KE+PE

  U -internal energy

  KE - the kinetic energy

  PE - the potential energy

  For the whole process A $\rightarrow$ $Q-W=E_2-E_1$

  Similarly for the process B $\rightarrow$ $Q-W=E_1-E_2$

- An isolated system which does not interact with the surroundings Q = 0 and W = 0. Therefore, E remains constant for such a system.
- The Zeroth Law deals with thermal equilibrium and provides a means for measuring temperatures.
- The First Law deals with the conservation of energy and introduces the concept of internal energy.
- The Second Law of thermodynamics provides with the guidelines on the conversion heat energy of matter into work. It also introduces the concept of entropy.
- The Third Law of thermodynamics defines the absolute zero of entropy. The entropy of a pure crystalline substance at absolute zero temperature is zero.

Summation of 3 Laws

- Firstly, there isn't a meaningful temperature of the source from which we can get the full conversion of heat to work. Only at infinite temperature one can dream of getting the full 1 kW work output.
- Secondly, more interestingly, there isn't enough work available to produce 0K. In other words, 0 K is unattainable. This is precisely the Third law.

- Because, we don't know what 0 K looks like, we haven't got a starting point for the temperature scale!! That is why all temperature scales are at best empirical.

You can't get something for nothing:

To get work output you must give some thermal energy.

You can't get something for very little:

To get some work output there is a minimum amount of thermal energy that needs to be given.

You can't get every thing:

However much work you are willing to give 0 K can't be reached.

Violation of all 3 laws:

Try to get everything for nothing.

Questions with Solution P. K. Nag

Q4.1 **An engine is tested by means of a water brake at 1000 rpm. The measured torque of the engine is 10000 mN and the water consumption of the brake is 0.5 m³/s, its inlet temperature being 20°C. Calculate the water temperature at exit, assuming that the whole of the engine power is ultimately transformed into heat which is absorbed by the cooling water.**

(**Ans.** 20.5°C)

Solution: Power = T.ω

$$= 10000 \times \left(\frac{2\pi \times 1000}{60}\right)$$

$$= 1.0472 \times 10^6 \text{ W}$$

$$= 1.0472\text{MW}$$

Let final temperature = t°C

$\therefore$ Heat absorb by cooling water / unit $= \dot{m}\ s\ \Delta t$

$$= \dot{v}\rho s\ \Delta t$$

$$= 0.5 \times 1000 \times 4.2 \times (t - 20)$$

$\therefore$ $0.5 \times 1000 \times 4.2 \times (t - 20) = 1.0472 \times 10^6$

$\therefore$ $t - 20 = 0.4986 \approx 0.5$

$\therefore$ $t = 20.5°C$

Q4.2 **In a cyclic process, heat transfers are + 14.7 kJ, – 25.2 kJ, – 3.56 kJ and + 31.5 kJ. What is the net work for this cyclic process?**

(**Ans.** 17.34 kJ)

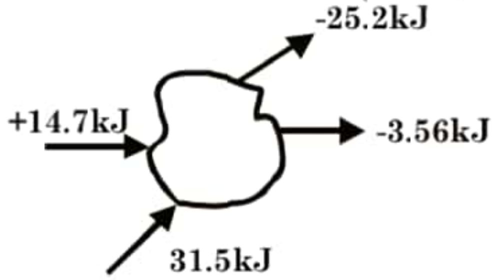
Solution : $\sum Q = (14.7 + 31.5 - 25.2 - 3.56)$ kJ

$= 17.44$ kJ

From first law of thermodynamics (for a cyclic process)

$$\sum Q = \sum W$$

$$\therefore \sum W = 17.44 \text{ kJ}$$



Q4.3 **A slow chemical reaction takes place in a fluid at the constant pressure of 0.1 MPa. The fluid is surrounded by a perfect heat insulator during the reaction which begins at state 1 and ends at state 2. The insulation is then removed and 105 kJ of heat flow to the surroundings as the fluid goes to state 3. The following data are observed for the fluid at states 1, 2 and 3.**

State	v (m³)	t (°C)
1	0.003	20
2	0.3	370
3	0.06	20

For the fluid system, calculate E_2 and E_3, if $E_1 = 0$

(**Ans.** $E_2 = -29.7$ kJ, $E_3 = -110.7$ kJ)

Solution: From first law of thermodynamics

$$\cancel{d}Q = \Delta E + pdV$$

$$\therefore \quad Q = \Delta E + \int pdV$$

$$\therefore \quad Q_{1-2} = (E_2 - E_1) + \int_1^2 pdV$$

$$\text{or} \quad = (E_2 - E_1) + 0.1 \times 10^3 (0.3 - 0.003) \qquad [\text{as insulated } Q_{2-3} = 0]$$

$$\text{or} \quad E_2 = -29.7 \text{ kJ}$$

$$Q_{2-3} = (E_3 - E_2) + \int_2^3 pdV$$

$$\text{or} \quad -105 = (E_3 - E_2) + 0.1 \times 10^3 (0.06 - 0.3)$$

$$\text{or} \quad -105 = E_3 + 29.7 + 0.1 \times 10^3 (0.06 - 0.3)$$

$$\text{or} \quad -105 = E_3 + 29.7 - 24$$

$$\text{or} \quad E_3 = -105 - 29.7 + 24$$

$$= -110.7 \text{ kJ}$$

Q4.4 **During one cycle the working fluid in an engine engages in two work interactions: 15 kJ to the fluid and 44 kJ from the fluid, and three heat interactions, two of which are known: 75 kJ to the fluid and 40 kJ from the fluid. Evaluate the magnitude and direction of the third heat transfer.**

(**Ans.** – 6 kJ)

Solution: From first law of thermodynamics

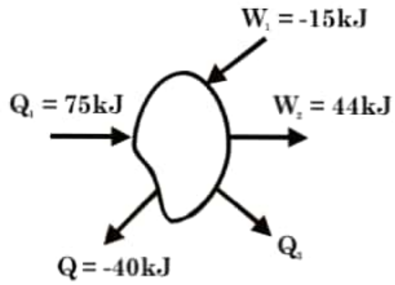
$$\sum \cancel{d}Q = \sum \cancel{d}W$$

$$\therefore \quad Q_1 + Q_2 + Q_3 = W_1 + W_2$$

$$\text{or} \quad 75 - 40 + Q_3 = -15 + 44$$

$$Q_3 = -6\text{kJ}$$

i.e. 6kJ from the system



Q4.5 **A domestic refrigerator is loaded with food and the door closed. During a certain period the machine consumes 1 kWh of energy and the internal energy of the system drops by 5000 kJ. Find the net heat transfer for the system.**

(**Ans.** – 8.6 MJ)

Solution:

$$Q = \Delta E + W$$

$$Q_{2-1} = (E_2 - E_1) + W_{2-1}$$

$$= -5000\text{kJ} + \frac{-1000 \times 3600}{1000} \text{kJ}$$

$$= -8.6\text{MJ}$$

Q4.6 **1.5 kg of liquid having a constant specific heat of 2.5 kJ/kg K is stirred in a well-insulated chamber causing the temperature to rise by 15°C. Find ΔE and W for the process.**

(**Ans.** Δ E = 56.25 kJ, W = – 56.25 kJ)

Solution: Heat added to the system = 1.5 × 2.5 × 15kJ

= 56.25 kJ

$\therefore$ ΔE rise = 56.25kJ

As it is insulated then $\cancel{d}Q = 0$

$\therefore$ $\Delta Q = \Delta E + W$

or $0 = 56.25 + W$

or $W = -56.25$ kJ

Q4.7 **The same liquid as in Problem 4.6 is stirred in a conducting chamber. During the process 1.7 kJ of heat are transferred from the liquid to the surroundings, while the temperature of the liquid is rising to 15°C. Find ΔE and W for the process.**

(**Ans.** Δ E = 54.55 kJ, W = 56.25 kJ)

Solution: As temperature rise is same so internal energy is same

$\Delta E = 56.25$ kJ

As heat is transferred form the system so we have to give more work = 1.7 kJ to the system

So $W = -56.25 - 1.7$ kJ

$= -57.95$ kJ

Q4.8 **The properties of a certain fluid are related as follows:**

$$\boldsymbol{u = 196 + 0.718t}$$

$$\boldsymbol{pv = 0.287\ (t + 273)}$$

Where u is the specific internal energy (kJ/kg), t is in °C, p is pressure (kN/m²), and v is specific volume (m³/kg). For this fluid, find c_v and c_p.

(**Ans.** 0.718, 1.005 kJ/kg K)

Solution:

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p$$

$$= \left[\frac{\partial(u + pV)}{\partial T}\right]_p$$

$$= \left[\frac{\partial\{196 + 0.718t + 0.287(t + 273)\}}{\partial T}\right]_p$$

$$= \left[\frac{0 + 0.718\ \partial t + 0.287\ \partial t + 0}{\partial T}\right]_p$$

$$= \left[1.005\frac{\partial t}{\partial T}\right]_p \qquad \begin{bmatrix} T = t + 273 \\ \therefore \partial T = \partial t \end{bmatrix}$$

$$= 1.005\ \text{kJ/kg} - \text{K}$$

$$
\begin{aligned}
c_v &= \left(\frac{\partial u}{\partial T}\right)_v \\
&= \left[\frac{\partial(196 + 0.718t)}{\partial T}\right]_v \\
&= \left[0 + 0.718\frac{\partial t}{\partial T}\right]_v \\
&= 0.718 \text{ kJ/kg} - \text{K}
\end{aligned}
$$

Q4.9 **A system composed of 2 kg of the above fluid expands in a frictionless piston and cylinder machine from an initial state of 1 MPa, 100°C to a final temperature of 30°C. If there is no heat transfer, find the net work for the process.**

(**Ans.** 100.52 kJ)

Solution: Heat transfer is not there so

$$
\begin{aligned}
Q &= \Delta E + W \\
W &= -\Delta E \\
&= -\Delta U \\
&= -\int_1^2 C_v dT \\
&= -0.718(T_2 - T_1) \\
&= -0.718(100 - 30) \\
&= -50.26 \text{ kJ/kg}
\end{aligned}
$$

$\therefore$ Total work (W) = 2 × (-50.26) = -100.52 kJ

Q 4.10 **If all the work in the expansion of Problem 4.9 is done on the moving piston, show that the equation representing the path of the expansion in the *pv*-plane is given by $pv^{1.4}$ = constant.**

Solution: Let the process is pV^n = constant.

Then

$$\text{Work done} = \frac{p_1V_1 - p_2V_2}{n-1} \qquad [\therefore \; pV = mRT]$$

$$= \frac{mRT_1 - mRT_2}{n-1} \qquad \begin{bmatrix} R = (c_p - c_v) \\ = 1.005 - 0.718 \\ = 0.287 \text{ kJ/kg} - \text{K} \end{bmatrix}$$

$$= \frac{mR}{n-1}(T_1 - T_2)$$

or $$= \frac{2 \times 0.287 \times (100 - 30)}{n-1} = 100.52$$

or $n - 1 = 0.39972$

or $n = 1.39972 \approx 1.4$

Q4.11 **A stationary system consisting of 2 kg of the fluid of Problem 4.8 expands in an adiabatic process according to $pv^{1.2}$ = constant. The initial**

conditions are 1 MPa and 200°C, and the final pressure is 0.1 MPa. Find W and ΔE for the process. Why is the work transfer not equal to $\int pdV$?

(**Ans.** $W = 217.35$, $\Delta E = -217.35$ kJ, $\int pdV = 434.4$ kJ)

Solution: $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{0.1}{1}\right)^{\frac{1.2-1}{1.2}}$

$$\therefore \quad T_2 = T_1 \times (0.10)^{\frac{0.2}{1.2}}$$
$$= 322.251$$
$$= 49.25°C$$

From first law of thermodynamics

$$đQ = \Delta E + đW$$
$$\therefore \quad 0 = \int C_v dT + đW$$
$$\therefore \quad đW = -\int C_v dT$$
$$= -0.718 \times \int_1^2 dT = -0.718 \times (200 - 49.25) \text{ kJ/kg}$$
$$W = -2 \times W$$
$$= -2 \times 108.2356 \text{kJ}$$
$$= -216.5 \text{kJ}$$
$$\therefore \quad \Delta E = 216.5 \text{kJ}$$
$$\int pdV = \frac{p_1V_1 - p_2V_2}{n-1}$$
$$= \frac{mRT_1 - mRT_2}{n-1}$$
$$= \frac{mR(T_1 - T_2)}{n-1}$$
$$= \frac{2 \times 0.287(200 - 49.25)}{(1.2-1)}$$
$$= 432.65 \text{kJ}$$

As this is not quasi-static process so work is not $\int pdV$.

Q4.12 **A mixture of gases expands at constant pressure from 1 MPa, 0.03 m³ to 0.06 m³ with 84 kJ positive heat transfer. There is no work other than that done on a piston. Find DE for the gaseous mixture.**

(**Ans.** 54 kJ)

The same mixture expands through the same state path while a stirring device does 21 kJ of work on the system. Find ΔE, W, and Q for the process.

(**Ans.** 54 kJ, – 21 kJ, 33 kJ)

Solution: Work done by the gas $(W) = \int pdV$

$$= p(V_2 - V_1)$$
$$= 1 \times 10^3 (0.06 - 0.03) \text{kJ}$$
$$= 30\text{kJ}$$

Heat added = 89kJ

$\therefore \quad Q = \Delta E + W$

or $\quad \Delta E = Q - W = 89 - 30 = 54\text{kJ}$

Q4.13 **A mass of 8 kg gas expands within a flexible container so that the *p–v* relationship is of the from $pv^{1.2}$ = constant. The initial pressure is 1000 kPa and the initial volume is 1 m³. The final pressure is 5 kPa. If specific internal energy of the gas decreases by 40 kJ/kg, find the heat transfer in magnitude and direction.**

(**Ans.** + 2615 kJ)

Solution: $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

$\therefore$ $$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{n}$$

or $$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

or $$V_2 = V_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

$$= 1 \times \left(\frac{1000}{5}\right)^{\frac{1}{1.2}} = 82.7\,\text{m}^3$$

$\therefore$ $$W = \frac{p_1V_1 - p_2V_2}{n-1}$$

$$= \frac{1000 \times 1 - 5 \times 82.7}{1.2 - 1} = 2932.5\text{kJ}$$

$$\Delta E = -8 \times 40 = -320 \text{ kJ}$$

$\therefore$ $$Q = \Delta E + W = -320 + 2932.5 = 2612.5\text{kJ}$$

Q4.14 **A gas of mass 1.5 kg undergoes a quasi-static expansion which follows a relationship $p = a + bV$, where a and b are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and the corresponding volumes are 0.20 m³ and 1.20 m³. The specific internal energy of the gas is given by the relation**

$$u = 1.5\, pv - 85 \text{ kJ/kg}$$

Where p is the kPa and v is in m³/kg. Calculate the net heat transfer and the maximum internal energy of the gas attained during expansion.

(**Ans.** 660 kJ, 503.3 kJ)

Solution:

$$1000 = a + b \times 0.2 \quad ....(i)$$

$$\underline{200 = a + b \times 1.2} \quad ...(ii)$$

(ii) − (i) gives

$$-800 = b$$

$$\therefore \quad a = 1000 + 2 \times 800 = 1160$$

$$\therefore \quad p = 1160 - 800V$$

$$\therefore \quad W = \int_{V_1}^{V_2} pdV$$

$$= \int_{0.2}^{1.2} (1160 - 800V)dV$$

$$= \left[1160V - 400V^2\right]_{0.2}^{1.2}$$

$$= 1160 \times (1.2 - 0.2) - 400\left(1.2^2 - .2^2\right) \text{kJ}$$

$$= 1160 - 560\text{kJ} = 600\text{kJ}$$

$$u_1 = 1.5 \times 1000 \times \frac{0.2}{1.5} - 85 = 215\text{kJ/kg}$$

$$u_2 = 1.5 \times 200 \times \frac{1.2}{1.5} - 85 = 155\text{kJ/kg}$$

$$\therefore \ \Delta u = u_2 - u_1 = (275 - 215) = 40\text{kJ/kg}$$

$$\therefore \ \Delta U = m\Delta u = 40 \times 1.5 = 60\text{kJ}$$

$$\therefore \quad Q = \Delta U + W = 60 + 600 = 660\text{kJ}$$

$$\Rightarrow \quad u = 1.5pv - 85\text{kJ/kg}$$

$$= 1.5\left(\frac{1160 - 800v}{1.5}\right)v - 85\text{kJ/kg}$$

$$= 1160v - 800v^2 - 85\text{kJ/kg}$$

$$\frac{\partial u}{\partial v} = 1160 - 1600v$$

for maximum u, $\quad \frac{\partial u}{\partial v} = 0 \therefore \quad v = \frac{1160}{1600} = 0.725$

$$\therefore \qquad u_{max.} = 1160 \times 0.725 - 800 \times (0.725)^2 - 85\text{kJ/kg}$$

$$= 335.5\text{kJ/kg}$$

$$U_{max} = 1.5u_{max} = 503.25\text{kJ}$$

Q4.15 **The heat capacity at constant pressure of a certain system is a function of temperature only and may be expressed as**

$$C_p = 2.093 + \frac{41.87}{t + 100} \text{ J/°C}$$

Where t is the temperature of the system in °C. The system is heated while it is maintained at a pressure of 1 atmosphere until its volume increases from 2000 cm³ to 2400 cm³ and its temperature increases from 0°C to 100°C.

(a) Find the magnitude of the heat interaction.

(b) How much does the internal energy of the system increase?

(**Ans.** (a) 238.32 J (b) 197.79 J)

Solution: $Q = \int_{273}^{373} C_p dT$

$t = T - 273$

$\therefore t + 100 = T - 173$

$$= \int_{273}^{373} \left(2.093 + \frac{41.87}{T-173}\right) dT$$

$$= \left[2.093T + 41.87 \ln|T - 173|\right]_{273}^{373}$$

$$= 2.093(373 - 273) + 41.87 \ln\left(\frac{200}{100}\right)$$

$$= 209.3 + 41.87 \ln 2$$

$$= 238.32 J$$

$$Q = \Delta E + \int pdV$$

$$\Delta E = Q - \int pdV$$

$$= Q - p(V_2 - V_1)$$

$$= 238.32 - 101.325(0.0024 - 0.0020) \times \times 1000 J$$

$$= (238.32 - 40.53) J$$

$$= 197.79 J$$

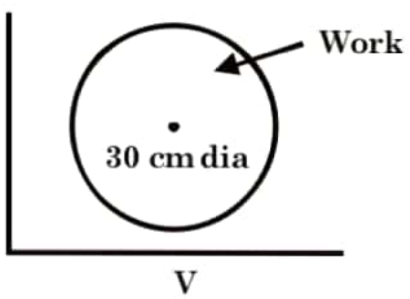
Q4.16 **An imaginary engine receives heat and does work on a slowly moving piston at such rates that the cycle of operation of 1 kg of working fluid can be represented as a circle 10 cm in diameter on a *p–v* diagram on which 1 cm = 300 kPa and 1 cm = 0.1 m³/kg.**

(a) How much work is done by each kg of working fluid for each cycle of operation?

(b) The thermal efficiency of an engine is defined as the ratio of work done and heat input in a cycle. If the heat rejected by the engine in a cycle is 1000 kJ per kg of working fluid, what would be its thermal efficiency?

(**Ans.** (a) 2356.19 kJ/kg, (b) 0.702)

Solution: Given Diameter = 10 cm



$$\therefore \text{Area} = \frac{\pi \times 10^2}{4} cm^2 = 78.54 \ cm^2$$

$$1 \ cm^2 \equiv 300 kPa \times 0.1 m^3 / kg$$

$$= 30 kJ$$

$$\therefore \text{Total work done} = 78.54 \times 30 kJ / kg$$

$$= 2356.2 \ kJ / kg$$

$$\text{Heat rejected} = 1000 kJ$$

$$\text{Therefore, } \eta = \frac{2356.2}{2356.2 + 1000} \times 100\%$$

$$= 70.204\%$$

Q4.17 **A gas undergoes a thermodynamic cycle consisting of three processes beginning at an initial state where p_1 = 1 bar, V_1 = 1.5 m³ and U_1 = 512 kJ. The processes are as follows:**

(i) Process 1–2: Compression with pV = constant to p_2 = 2 bar, U_2 = 690 kJ

(ii) Process 2–3: W_{23} = 0, Q_{23} = –150 kJ, and

(iii) Process 3–1: W_{31} = +50 kJ. Neglecting KE and PE changes, determine the heat interactions Q_{12} and Q_{31}.

(**Ans.** 74 kJ, 22 kJ)

Solution:

$$Q_{1-2} = \Delta E + \int pdV$$

$$Q_{1-2} = (u_2 - u_1) + p_1V_1 \int_{V_1}^{V_2} \frac{dV}{V}$$

$$= (690 - 512) + 100 \times 1.5 \times \ln\left(\frac{p_1}{p_2}\right)$$

$$= 178 - 103.972$$

$$= 74.03 \text{kJ}$$

As $W_{2\text{-}3}$ is ZERO so it is constant volume process. As W_{31} is +ive (positive) so expansion is done.

$$\therefore \ u_3 = u_2 - 150 = 540 \text{kJ}$$

$$\therefore Q_{31} = u_1 - u_3 + W$$

$$= \Delta E + W = -(540 - 512) + 50$$

$$= -28 + 50 = 22 \text{kJ}$$

Q4.18 **A gas undergoes a thermodynamic cycle consisting of the following processes:**

(i) Process 1–2: Constant pressure p = 1.4 bar, V_1 = 0.028 m³, W_{12} = 10.5 kJ

(ii) Process 2–3: Compression with pV = constant, $U_3 = U_2$

(iii) Process 3–1: Constant volume, $U_1 - U_3$ = – 26.4 kJ. There are no significant changes in KE and PE.

(a) Sketch the cycle on a p–V diagram

(b) Calculate the net work for the cycle in kJ

(c) Calculate the heat transfer for process 1–2

(d) Show that $\sum_{\text{cycle}} Q = \sum_{\text{cycle}} W$.

(**Ans.** (b) – 8.28 kJ, (c) 36.9 kJ)

Solution: (b) W_{12} = 10.5 kJ

$$W_{23} = \int_2^3 p\,dV$$

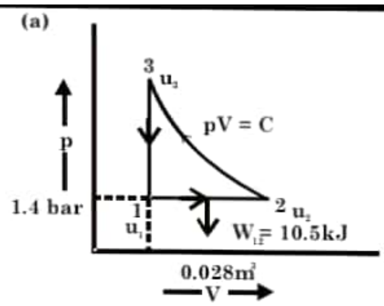
$$= p_2 V_2 \int_2^3 \frac{dV}{V}$$

$$= p_2 V_2 \ln\left(\frac{V_3}{V_2}\right)$$

$$= p_2 V_2 \ln\left(\frac{V_1}{V_2}\right)$$

$$= 1.4 \times 100 \times 0.103 \times \ln\left(\frac{0.028}{0.103}\right)$$

$$= -18.783\text{kJ}$$



$u_2 = u_3$

$u_1 - u_3 = -26.4\text{kJ}$

$\therefore\ u_1 = u_3 - 26.4\text{kJ}$

$= u_2 - 26.4\text{kJ}$

$u_2 = u_1 + 26.4\text{kJ}$

$$\left[\begin{array}{ll} \text{as} & W_{12} = p(V_2 - V_1) \\ & 10.5 = 1.4 \times 100(V_2 - 0.028) \\ \therefore & V_2 = 0.103\ \text{m}^3 \end{array}\right]$$

$W_{31} = 0$ as constant volume

$\therefore$ Net work output $= -8.283$ kJ   ans.(b)

(c) $Q_{12} = U_2 - U_1 + W_{12}$

$= 26.4 + 10.5\text{kJ} = 36.9\text{kJ}$

(d) $Q_{23} = U_3 - U_2 + W_{23}$

$= 0 - 18.783\text{kJ} = -18.783\text{kJ}$

$Q_{31} = U_2 - U_3 + 0 = -26.4\text{kJ}$

$\therefore \sum Q = Q_{12} + Q_{23} + Q_{31} = 36.9\text{kJ} - 18.783 - 26.4$

$= -8.283\text{kJ}$

$\therefore \sum W = \sum Q$ Proved.

5. First Law Applied to Flow Process

Some Important Notes

- **S.F.E.E. per unit mass basis**

$$h_1 + \frac{V_1^2}{2} + gZ_1 + \frac{\cancel{d}Q}{dm} = h_2 + \frac{V_2^2}{2} + gZ_2 + \frac{\cancel{d}W}{dm}$$

[h, W, Q should be in **J/kg** and C in m/s and g in m/s²]

$$h_1 + \frac{V_1^2}{2000} + \frac{gZ_1}{1000} + \frac{\cancel{d}Q}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{gZ_2}{1000} + \frac{\cancel{d}W}{dm}$$

[h, W, Q should be in **kJ/kg** and C in m/s and g in m/s²]

- **S.F.E.E. per unit time basis**

$$w_1\left(h_1 + \frac{V_1^2}{2} + Z_1 g\right) + \frac{\cancel{d}Q}{d\tau}$$

$$= w_2\left(h_2 + \frac{V_2^2}{2} + Z_2 g\right) + \frac{\cancel{d}W_x}{d\tau}$$

Where, w = mass flow rate (kg/s)

- **Steady Flow Process Involving Two Fluid Streams at the Inlet and Exit of the Control Volume**

Mass balance

$$w_1 + w_2 = w_3 + w_4$$

$$\frac{A_1 V_1}{v_1} + \frac{A_2 V_2}{v_2} = \frac{A_3 V_3}{v_3} + \frac{A_4 V_4}{v_4}$$

Where, v = specific volume (m³/kg)

Energy balance

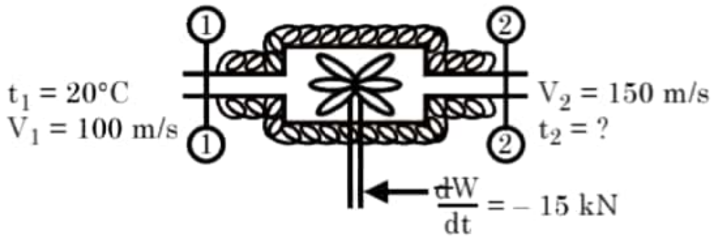
$$w_1\left(h_1+\frac{V_1^2}{2}+Z_1g\right)+w_2\left(h_2+\frac{V_2^2}{2}+Z_2g\right)+\frac{đQ}{d\tau}$$
$$=w_3\left(h_3+\frac{V_3^2}{2}+Z_3g\right)+w_4\left(h_4+\frac{V_4^2}{2}+Z_4g\right)+\frac{đW_x}{d\tau}$$

Questions with Solution P. K. Nag

Q5.1 **A blower handles 1 kg/s of air at 20°C and consumes a power of 15 kW. The inlet and outlet velocities of air are 100 m/s and 150 m/s respectively. Find the exit air temperature, assuming adiabatic conditions. Take c_p of air is 1.005 kJ/kg-K.**

(**Ans.** 28.38°C)

Solution:



From S.F.E.E.

$$w_1\left(h_1+\frac{V_1^2}{2000}+\frac{gZ_1}{1000}\right)+\frac{đQ}{dt}=w_2\left(h_2+\frac{V_2^2}{2000}+\frac{gZ_2}{1000}\right)+\frac{đW}{dt}$$

Here $w_1 = w_2 = 1\ \text{kg/s}$; $Z_1 = Z_2$; $\frac{đQ}{dt}=0.$

$$\therefore \quad h_1+\frac{100^2}{2000}+0=h_2+\frac{150^2}{2000}-15$$

$$\therefore \quad h_2-h_1=\left(15+\frac{100^2}{2000}-\frac{150^2}{2000}\right)$$

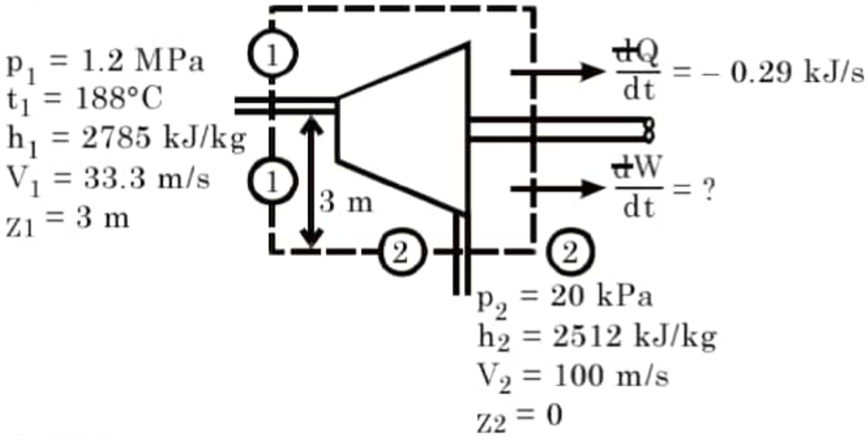
or $$C_p\left(t_2-t_1\right)=8.75$$

or $$t_2=20+\frac{8.75}{1.005}=28.7°\text{C}$$

Q5.2 **A turbine operates under steady flow conditions, receiving steam at the following state: Pressure 1.2 MPa, temperature 188°C, enthalpy 2785 kJ/kg, velocity 33.3 m/s and elevation 3 m. The steam leaves the turbine at the following state: Pressure 20 kPa, enthalpy 2512 kJ/kg, velocity 100 m/s, and elevation 0 m. Heat is lost to the surroundings at the rate of 0.29 kJ/s. If the rate of steam flow through the turbine is 0.42 kg/s, what is the power output of the turbine in kW?**

(**Ans.** 112.51 kW)

Solution: $w_1 = w_2 = 0.42\,\text{kg/s}$



By S.F.E.E.

$$w_1\left(h_1 + \frac{V_1^2}{2000} + \frac{gZ_1}{1000}\right) + \frac{\text{đ}Q}{dt} = w_2\left(h_2 + \frac{V_2^2}{2000} + \frac{gZ_2}{1000}\right) + \frac{\text{đ}W}{dt}$$

or $$0.42\left\{2785 + \frac{33.3^2}{2000} + \frac{9.81\times 3}{1000}\right\} - 0.29 = 0.42\left\{2512 + \frac{100^2}{2000} + 0\right\} + \frac{\text{đ}W}{dt}$$

or $$1169.655 = 1057.14 + \frac{\text{đ}W}{dt}$$

or $$\frac{\text{đ}W}{dt} = 112.515\ \text{kW}$$

Q5.3 **A nozzle is a device for increasing the velocity of a steadily flowing stream. At the inlet to a certain nozzle, the enthalpy of the fluid passing is 3000 kJ/kg and the velocity is 60 m/s. At the discharge end, the enthalpy is 2762 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.**

(a) Find the velocity at exists from the nozzle.

(b) If the inlet area is 0.1 m² and the specific volume at inlet is 0.187 m³/kg, find the mass flow rate.

(c) If the specific volume at the nozzle exit is 0.498 m³/kg, find the exit area of the nozzle.

(**Ans.** (a) 692.5 m/s, (b) 32.08 kg/s (c) 0.023 m²)

Solution: (a) Find V_2 i.e. Velocity at exit from S.F.E.E.

$$h_1 + \frac{V_1^2}{2000} + \frac{gZ_1}{1000} + \frac{\text{đ}Q}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{gZ_2}{1000} + \frac{\text{đ}W}{dm}$$

Here $Z_1 = Z_2$ and $\frac{\text{đ}Q}{dm} = 0$ and $\frac{\text{đ}W}{dm} = 0$

$$\therefore \quad h_1 + \frac{V_1^2}{2000} = h_2 + \frac{V_2^2}{2000}$$

$$\text{or} \quad \frac{V_2^2 - V_1^2}{2000} = (h_1 - h_2)$$

$$\text{or} \quad V_2^2 = V_1^2 + 2000(h_1 - h_2)$$

$$\text{or} \quad V_2 = \sqrt{V_1^2 + 2000(h_1 - h_2)}$$

$$= \sqrt{60^2 + 2000(3000 - 2762)} \text{m/s}$$

$$= 692.532 \text{ m/s}$$

(b) Mass flow rate $(w) = \frac{A_1V_1}{v_1}$

$$= \frac{0.1 \times 60}{0.187} \text{kg/s} = 32.1 \text{kg/s}$$

(c) Mass flow rate is same so

$$32.0855613 = \frac{A_2 \times 692.532}{0.498}$$

$$\text{or} \quad A_2 = 8.023073 \text{ m}^2$$

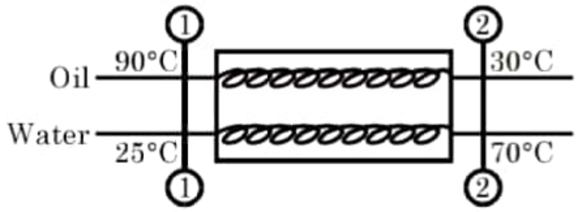
Q5.4 **In oil cooler, oil flows steadily through a bundle of metal tubes submerged in a steady stream of cooling water. Under steady flow conditions, the oil enters at 90°C and leaves at 30°C, while the water enters at 25°C and leaves at 70°C. The enthalpy of oil at t°C is given by**

$$\mathbf{h = 1.68\,t + 10.5 \times 10^{-4}\,t^2 \text{ kJ/kg}}$$

What is the cooling water flow required for cooling 2.78 kg/s of oil?

(**Ans.** 1.47 kg/s)

Solution: $w_o\,(h_{oi} + 0 + 0) + w_{H_2O}\,(h_{H_2O_i} + 0 + 0) + 0\; w_o\,(h_{o,o} + 0 + 0) + w_{H_2O}(h_{H_2O_o} + 0 + 0) + 0$



$$\therefore \quad w_o(h_{oi} - h_{o,o}) = w_{H_20}(h_{H_2O_o} - h_{H_2O_i})$$

$h_{oi} = 1.68 \times 90 + 10.5 \times 10^{-4} \times 90^2$ kJ/kg $= 159.705$ kJ/kg

$h_{o,o} = 1.68 \times 30 + 10.5 \times 10^{-4} \times 36^2$ kJ/kg $= 51.395$ kJ/kg

$$\therefore \quad W_{H_2o} = \frac{2.78 \times 108.36}{4.187\,(70 - 25)} \text{ kg/s}$$

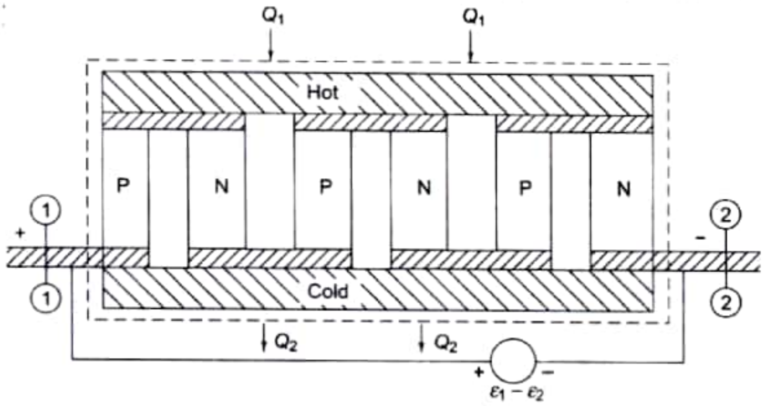
$$= 1.598815 \text{ kg/s} \simeq 1.6 \text{ kg/s}$$

Q5.5 **A thermoelectric generator consists of a series of semiconductor elements (Figure) heated on one side and cooled on the other. Electric current flow is produced as a result of energy transfer as heat. In a**

particular experiment the current was measured to be 0.5 amp and the electrostatic potential at

(1) Was 0.8 volt above that at

(2) Energy transfer as heat to the hot side of the generator was taking place at a rate of 5.5 watts. Determine the rate of energy transfer as heat from the cold side and the energy conversion efficiency.



(**Ans.** Q_2 = 5.1 watts, η = 0.073)

Solution:

$$\dot{Q}_1 = \dot{E} + \dot{Q}_2$$

or $$5.5 = 0.5 \times 0.8 + \dot{Q}_2$$

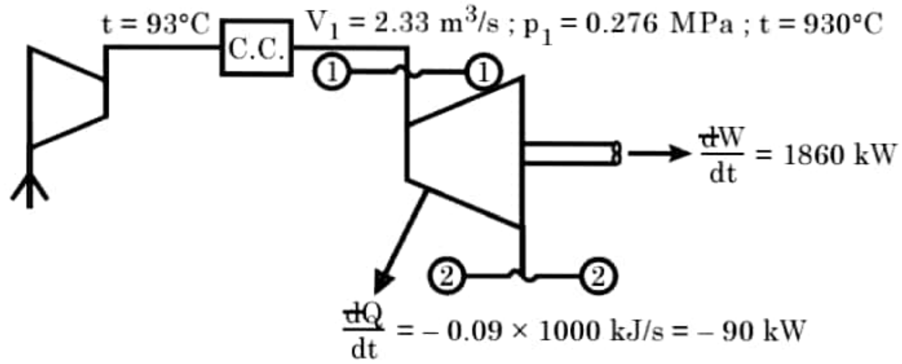
or $$\dot{Q}_2 = 5.1 \text{ watt}$$

$$\eta = \frac{5.5 - 5.1}{5.5} \times 100\% = 7.273\%$$

Q5.6 **A turbo compressor delivers 2.33 m³/s at 0.276 MPa, 43°C which is heated at this pressure to 430°C and finally expanded in a turbine which delivers 1860 kW. During the expansion, there is a heat transfer of 0.09 MJ/s to the surroundings. Calculate the turbine exhaust temperature if changes in kinetic and potential energy are negligible.**

(**Ans.** 157°C)

Solution:



$$w_1 h_1 + \frac{dQ}{dt} = w_2 h_2 + \frac{dW}{dt}$$

$$\therefore w_1 (h_1 - h_2) = \frac{dW}{dt} - \frac{dQ}{dt}$$

or $$= 1860 - (-90) = 1950 \text{ kW}$$

$$p_1 V_1 = m_1 R T_1$$

$$\therefore \quad \dot{m}_1 = \frac{p_1 V_1}{RT_1} = \frac{276 \text{ kPa} \times 2.33 \text{ m}^3/\text{s}}{0.287 \text{ kJ/kg} \times 316 K} = 7.091 \text{ kg/s}$$

Or $\quad h_1 - h_2 = 275$

$\therefore \quad C_p (t_1 - t_2) = 275$

or $\quad t_1 - t_2 = \dfrac{275}{1.005} \; 273.60$

$\therefore \quad t_2 = 430 - 273.60$

$= 156.36^\circ$ C

Q5.7 **A reciprocating air compressor takes in 2 m³/min at 0.11 MPa, 20°C which it delivers at 1.5 MPa, 111°C to an aftercooler where the air is cooled at constant pressure to 25°C. The power absorbed by the compressor is 4.15 kW. Determine the heat transfer in**

(a) The compressor

(b) The cooler

State your assumptions.

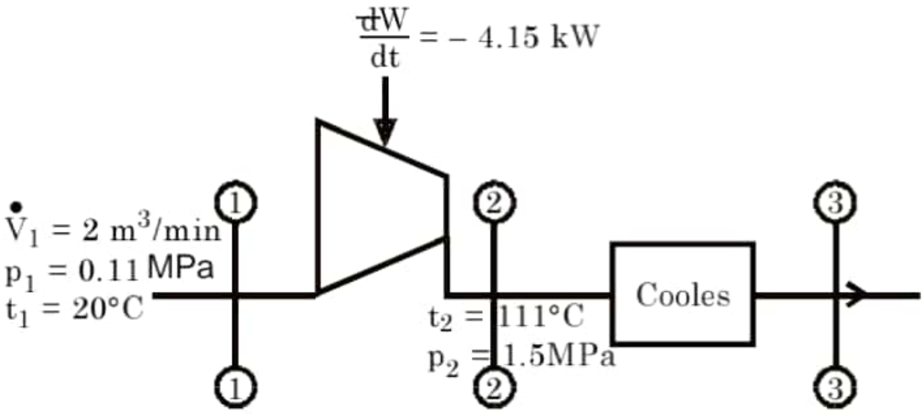
(Ans. – 0.17 kJ/s, – 3.76 kJ/s)

Solution:

(a) $\quad \therefore \quad w_1(h_1 + 0 + 0) + \dfrac{dQ}{dt} = w_1 h_2 + \dfrac{dW}{dt}$

$\therefore \quad 0.0436\,(111.555 - 20.1) - 4.15 = \left(\dfrac{dQ}{dt}\right)$

$\dfrac{dQ}{dt} = -0.1622$ kW $\quad$ i.e. 1622 kW loss by compressor



$$\text{Compressor work} = \frac{n}{n \cdot 1}(p_2 V_2 \;\cdot\; p_1 V_1) \;=\; \frac{n}{n-1}(mRT_2 - mRT_1)$$

$$= \frac{1.4}{0.4} \times 0.0436 \times 0.287(111 - 20)\,\text{kW}$$

$$= 3.9854 \text{ kW}$$

$$\therefore \frac{dQ}{dt} = 3.9854 - 4.15 = -0.165 \text{ kW}$$

(b) $\quad \dfrac{dQ}{dt}$ For cooler

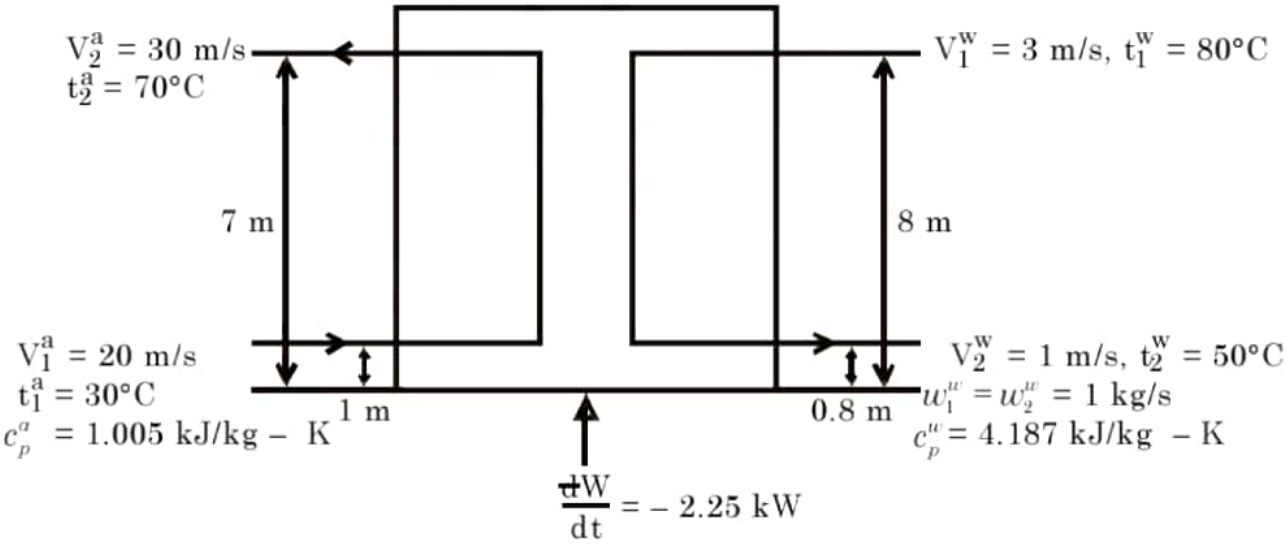
$$= \dot{m}\, c_p(t_2 - t_1)$$
$$= 0.0436 \times 1.005 \times (111 - 25) \text{ kJ/s}$$
$$= 3.768348 \text{ kW to surroundings}$$

Q5.8 **In water cooling tower air enters at a height of 1 m above the ground level and leaves at a height of 7 m. The inlet and outlet velocities are 20 m/s and 30 m/s respectively. Water enters at a height of 8 m and leaves at a height of 0.8 m. The velocity of water at entry and exit are 3 m/s and 1 m/s respectively. Water temperatures are 80°C and 50°C at the entry and exit respectively. Air temperatures are 30°C and 70°C at the entry and exit respectively. The cooling tower is well insulated and a fan of 2.25 kW drives the air through the cooler. Find the amount of air per second required for 1 kg/s of water flow. The values of c_p of air and water are 1.005 and 4.187 kJ/kg K respectively.**

(**Ans.** 3.16 kg/s)

Solution: Let air required is w_1^a kg/s

$$\therefore\ w_1^a\left(h_1^a + \frac{V_1^{a2}}{2000} + \frac{gZ_1^a}{1000}\right) + w_1^w\left(h_1^w + \frac{V_1^{w^2}}{2000} + \frac{gZ_1^w}{1000}\right) + \frac{đQ}{dt}$$
$$= w_2^a\left(h_2^a + \frac{V_2^{a2}}{2000} + \frac{gZ_2^a}{1000}\right) + w_2^w\left(h_2^w + \frac{V_2^{w^2}}{2000} + \frac{gZ_2^w}{1000}\right) + \frac{đW}{dt}$$
$$\therefore\quad w_1^a = w_2^a = \text{w (say) and } \frac{đQ}{dt} = 0\ \ w_1^w = w_2^w = 1 \text{ kg/s}$$



$$\therefore\quad \left\{(h_1^a - h_2^a) + \frac{V_1^{a^2} - V_2^{a^2}}{2000} + \frac{g}{1000}(Z_1^a - Z_2^a)\right\}$$
$$= \left\{(h_2^w - h_1^w) + \frac{V_2^{w^2} - V_1^{w^2}}{2000} + \frac{g}{1000}(Z_1^w - Z_2^w)\right\} + \frac{đW}{dt}$$
$$\text{Or}\quad w\left\{1.005 \times (30 - 70) + \frac{20^2 - 30^2}{2000} + \frac{9.81}{1000}(1 - 7)\right\}$$

$$= 4.187\,(50 - 80) + \frac{1^2 - 3^2}{2000} + \frac{9.81}{1000} \times (0.8 - 8) - 2.25$$

or $\quad - w \times 40.509 = -127.9346$

$\therefore \quad w = \dfrac{127.9346}{40.509} = 3.1582 \text{ kg/s} \approx 3.16 \text{ kg/s}$

Q5.9 **Air at 101.325 kPa, 20°C is taken into a gas turbine power plant at a velocity of 140 m/s through an opening of 0.15 m² cross-sectional area. The air is compressed heated, expanded through a turbine, and exhausted at 0.18 MPa, 150°C through an opening of 0.10 m² cross-sectional area. The power output is 375 kW. Calculate the net amount of heat added to the air in kJ/kg. Assume that air obeys the law**

$$\mathbf{pv = 0.287\,(t + 273)}$$

Where p is the pressure in kPa, v is the specific volume in m³/kg, and t is the temperature in °C. Take c_p = 1.005 kJ/kg K.

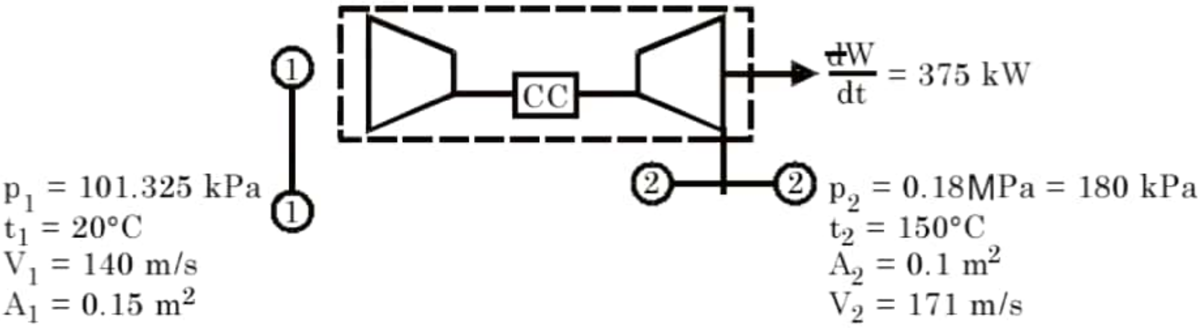
(**Ans.** 150.23 kJ/kg)

Solution: Volume flow rate at inlet $(\dot{V})_1 = V_1A_1 \text{ m}^3\text{/s} = 21 \text{ m}^3\text{/s}$

Inlet mass flow rate $(w_1) = \dfrac{p_1 \dot{V}_1}{R\,T_1} = \dfrac{101.325 \times 21}{0.287 \times 293} = 25.304 \text{ kg/s}$

Volume flow rate at outlet $= (\dot{V}_2) = \dfrac{w_2 RT_2}{p_2}$

$$= \frac{25.304 \times 0.287 \times 423}{180} = 17 \text{ m}^3\text{/s}$$



Velocity at outlet $= \dfrac{\dot{V}_2}{A_2} = \dfrac{17}{0.1} = 170.66 \text{ m/s}$

$\therefore$ Using S.F.E.E.

$$w_1\left(h_1 + \frac{V_1^2}{2000} + 0\right) + \frac{dQ}{dt} = w_2\left(h_2 + \frac{V_2^2}{2000} + 0\right) + \frac{dW}{dt}$$

$$w_1 = w_2 = w = 25.304 \text{ kg/s}$$

$$\therefore \quad \frac{dQ}{dt} = w\left\{(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2000}\right\} + \frac{dW}{dt}$$

$$= w\left\{C_p(t_2 - t_1) + \frac{V_2^2 - V_1^2}{2000}\right\} + \frac{dW}{dt}$$

$$= 25.304\left\{1.005\,(150-20)+\frac{171^2-140^2}{2000}\right\}+375 \text{ kW}$$

$$= 3802.76 \text{ kW}$$

$$\frac{đQ}{dm} = \frac{đQ}{dt}\Big/w = \frac{3802.76}{25.304} = 150.28 \text{ kJ/kg}$$

Q5.10 **A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 16°C, a pressure of 100 kPa, and an enthalpy of 391.2 kJ/kg. The gas leaves the compressor at a temperature of 245°C, a pressure of 0.6 MPa, and an enthalpy of 534.5 kJ/kg. There is no heat transfer to or from the gas as it flows through the compressor.**

(a) Evaluate the external work done per unit mass of gas assuming the gas velocities at entry and exit to be negligible.

(b) Evaluate the external work done per unit mass of gas when the gas velocity at entry is 80 m/s and that at exit is 160 m/s.

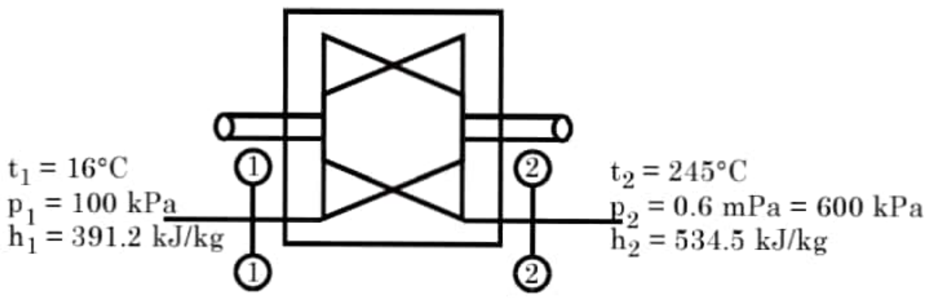
(Ans. 143.3 kJ/kg, 152.9 kJ/kg)

Solution: (a) $$h_1+\frac{V_1^2}{2000}+\frac{gZ_1}{1000}+\frac{đQ}{dm} = h_2+\frac{V_2^2}{2000}+\frac{gZ_2}{1000}+\frac{đW}{dm}$$

For V_1 and V_2 negligible and $Z_1 = Z_2$ so

$$\frac{đW}{dm} = h_1 - h_2 = (391.2 - 5345) \text{ kJ/kg}$$

$= -143.3$ kJ/kg i.e. work have to give



$$\therefore \frac{đQ}{dt} = 0$$

(b) V_1 = 80 m/s; V_2 = 160 m/s

So $$\frac{đW}{dm} = (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2000}$$

$$= -143.3 + \frac{80^2 - 160^2}{2000} \text{ kJ/kg} = (-143.3 - 9.6) \text{ kJ/kg}$$

$= -152.9$ kJ/kg i.e. work have to give

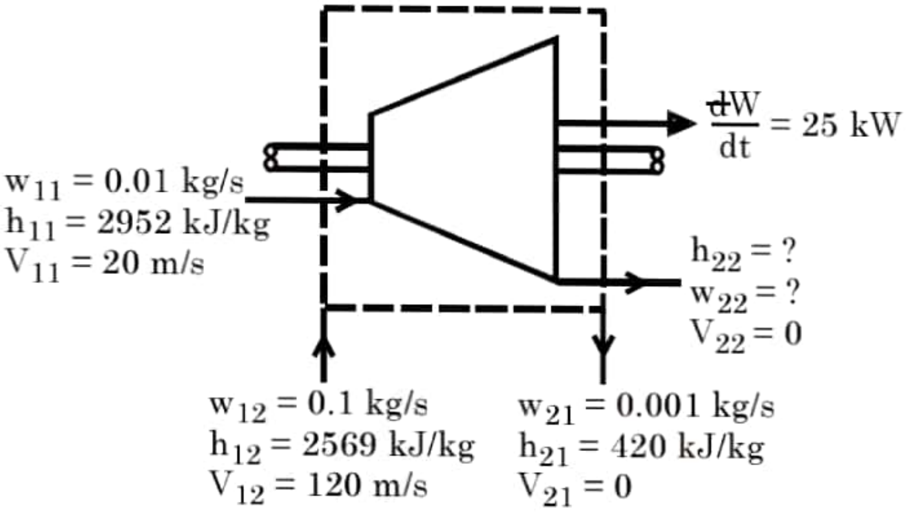
Q5.11 **The steam supply to an engine comprises two streams which mix before entering the engine. One stream is supplied at the rate of 0.01 kg/s with an enthalpy of 2952 kJ/kg and a velocity of 20 m/s. The other stream is supplied at the rate of 0.1 kg/s with an enthalpy of 2569 kJ/kg and a velocity of 120 m/s. At the exit from the engine the fluid leaves as two**

streams, one of water at the rate of 0.001 kg/s with an enthalpy of 420 kJ/kg and the other of steam; the fluid velocities at the exit are negligible. The engine develops a shaft power of 25 kW. The heat transfer is negligible. Evaluate the enthalpy of the second exit stream.

(**Ans.** 2402 kJ/kg)

Solution: $\therefore \frac{dQ}{dt} = 0$

By mass balance



$W_{11} + W_{12} = W_{21} + W_{22}$

$\therefore \quad W_{22} = 0.01 + 0.1 - 0.001 \text{ kg/s} = 0.109 \text{ kg/s}$

$$\therefore \quad W_{11}\left(h_{11} + \frac{V_{11}^2}{2000}\right) + W_{12}\left(h_{12} + \frac{V_{12}^2}{2000}\right) + \frac{dQ}{dt}$$

$$= W_{21}(h_{21}) + W_{22} \times h_{22} + \frac{dW}{dt}$$

$$\therefore \quad 0.01\left(2952 + \frac{20^2}{2000}\right) + 0.1\left(2569 + \frac{120^2}{2000}\right) + 0$$

$$= 0.001 \times 420 + 0.109 \times h_{22} + 25$$

or $\quad 29.522 + 257.62 = 0.42 + 0.109 \times h_{22} + 25$

or $\quad 286.722 = 0.109 \times h_{22} + 25$

or $\quad h_{22} = 2401.2$ kJ/kg

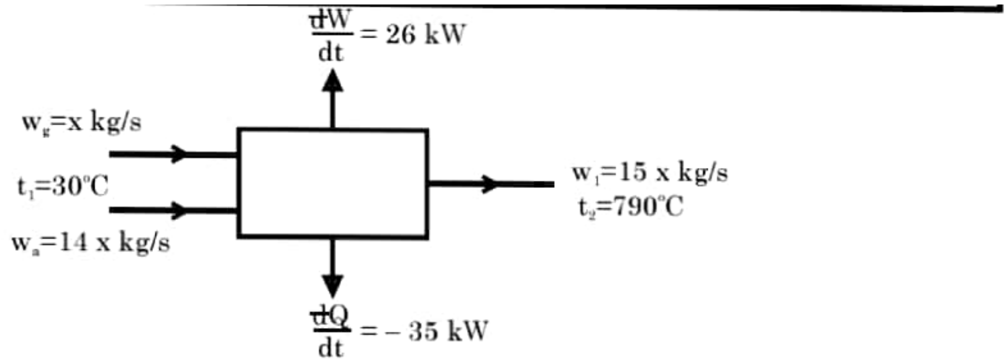
Q5.12 **The stream of air and gasoline vapour, in the ratio of 14: 1 by mass, enters a gasoline engine at a temperature of 30°C and leaves as combustion products at a temperature of 790°C. The engine has a specific fuel consumption of 0.3 kg/kWh. The net heat transfer rate from the fuel-air stream to the jacket cooling water and to the surroundings is 35 kW. The shaft power delivered by the engine is 26 kW. Compute the increase in the specific enthalpy of the fuel air stream, assuming the changes in kinetic energy and in elevation to be negligible.**

(**Ans.** – 1877 kJ/kg mixture)

Solution: In 1 hr. this m/c will produce 26 kWh for that we need fuel

$= 0.3 \times 26 = 7.8$ kg fuel/hr.

$\therefore$ Mass flow rate of fuel vapor and air mixture



$$w_1 = \frac{15 \times 7.8}{3600} \text{ kg/s} = 0.0325 \text{ kg/s}$$

Applying S.F.E.E.

$$w_1 h_1 + \frac{đQ}{dt} = w_1 h_2 + \frac{đW}{dt}$$

or $$w_1 (h_2 - h_1) = \frac{đQ}{dt} - \frac{đW}{dt}$$

$\therefore$ $$h_2 - h_1 = \frac{\frac{đQ}{dt} - \frac{đW}{dt}}{w_1}$$

$$= \frac{-35 - 26}{0.0325} = -1877 \text{ kJ/kg of mixture.}$$

Q5.13 **An air turbine forms part of an aircraft refrigerating plant. Air at a pressure of 295 kPa and a temperature of 58°C flows steadily into the turbine with a velocity of 45 m/s. The air leaves the turbine at a pressure of 115 kPa, a temperature of 2°C, and a velocity of 150 m/s. The shaft work delivered by the turbine is 54 kJ/kg of air. Neglecting changes in elevation, determine the magnitude and sign of the heat transfer per unit mass of air flowing. For air, take c_p = 1.005 kJ/kg K and the enthalpy h = c_p t.**

(**Ans.** + 7.96 kJ/kg)

Solution:

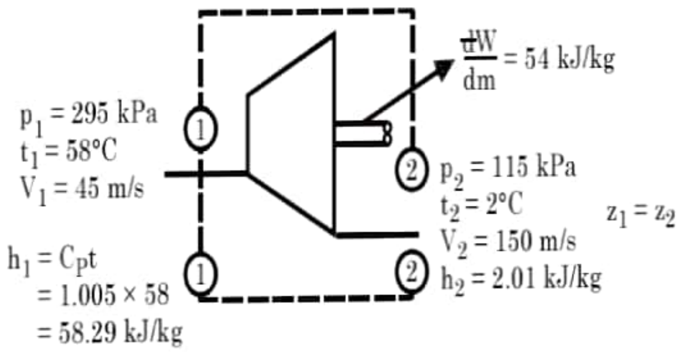
$$h_1 + \frac{V_1^2}{2000} + \frac{đQ}{dm} = h_2 + \frac{V_2^2}{2000} + \frac{đW}{dm}$$

or $$\frac{đQ}{dm} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2000} + \frac{đW}{dm}$$

$$= (2.01 - 58.29) + \frac{150^2 - 45^2}{2000} + 54 \text{ kJ/kg}$$

$$= -56.28 + 10.2375 + 54 \text{ kJ/kg}$$

$= 7.9575$ kJ/kg (have to give to the system)



6. Second Law of Thermodynamics

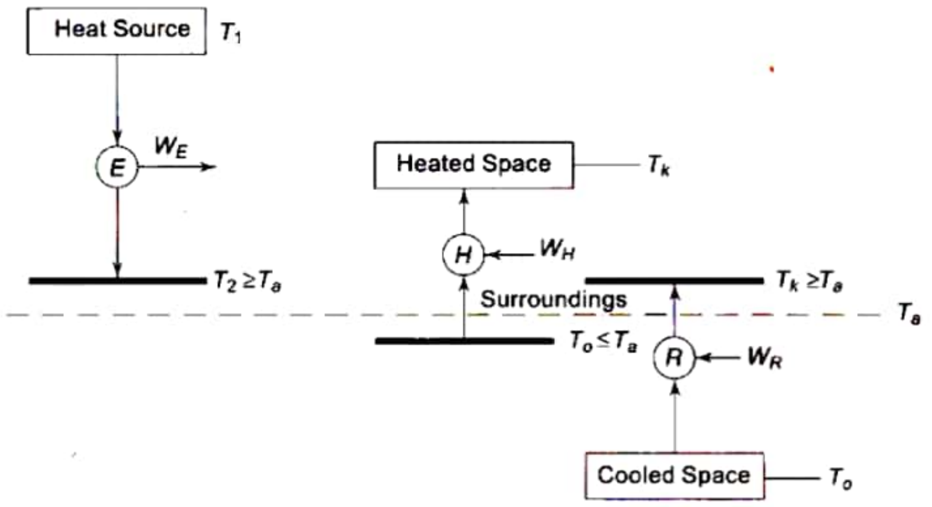
Some Important Notes

Regarding Heat Transfer and Work Transfer

- Heat transfer and work transfer are the energy interactions.
- Both heat transfer and work transfer are boundary phenomena.
- It is wrong to say 'total heat' or 'heat content' of a closed system, because heat or work is not a property of the system.
- Both heat and work are path functions and inexact differentials.
- Work is said to be a high grade energy and heat is low grade energy.
- HEAT and WORK are NOT properties because they depend on the path and end states.
- HEAT and WORK are not properties because their net change in a cycle is not zero.
- **Clausius' Theorem:** The cyclic integral of $đ$ Q/T for a reversible cycle is equal to zero.

  or $$\oint_R \frac{đQ}{T} = 0$$

- The more effective way to increase the cycle efficiency is **to decrease T_2.**
- **Comparison of heat engine, heat pump and refrigerating machine**



$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$

$$hence,\ \eta_{Carnot,HE} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$$

$$COP_{Carnot,HP} = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

$$COP_{Carnot,R} = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

Questions with Solution P. K. Nag

Q6.1 **An inventor claims to have developed an engine that takes in 105 MJ at a temperature of 400 K, rejects 42 MJ at a temperature of 200 K, and delivers 15 kWh of mechanical work. Would you advise investing money to put this engine in the market?**

(**Ans.** No)

Solution: Maximum thermal efficiency of his engine possible

$$\eta_{max} = 1 - \frac{200}{400} = 50\%$$

∴ That engine and deliver output = η × input

$$= 0.5 \times 105 \text{ MJ}$$
$$= 52.5 \text{ MJ} = 14.58 \text{ kWh}$$

As he claims that his engine can deliver more work than ideally possible so I would not advise to investing money.

Q6.2 **If a refrigerator is used for heating purposes in winter so that the atmosphere becomes the cold body and the room to be heated becomes the hot body, how much heat would be available for heating for each kW input to the driving motor? The COP of the refrigerator is 5, and the electromechanical efficiency of the motor is 90%. How does this compare with resistance heating?**

(**Ans.** 5.4 kW)

Solution:
$$\text{COP} = \frac{\text{desired effect}}{\text{input}}$$

$$(\text{COP})_{\text{ref.}} = (\text{COP})_{\text{H.P}} - 1 \qquad \therefore (\text{COP})_{\text{H.P.}} = 6$$

or $$6 = \frac{\text{H}}{\text{W}}$$

So input $$(\text{W}) = \frac{\text{H}}{6}$$

But motor efficiency 90% so

Electrical energy require $$(\text{E}) = \frac{\text{W}}{0.9} = \frac{\text{H}}{0.9 \times 6}$$

$$= 0.1852 \text{ H}$$
$$= 18.52\% \text{ of Heat (direct heating)}$$

$$\text{H} = \frac{100}{18.52} \frac{\text{kW}}{\text{kW of work}} = 5.3995 \text{ kW}$$

Q6.3 **Using an engine of 30% thermal efficiency to drive a refrigerator having a COP of 5, what is the heat input into the engine for each MJ removed from the cold body by the refrigerator?**

(**Ans.** 666.67 kJ)

If this system is used as a heat pump, how many MJ of heat would be available for heating for each MJ of heat input to the engine?

(**Ans.** 1.8 MJ)

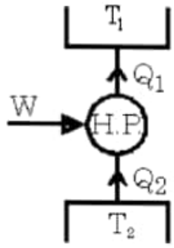
Solution: COP of the Ref. is 5

So for each MJ removed from the cold body we need work

$$= \frac{1\,MJ}{5} = 200 \text{ kJ}$$

For 200 kJ work output of heat engine hair $\eta = 30\%$

We have to supply heat $= \dfrac{200\,\text{kJ}}{0.3} = 666.67$ kJ



Now

COP of H.P. = COP of Ref. + 1

= 5 + 1 = 6

Heat input to the H.E. = 1 MJ

$\therefore$ Work output (W) = 1 × 0.3 MJ = 300 kJ

That will be the input to H.P.

$$\therefore (COP)_{H.P} = \frac{Q_1}{W}$$

$$\therefore \quad Q_1 = (COP)_{H.P.} \times W = 6 \times 300 \text{ kJ} = 1.8 \text{ MJ}$$

Q6.4 **An electric storage battery which can exchange heat only with a constant temperature atmosphere goes through a complete cycle of two processes. In process 1–2, 2.8 kWh of electrical work flow into the battery while 732 kJ of heat flow out to the atmosphere. During process 2–1, 2.4 kWh of work flow out of the battery.**

(a) Find the heat transfer in process 2–1.

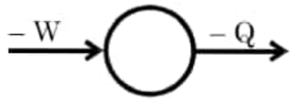
(b) If the process 1–2 has occurred as above, does the first law or the second law limit the maximum possible work of process 2–1? What is the maximum possible work?

(c) If the maximum possible work were obtained in process 2–1, what will be the heat transfer in the process?

(**Ans.** (a) – 708 kJ (b) Second law, W_{2-1} = 9348 kJ (c) Q_{2-1} = 0)

Solution: **From the first Law of thermodynamics**

(a) For process 1–2



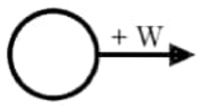
$$Q_{1-2} = E_2 - E_1 + W_{1-2}$$

$$-732 = (E_2 - E_1) - 10080$$

$$[2.8 \text{ kWh} = 2.8 \times 3600 \text{ kJ}]$$

$$\therefore \quad E_2 - E_1 = 9348 \text{ kJ}$$

For process 2–1



$$Q_{21} = E_1 - E_2 + W_{21}$$

$$= -9348 + 8640$$

= –708 kJ i.e. Heat flow out to the atmosphere.

(b) Yes Second Law limits the maximum possible work. As Electric energy stored in a battery is High grade energy so it can be completely converted to the work. Then,

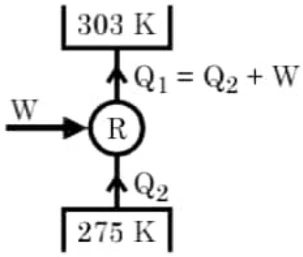
$$W = 9348 \text{ kJ}$$

(c) $Q_{21} = -9348 + 9348 = 0$ kJ

Q6.5 **A household refrigerator is maintained at a temperature of 2°C. Every time the door is opened, warm material is placed inside, introducing an average of 420 kJ, but making only a small change in the temperature of the refrigerator. The door is opened 20 times a day, and the refrigerator operates at 15% of the ideal COP. The cost of work is Rs. 2.50 per kWh. What is the monthly bill for this refrigerator? The atmosphere is at 30°C.**

(**Ans.** Rs. 118.80)

Solution: Ideal COP of Ref. $= \dfrac{275}{30-2} = \dfrac{275}{28} = 9.82143$



Actual COP $= 0.15 \times \text{COP}_{\text{ideal}} = 1.4732$

Heat to be removed in a day

$(Q_2) = 420 \times 20$ kJ
$= 8400$ kJ

$\therefore$ Work required $= 5701.873$ kJ/day
$= 1.58385$ kWh/day

Electric bill per month $= 1.58385 \times 0.32 \times 30$ Rupees
$=$ Rs. 15.20

Q6.6 **A heat pump working on the Carnot cycle takes in heat from a reservoir at 5°C and delivers heat to a reservoir at 60°C. The heat pump is driven by a reversible heat engine which takes in heat from a reservoir at 840°C and rejects heat to a reservoir at 60°C. The reversible heat engine also drives a machine that absorbs 30 kW. If the heat pump extracts 17 kJ/s from the 5°C reservoir, determine**

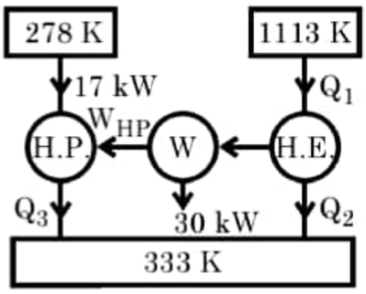
(a) The rate of heat supply from the 840°C source

(b) The rate of heat rejection to the 60°C sink.

(**Ans.** (a) 47.61 kW; (b) 34.61 kW)

Solution:

COP of H.P.



$$= \frac{333}{333-278} = 6.05454$$

$$Q_3 = W_{H.P.} + 17$$

$$\therefore \quad \frac{W_{H.P.} + 17}{W_{H.P.}} = 6.05454$$

$$\therefore \quad \frac{17}{W_{H.P.}} = 5.05454$$

$$\therefore \quad W_{H.P.} = \frac{17}{5.05454} = 3.36 \text{ kW}$$

$\therefore$ Work output of the Heat engine

$W_{H.E.} = 30 + 3.36 = 33.36$ kW

$$\eta \text{ of the H.E.} = 1 - \frac{333}{1113} = 0.7$$

(a) $\therefore \quad \dfrac{W}{Q_1} = 0.7$

$\therefore \quad Q_1 = \dfrac{W}{0.7} = 47.61$ kW

(b) Rate of heat rejection to the 333 K

(i) From H.E. = $Q_1 - W = 47.61 - 33.36 = 14.25$ kW

(ii) For H.P. = $17 + 3.36 = 20.36$ kW

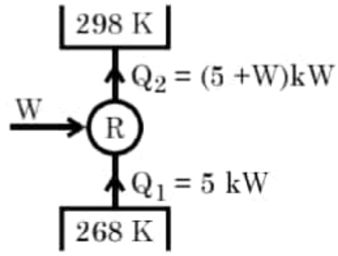
$\therefore$ Total = 34.61 kW

Q6.7 **A refrigeration plant for a food store operates as a reversed Carnot heat engine cycle. The store is to be maintained at a temperature of – 5°C and the heat transfer from the store to the cycle is at the rate of 5 kW. If heat is transferred from the cycle to the atmosphere at a temperature of 25°C, calculate the power required to drive the plant.**

(**Ans.** 0.56 kW)

Solution: $(\text{COP})_R = \dfrac{268}{298 - 268} = 8.933$

$= \dfrac{5 \text{ kW}}{W}$



$\therefore \quad W = \dfrac{5}{8.933}$ kW $= 0.56$ kW

Q6.8 **A heat engine is used to drive a heat pump. The heat transfers from the heat engine and from the heat pump are used to heat the water circulating through the radiators of a building. The efficiency of the heat engine is 27% and the COP of the heat pump is 4. Evaluate the ratio of the heat transfer to the circulating water to the heat transfer to the heat engine.**

(**Ans.** 1.81)

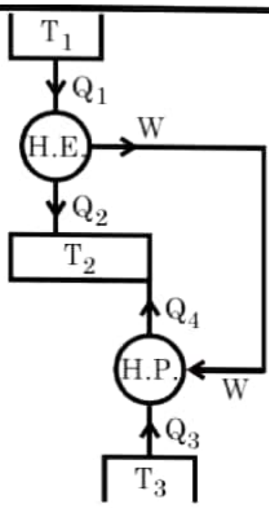
Solution: For H.E.

$1 - \dfrac{Q_2}{Q_1} = 0.27$

$$\frac{Q_2}{Q_1} = 0.73$$

$$Q_2 = 0.73\ Q_1$$

$$W = Q_1 - Q_2 = 0.27\ Q_1$$



For H.P.

$$\frac{Q_4}{W} = 4$$

$\therefore$ $Q_4 = 4W = 1.08\ Q_1$

$\therefore$ $Q_2 + Q_4 = (0.73 + 1.08)\ Q_1 = 1.81\ Q_1$

$\therefore$ $\dfrac{\text{Heat transfer to the circulating water}}{\text{Heat for to the Heat Engine}}$

$$= \frac{1.81\ Q_1}{Q_1} = 1.81$$

Q6.9 **If 20 kJ are added to a Carnot cycle at a temperature of 100°C and 14.6 kJ are rejected at 0°C, determine the location of absolute zero on the Celsius scale.**

(**Ans.** – 270.37°C)

Solution:

$$\frac{Q_1}{Q_2} = \frac{\phi(t_1)}{\phi(t_2)}$$

Let ϕ (t) = at + b

$\therefore$ $\dfrac{Q_1}{Q_2} = \dfrac{at_1 + b}{at_2 + b}$

or $\dfrac{20}{14.6} = \dfrac{a \times 100 + b}{a \times 0 + b} = \dfrac{a}{b} \times 100 + 1$

$\therefore$ $\dfrac{a}{b} = 3.6986 \times 10^{-3}$

For absolute zero, $Q_2 = 0$

$\therefore$ $\dfrac{Q_1}{0} = \dfrac{a \times 100 + b}{a \times t + b}$

or $a \times t + b = 0$

or $t = \dfrac{-b}{a} = -\dfrac{1}{3.6986 \times 10^{-3}} = -270.37^{\circ}\ C$

Q6.10 **Two reversible heat engines *A* and *B* are arranged in series, A rejecting heat directly to *B*. Engine *A* receives 200 kJ at a temperature of 421°C from a hot source, while engine *B* is in communication with a cold sink at a temperature of 4.4°C. If the work output of *A* is twice that of *B*, find**

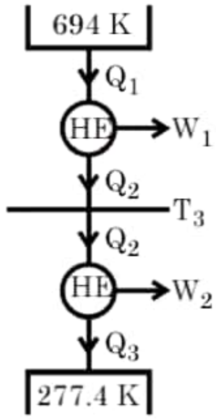
(a) The intermediate temperature between *A* and *B*

(b) The efficiency of each engine

(c) The heat rejected to the cold sink

(**Ans.** 143.4°C, 40% and 33.5%, 80 kJ)

Solution: $\dfrac{Q_1}{694} = \dfrac{Q_2}{T} = \dfrac{Q_1 - Q_2}{694 - T} = \dfrac{Q_3}{277.4} = \dfrac{Q_2 - Q_3}{T - 277.4}$



Hence $Q_1 - Q_2 = 2\,W_2$

$Q_2 - Q_3 = W_2$

$\therefore \quad \dfrac{2}{694 - T} = \dfrac{1}{T - 277.4}$

or $\quad 2T - 277.4 \times 2 = 694 - T$

or $\quad T = 416.27\text{ K} = 143.27^\circ\text{ C}$

(b) $\quad \eta_1 = 40\%$

$\eta_2 = 1 - \dfrac{277.4}{416.27} = 33.36\%$

(c) $\quad Q_2 = \dfrac{416.27}{694} \times 200\text{ kJ} = 119.96\text{ kJ}$;

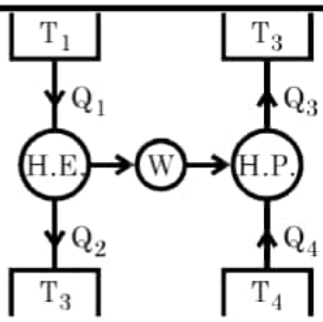
$Q_1 = \dfrac{277.4}{416.27} \times 119.96 = 79.94\text{ kJ}$

Q6.11 **A heat engine operates between the maximum and minimum temperatures of 671°C and 60°C respectively, with an efficiency of 50% of the appropriate Carnot efficiency. It drives a heat pump which uses river water at 4.4°C to heat a block of flats in which the temperature is to be maintained at 21.1°C. Assuming that a temperature difference of 11.1°C exists between the working fluid and the river water, on the one hand, and the required room temperature on the other, and assuming the heat pump to operate on the reversed Carnot cycle, but with a COP of 50% of the ideal COP, find the heat input to the engine per unit heat output from the heat pump. Why is direct heating thermodynamically more wasteful?**

(**Ans.** 0.79 kJ/kJ heat input)

Solution: Carnot efficiency (η) = $1 - \dfrac{273 + 60}{273 + 671} = 1 - \dfrac{333}{944} = 0.64725$

Actual (η) = $0.323623 = 1 - \dfrac{Q_1'}{Q_1}$



Work output (W) = $\frac{Q_1}{T_1}(T_1 - T_2)$

For H.P.

Work input (W) = $\frac{Q_4}{T_4}(T_3 - T_4)$

$$\therefore \quad \frac{Q_1}{T_1}(T_1 - T_2) = \frac{Q_4}{T_4}(T_3 - T_4)$$

or $$\boxed{\frac{Q_4}{Q_1} = \frac{T_4}{T_1}\left\{\frac{T_1 - T_2}{T_3 - T_4}\right\}}$$

This is the expression.

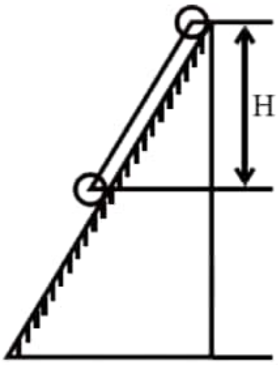
Q6.24 **Prove that the following propositions are logically equivalent:**
(a) A PMM2 is Impossible
(b) A weight sliding at constant velocity down a frictional inclined plane executes an irreversible process.

Solution: Applying First Law of Thermodynamics

$Q_{12} = E_2 - E_1 + W_{1\text{-}2}$

or $0 = E_2 - E_1 - mgh$

or $E_1 - E_2 = mgh$



7. Entropy

Some Important Notes

1. Clausius theorem: $\oint \left(\frac{đQ}{T}\right)_{rev.} = 0$

2. $S_f - S_i = \int_i^f \frac{đ\, Q_{rev.}}{T} = (\Delta S)_{\text{irrev. Path}}$

   Integration can be performed only on a reversible path.

3. **Clausius Inequality:** $\oint \frac{đQ}{T} \leq 0$

4. At the equilibrium state, the system is at the peak of the entropy hill. (Isolated)

5. $TdS = dU + pdV$

6. $TdS = dH - Vdp$

7. Famous relation $S = K \ln W$

   Where K = Boltzmann constant
   W = thermodynamic probability.

8. General case of change of entropy of a Gas

   $$S_2 - S_1 = m\left\{c_v \ln \frac{p_2}{p_1} + c_p \ln \frac{V_2}{V_1}\right\}$$

   Initial condition of gas p_1, V_1, T_1, S_1 and
   Final condition of gas p_2, V_2, T_2, S_2

Questions with Solution P. K. Nag

Q7.1. **On the basis of the first law fill in the blank spaces in the following table of imaginary heat engine cycles. On the basis of the second law classify each cycle as reversible, irreversible, or impossible.**

Cycle	*Temperature*		*Rate of Heat Flow*		*Rate of work*	*Efficiency*
	Source	*Sink*	*Supply*	*Rejection*	*Output*	
(a)	**327°C**	**27°C**	**420 kJ/s**	**230 kJ/s**	**...kW**	
(b)	**1000°C**	**100°C**	**...kJ/min**	**4.2 MJ/min**	**... kW**	**65%**
(c)	**750 K**	**300 K**	**...kJ/s**	**...kJ/s**	**26 kW**	**65%**
(d)	**700 K**	**300 K**	**2500 kcal/h**	**...kcal/h**	**1 kW**	**—**

(**Ans.** (a) Irreversible, (b) Irreversible, (c) Reversible, (d) Impossible)

Solution:

Cycle	**Temperature**		**Rate of Heat Flow**		**Rate of work**	**Effici-ency**	**Remark**
	Source	**Sink**	**Supply**	**Rejection**			
(a)	327°C	27°C	420 kJ/s	230 kJ/s	190kW	0.4523	η_{max} = 50%, irrev.possible
(b)	1000°C	100°C	12000 kJ/km	4.2 kJ/m	7800 kW	65%	η_{max}=70.7% irrev.possible
(c)	750 K	300 K	43.33 kJ/s	17.33 kJ/s	26 kW	60%	η_{max}= 60% rev. possible
(d)	700 K	300 K	2500 kcal/h	1640 kcal/h	1 kW	4.4%	η_{max}=57% irrev.possible

Q7.2 **The latent heat of fusion of water at 0°C is 335 kJ/kg. How much does the entropy of 1 kg of ice change as it melts into water in each of the following ways:**

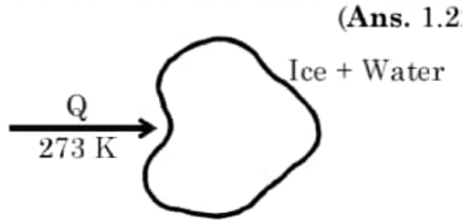
(a) Heat is supplied reversibly to a mixture of ice and water at 0°C.

(b) A mixture of ice and water at 0°C is stirred by a paddle wheel.

(**Ans.** 1.2271 kJ/K)

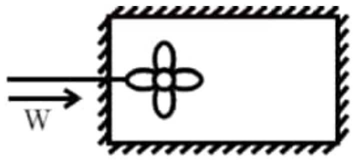
Solution : (a) $(\Delta S)_{\text{system}} = +\dfrac{1 \times 335}{273}$ kJ/ K

$= 1.227$ kJ/K



(b) $(\Delta S)_{\text{system}}$

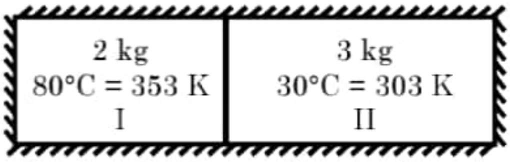
$$= \int_{273}^{273} m\, c_P \frac{dT}{T} = 0$$



Q7.3 **Two kg of water at 80°C are mixed adiabatically with 3 kg of water at 30°C in a constant pressure process of 1 atmosphere. Find the increase in the entropy of the total mass of water due to the mixing process (c_p of water = 4.187 kJ/kg K).**

(**Ans.** 0.0576 kJ/K)

Solution: If final temperature of mixing is T_f then



$$2 \times c_P \ (353 - T_f)$$
$$= 3 \times c_P \ (T_f - 303)$$

or $T_f = 323$ K

$$(\Delta S)_{system} = (\Delta S)_I + (\Delta S)_{II}$$

$$= \int_{353}^{323} m_1 c_P \frac{dT}{T} + \int_{303}^{323} m_1 c_P \frac{dT}{T}$$

$$= 2 \times 4.187 \ln\left(\frac{323}{353}\right) + 3 \times 4.187 \times \ln\frac{323}{303}$$

$$= 0.05915 \text{ kJ/K}$$

Q7.4 **In a Carnot cycle, heat is supplied at 350°C and rejected at 27°C. The working fluid is water which, while receiving heat, evaporates from liquid at 350°C to steam at 350°C. The associated entropy change is 1.44 kJ/kg K.**

(a) If the cycle operates on a stationary mass of 1 kg of water, how much is the work done per cycle, and how much is the heat supplied?

(b) If the cycle operates in steady flow with a power output of 20 kW, what is the steam flow rate?

(**Ans.** (a) 465.12, 897.12 kJ/kg, (b) 0.043 kg/s)

Solution: If heat required for evaporation is Q kJ/kg then

(a) $$\frac{Q}{(350 + 273)} = 1.44$$

or Q = 897.12 kJ/kg

It is a Carnot cycle so $\eta = 1 - \dfrac{(273 + 27)}{(350 + 273)}$

$\therefore$ $W = \eta.Q = 465.12$ kJ

(b) $$P = \dot{m}W \text{ or } \dot{m} = \frac{P}{W} = \frac{20}{465.12} \text{ kg/s} = 0.043 \text{ kg/s}$$

Q7.5 **A heat engine receives reversibly 420 kJ/cycle of heat from a source at 327°C, and rejects heat reversibly to a sink at 27°C. There are no other heat transfers. For each of the three hypothetical amounts of heat rejected, in (a), (b), and (c) below, compute the cyclic integral of dQ/T.**

from these results show which case is irreversible, which reversible, and which impossible:
(a) 210 kJ/cycle rejected
(b) 105 kJ/cycle rejected
(c) 315 kJ/cycle rejected

(**Ans.** (a) Reversible, (b) Impossible, (c) Irreversible)

Solution: (a) $$\oint \frac{dQ}{T} = \frac{+420}{(327+273)} - \frac{210}{(27+273)} = 0$$

$\therefore$ Cycle is Reversible, Possible

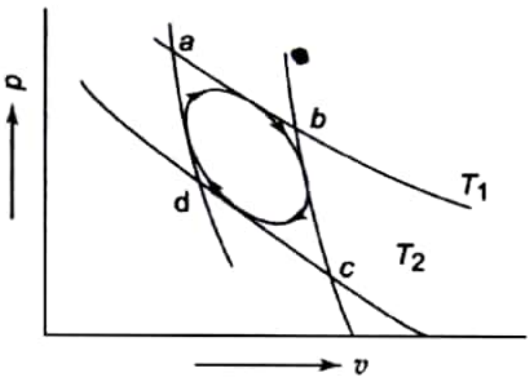
(b) $$\oint \frac{dQ}{T} = +\frac{420}{600} - \frac{105}{300} = 0.35$$

$\therefore$ Cycle is Impossible

(c) $$\oint \frac{dQ}{T} = +\frac{420}{600} - \frac{315}{300} = -0.35$$

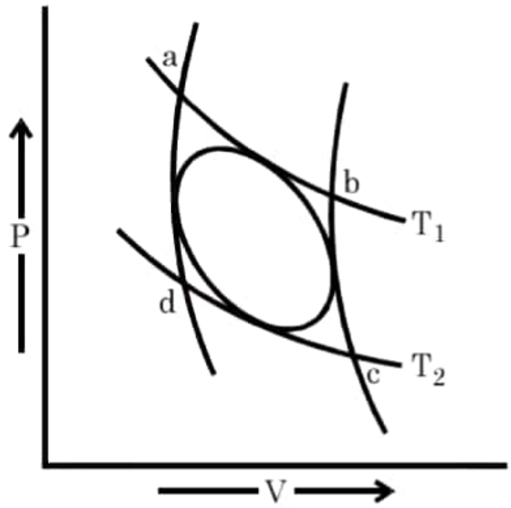
$\therefore$ Cycle is irreversible but possible.

Q7.6 **In Figure, *abed* represents a Carnot cycle bounded by two reversible adiabatic and two reversible isotherms at temperatures T_1 and T_2 ($T_1 > T_2$).**



The oval figure is a reversible cycle, where heat is absorbed at temperature less than, or equal to, T_1, and rejected at temperatures greater than, or equal to, T_2. Prove that the efficiency of the oval cycle is less than that of the Carnot cycle.

Solution:



Q7.7 **Water is heated at a constant pressure of 0.7 MPa. The boiling point is 164.97°C. The initial temperature of water is 0°C. The latent heat of evaporation is 2066.3 kJ/kg. Find the increase of entropy of water, if the final state is steam**

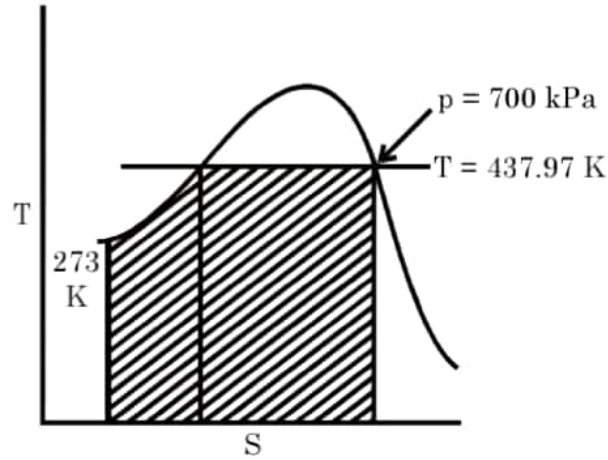
(**Ans.** 6.6967 kJ/kg K)

Solution: $(\Delta S)_{\text{Water}}$

$$= \int_{273}^{437.97} 1 \times 4187 \times \frac{dT}{T}$$

$$= 4.187 \ln\left(\frac{437.97}{273}\right) \text{kJ}/K$$

$= 1.979$ kJ/K



$(\Delta S)_{\text{Eva pour}}$

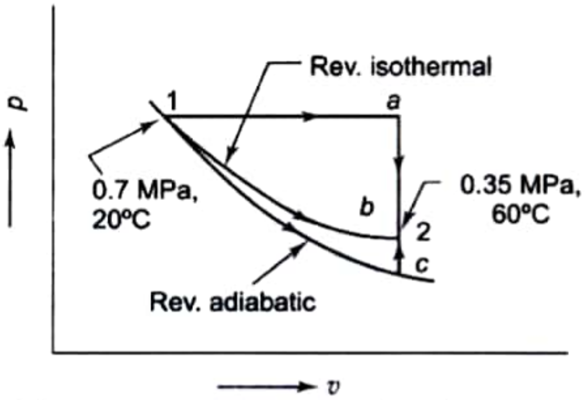
$$= \frac{1 \times 2066.3}{437.97} \text{kJ}/K$$

$= 4.7179$ kJ/K

$(\Delta s)_{\text{system}}$

$= 6.697$ kJ/kg – K

Q7.8 **One kg of air initially at 0.7 MPa, 20°C changes to 0.35 MPa, 60°C by the three reversible non-flow processes, as shown in Figure. Process 1: *a*-2 consists of a constant pressure expansion followed by a constant volume cooling, process 1: *b*-2 an isothermal expansion followed by a constant pressure expansion, and process 1: *c*-2 an adiabatic**



Expansion followed by a constant volume heating. Determine the change of internal energy, enthalpy, and entropy for each process, and find the work transfer and heat transfer for each process. Take c_p = 1.005 and c_v = 0.718 kJ/kg K and assume the specific heats to be constant. Also assume for air pv = 0.287 T, where p is the pressure in kPa, v the specific volume in m³/kg, and T the temperature in K.

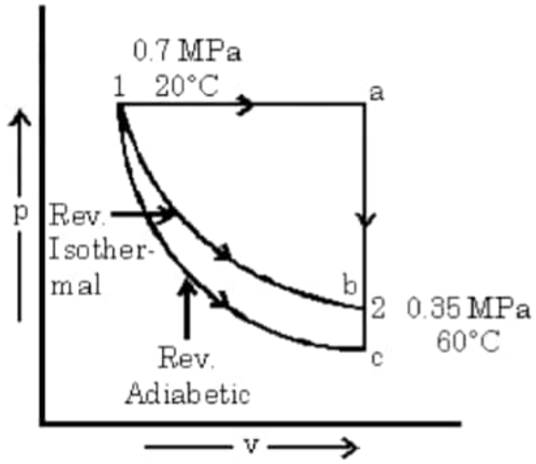
Solution: $p_1 = 0.7$ MPa = 700 kPa $\quad T_1 = 293$ K

$\therefore$ $v_1 = 0.12013$ m³/kg $\quad p_a = 700$ kPa

$\therefore$ $T_a = 666$ K $\quad v_a = 0.27306$ m³/kg

$p_2 = 350$ kPa $\quad T_2 = 333$ K

$\therefore$ $v_2 = 0.27306$ m³/kg



For process 1–a–2

$$Q_{1-a} = u_a \cdot u_1 + \int_{v_1}^{v_a} p\,dV$$

$$= u_a - u_1 + 700(0.27306 - 0.12013)$$

$$= u_a - u_1 + 107$$

$$Q_{a-2} = u_2 - u_a + 0$$

$\therefore$ $u_a - u_1 = 267.86$ kJ/kg

$$u_2 - u_a = -239 \text{ kJ/kg}$$

$$Q_{1-a} = \int_{T_1}^{T_a} c_p \, dT$$

$$= 1.005 \times (666 - 293)$$
$$= 374.865 \text{ kJ/kg}$$

$$Q_{a-2} = \int_{T_a}^{T_2} c_v \, dT$$

$$= 0.718 \, (333 - 666)$$
$$= -239 \text{ kJ/kg}$$

(i) $\Delta u = u_2 - u_1 = 28.766$ kJ/kg

(ii) $\Delta h = h_2 - h_1 = u_2 - u_1 + p_2 v_2 - p_1 v_1$

$$= 28.766 + 350 \times 0.27306 - 700 \times 0.12013 = 40.246 \text{ kJ/kg}$$

(iii) $Q = Q_2 + Q_1 = 135.865$ kJ/kg

(iv) $W = W_1 + W_2 = 107$ kJ/kg

(v) $\Delta s = s_2 - s_1 = (s_2 - s_a) + (s_a - s_1)$

$$= C_v \ln\left(\frac{T_2}{T_a}\right) + C_p \ln\left(\frac{T_a}{T_1}\right)$$

$$= 0.3275 \text{ kJ/kg} - \text{K}$$

Q7.9 **Ten grammes of water at 20°C is converted into ice at –10°C at constant atmospheric pressure. Assuming the specific heat of liquid water to remain constant at 4.2 J/gK and that of ice to be half of this value, and taking the latent heat of fusion of ice at 0°C to be 335 J/g, calculate the total entropy change of the system.**

(**Ans.** 16.02 J/K)

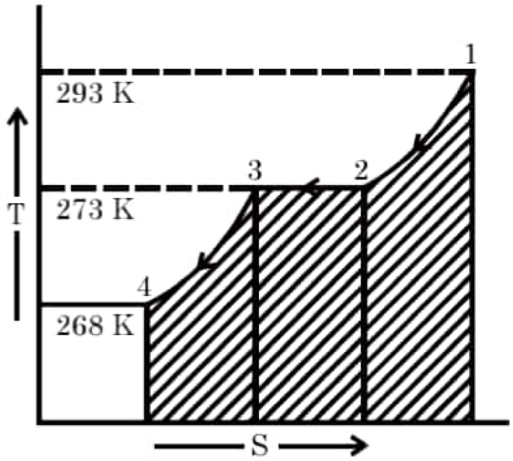
Solution:

$$S_2 - S_1 = \int_{293}^{273} \frac{m c_p \, dT}{T}$$

$$= 0.01 \times 4.2 \times \ln \frac{273}{293} \text{ kJ/} K$$
$$= -0.00297 \text{ kJ/K}$$
$$= -2.9694 \text{ J/K}$$

$$S_3 - S_2 = \frac{-mL}{T}$$

$$= \frac{-0.01 \times 335 \times 1000}{273}$$
$$= -12.271 \text{ J/K}$$



$$S_4 - S_3 = \int_{273}^{268} \frac{m\,c_p\,dT}{T} = 0.01 \times \left(\frac{4.2}{2}\right) \times \ln\frac{268}{273}\ kJ/K$$

$= -0.3882$ J/K

$\therefore$ $S_4 - S_1 = -15.63$ J/K

$\therefore$ Net Entropy change = 15.63 J/K

Q7.10 **Calculate the entropy change of the universe as a result of the following processes:**

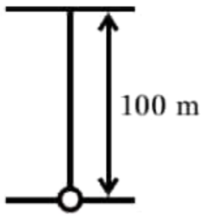
(a) A copper block of 600 g mass and with C_p of 150 J/K at 100°C is placed in a lake at 8°C.

(b) The same block, at 8°C, is dropped from a height of 100 m into the lake.

(c) Two such blocks, at 100 and 0°C, are joined together.

(**Ans.** (a) 6.69 J/K, (b) 2.095 J/K, (c) 3.64 J/K)

Solution:



(a) $$(\Delta S)_{copper} = \int_{373}^{281} m\,c_p \frac{dT}{T}$$

$$= 150 \ln\frac{281}{373}\ J/K$$

$= -42.48$ J/K

As unit of C_P is J/K there for

$\therefore$ It is heat capacity

i.e. $C_p = m\,c_p$

$$(\Delta S)_{lake} = \frac{C_p(100-8)}{281}\ J/K$$

$$= \frac{150(100-8)}{281}\ J/K = 49.11\ J/K$$

$(\Delta S)_{univ} = (\Delta S)_{COP} + (\Delta S)_{lake} = 6.63$ J/K

(b) Work when it touch water = 0.600 × 9.81 × 100 J = 588.6 J

As work dissipated from the copper

$(\Delta S)_{copper} = 0$

As the work is converted to heat and absorbed by water then

$$(\Delta S)_{lake} = \frac{W = Q}{281} = \frac{588.6}{281}\ J/K = 2.09466\ J/K$$

$\therefore$ $(\Delta S)_{univ} = 0 + 2.09466$ J/k $= 2.09466$ J/K

(c) Final temperature $(T_f) = \dfrac{100+0}{2} = 50^\circ C = 323$ K

$$(\Delta S)_I = C_p \int_{T_1}^{T_f} \frac{dT}{T} \quad ; \quad (\Delta S)_{II} = C_p \int_{T_2}^{T_f} \frac{dT}{T}$$

$$\therefore \quad (\Delta S)_{system} = 150 \ln\left(\frac{T_f}{T_1}\right) + 150 \ln\left(\frac{T_f}{T_2}\right)$$

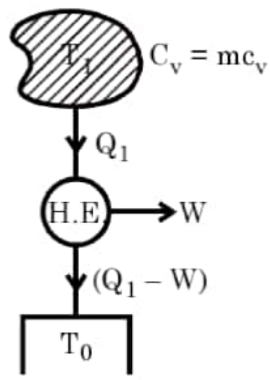
$$= 150 \left\{\ln \frac{323}{373} + \ln \frac{323}{273}\right\} J/K = 3.638 \text{ J/K}$$

Q7.11 **A system maintained at constant volume is initially at temperature T_1, and a heat reservoir at the lower temperature T_0 is available. Show that the maximum work recoverable as the system is cooled to T_0 is**

$$\boldsymbol{W = C_v\left[(T_1 - T_0) - T_0 \ln \frac{T_1}{T_0}\right]}$$

Solution:

For maximum work obtainable the process should be reversible



$$(\Delta S)_{body} = \int_{T_1}^{T_0} C_v \frac{dT}{T} = C_v \ln\left(\frac{T_0}{T_1}\right)$$

$$(\Delta S)_{resoir} = \frac{Q - W}{T_0}$$

$$(\Delta S)_{cycle} = 0$$

$$\therefore \quad (\Delta S)_{univ.} = C_v \ln\left(\frac{T_0}{T_1}\right) + \frac{Q - W}{T_0} \geq 0$$

$$\therefore \quad C_v \ln\left(\frac{T_0}{T_1}\right) + \frac{Q - W}{T_0} \geq 0$$

or $\quad C_v T_0 \ln\left(\frac{T_0}{T_1}\right) + Q - W \geq 0 \qquad \therefore \quad Q = C_v(T_1 - T_0)$

or $\quad W \leq Q + C_v T_0 \ln\left(\frac{T_0}{T_1}\right)$

or $\quad W \leq C_v (T_1 - T_0) + C_v T_0 \ln\left(\frac{T_0}{T_1}\right)$

or $\quad W \leq C_v \left\{(T_1 - T_0) + T_0 \ln\left(\frac{T_0}{T_1}\right)\right\}$

$\therefore \quad$ Maximum work $W_{max} = C_v \left\{(T_1 - T_0) + T_0 \ln\left(\frac{T_0}{T_1}\right)\right\}$

8. Availability & Irreversibility

Some Important Notes

1. Available Energy (A.E.)

$$W_{max} = Q_1\left(1 - \frac{T_0}{T_1}\right) = m\,c_p \int_{T_0}^{T}\left(1 - \frac{T_0}{T}\right) dT$$

$$= (T_1 - T_0)\,\Delta S$$

$$= u_1 - u_2 - T_0\,(s_1 - s_2)$$

(**For closed system**), it is not $(\phi_1 - \phi_2)$ because change of volume is present there.

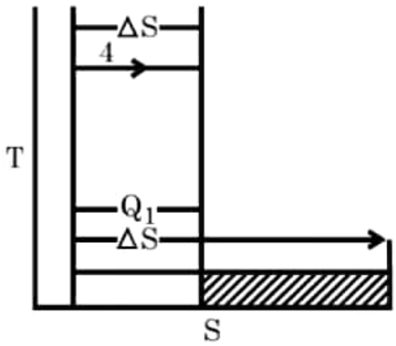
$$= h_1 - h_2 - T_0\,(s_1 - s_2)$$

(**For steady flow system**), it is $(A_1 - A_2)$ as in steady state no change in volume is CONSTANT VOLUME (i.e. change in availability in steady flow)

2. Decrease in Available Energy

$$= T_0\,[\Delta S' - \Delta S]$$

Take $\Delta S'$ & ΔS both +Ve Quantity



3. Availability function:

$$A = h - T_0 s + \frac{V^2}{2} + gZ$$

Availability = maximum useful work

For steady flow

Availability $= A_1 - A_0 = (h_1 - h_0) - T_0\,(s_1 - s_0) + \frac{V_1^2}{2} + gZ$ $\quad (\because V_0 = 0,\ Z_0 = 0)$

$$\phi = u - T_0 s + p_0 V$$

For closed system

Availability $= \phi_1 - \phi_0 = u_1 - u_0 - T_0\,(s_1 - s_0) + p_0\,(V_1 - V_0)$

Available energy is maximum work obtainable not USEFULWORK.

4. Unavailable Energy (U.E.)

$$= T_0\,(S_1 - S_2)$$

5. Increase in unavailable Energy = Loss in availability

$$= T_0\,(\Delta S)_{univ.}$$

6. **Irreversibility**

$$I = W_{max} - W_{actual}$$
$$= T_0(\Delta S)_{univ.}$$

7. **Irreversibility rate** = $\dot{I}$ rate of energy degradation

$$S_{gen} = \int_1^2 \dot{m}\, dS$$
$$= \text{rate of energy loss } (\dot{W}_{lost})$$
$$= T_0 \times \dot{S}_{gen} \quad \text{for all processes}$$

8. $\mathbf{W_{actual}} \Rightarrow đQ = du + đW_{act}$ **this for closed system**

$$h_1 + \frac{V_1^2}{2} + g Z_1 + \frac{đQ}{dm} = h_2 + \frac{V_2^2}{2} + g Z_2 + \frac{đW_{act}}{dm}$$ **this for steady flow**

9. **Helmholtz function,** $F = U - TS$

10. **Gibb's function,** $G = H - TS$

11. **Entropy Generation number (Ns)** = $\dfrac{\dot{S}_{gen}}{\dot{m}\, c_p}$

12. **Second law efficiency**

$$\eta_{II} = \frac{\text{Minimum exergy intake to perform the given task } (X_{min})}{\text{Actual exergy intake to perform the given task } (X)} = \eta_1 / \eta_{Carnot}$$

$\mathbf{X_{min}} = W$, if work is involved

$= Q\left(1 - \dfrac{T_0}{T}\right)$ if Heat is involved.

13. **To Calculate dS**

i) Use $S_2 - S_1 = m\left[c_v \ln \dfrac{p_2}{p_1} + c_p \ln \dfrac{V_2}{V_1}\right]$

For closed system

$$TdS = dU + pdV$$

or
$$dS = m\, c_v \frac{dT}{T} + \frac{p}{T} dV$$
$$= m\, c_v \frac{dT}{T} + mR \frac{dV}{V}$$
$$\int_1^2 dS = m\, c_v \int_1^2 \frac{dT}{T} + mR \int_1^2 \frac{dV}{V}$$

For steady flow system

$$TdS = dH - Vdp$$

or
$$dS = m\, c_p \frac{dT}{T} - \frac{V}{T} dp \qquad pV = mRT$$
$$\int_1^2 dS = m\, c_p \int_1^2 \frac{dT}{T} \cdot mR \int_1^2 \frac{dp}{p} \qquad \frac{V}{T} = \frac{mR}{p}$$

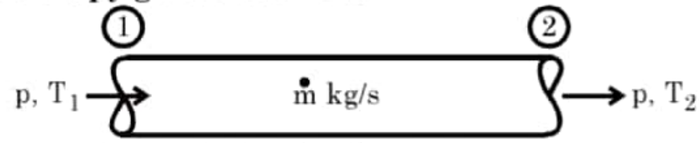
But Note that

$$TdS = dU + pdV$$

And $$TdS = dH - Vdp$$

Both valid for closed system only

14. In Pipe Flow Entropy generation rate



Due to lack of insulation it may be

$T_1 > T_2$ for hot fluid $T_1 < T_2$ for cold fluid

$$\dot{S}_{gen} = \dot{S}_{sys} - \frac{\dot{Q}}{T_0}$$

$$= \dot{m}(S_2 - S_1) - \frac{\dot{m}\, c_p (T_2 - T_1)}{T_0}$$

$\therefore$ Rate of Irreversibility $(\dot{I}) = T_0\, \dot{S}_{gen}$

15. Flow with friction

Decrease in availability = $\dot{m}\, RT_0 \times \dfrac{\Delta p}{p_1}$

Q8.1 **What is the maximum useful work which can be obtained when 100 kJ are abstracted from a heat reservoir at 675 K in an environment at 288 K? What is the loss of useful work if**

(a) A temperature drop of 50°C is introduced between the heat source and the heat engine, on the one hand, and the heat engine and the heat sink, on the other

(b) The source temperature drops by 50°C and the sink temperature rises by 50°C during the heat transfer process according to the linear law $\frac{\bar{d}Q}{dT} = \pm$ constant?

(**Ans.** (a) 11.2 kJ, (b) 5.25 kJ)

Solution:

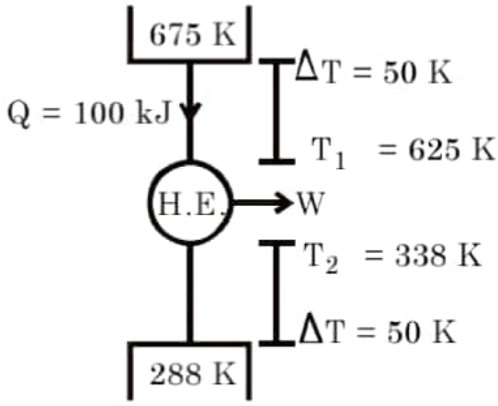
Entropy change for this process

$$\Delta S = \frac{-100}{675} \text{ kJ}/K$$

$$= 0.14815 \text{ kJ/K}$$

$$W_{max} = (T - T_0)\,\Delta S$$
$$= (675 - 288)\,\Delta S = 57.333 \text{ kJ}$$

(a) Now maximum work obtainable



$$W'_{max} = 100\left(1 - \frac{338}{625}\right)$$
$$= 45.92 \text{ kJ}$$

$\therefore$ Loss of available work = 57.333 – 45.92
= 11.413 kJ

(b) Given $\frac{\bar{d}Q}{dT} = \pm$ constant

Let $\bar{d}Q = \pm\, mc_p\, dT$

$\therefore$ When source temperature is (675 – T) and since temperature (288 + T) at that time if $\bar{d}Q$ heat is flow then maximum. Available work from that $\bar{d}Q$ is $\bar{d}W$.

$$\therefore\ \bar{d}W_{max.} = \bar{d}Q\left(1 - \frac{288 + T}{675 - T}\right)$$

$$= \left(1 - \frac{288 + T}{675 - T}\right) m\, c_p\, dT$$

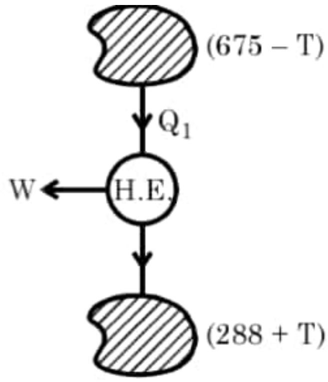
$$\therefore\quad W_{max} = m\, c_p \int_0^{50}\left(1 - \frac{288 + T}{675 - T}\right) dT$$

$$\left\{\frac{-288 - T}{675 - T} = \frac{-963 + 675 - T}{675 - T}\right\}$$

$$= m\,c_p \int_0^{50} \left\{1 + 1 - \frac{963}{675 - T}\right\} dT$$

$$= \quad m\,c_p \left\{2(50 - 0) + 963 \ln\left(\frac{675 - 50}{675 - 0}\right)\right\}$$

$$= 25.887 \ mc_p \quad kJ$$



$m\ c_p \ \times 50 = 100$ kJ

$= 51.773$ kJ

$\therefore \quad mc_p \ = 2$ kJ/K

$\therefore$ Loss of availability = (57.333 – 51.773) kJ

= 5.5603 kJ

Q 8.2 **In a steam generator, water is evaporated at 260°C, while the combustion gas (c_p = 1.08 kJ/kg K) is cooled from 1300°C to 320°C. The surroundings are at 30°C. Determine the loss in available energy due to the above heat transfer per kg of water evaporated (Latent heat of vaporization of water at 260°C = 1662.5 kJ/kg).**

(Ans. 443.6 kJ)

Solution: Availability decrease of gas

$$A_{gas} = h_1 - h_2 - T_0\,(s_1 - s_2)$$

$$= mc_p(T_1 - T_2) \quad - T_0\ mc_p \ln\left(\frac{T_1}{T_2}\right)$$

$$= m\,c_p\left[(T_1 - T_2) - T_0 \ln\frac{T_1}{T_2}\right]$$

$\therefore T_1 = 1573$ K; $T_2 = 593$ K; $T_0 = 303$ K

$= m \times 739.16$ kJ

Availability increase of water

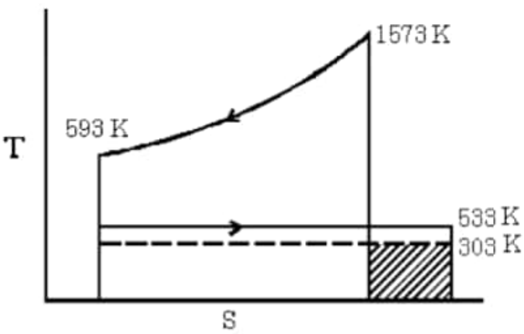
$A_w \quad = (T_1 - T_0)\ \Delta S$

$$= (T_1 - T_0) \times \frac{mL}{T_1}$$

$$= 1 \times 1662.5\left\{1 - \frac{303}{533}\right\}$$

$= 717.4$ kJ

For mass flow rate of gas (m)



$$m_g\ c_{p_g}(T_2 - T_1) = m_w \times L$$

$$\therefore \quad m_g \times 1.08 \times (1300 - 320) = 1 \times 1662.5$$

$\dot{m}_g$ = 1.5708 kg/ of water of evaporator

$A_{gas} = 1161.1$ kJ

Loss of availability $= \dot{A}_{gas} - A_w$

= (1161.1 – 717.4) kJ

= 443.7 kJ

Q8.6 **Eighty kg of water at 100°C are mixed with 50 kg of water at 60°C, while the temperature of the surroundings is 15°C. Determine the decrease in available energy due to mixing.**

(Ans. 236 kJ)

Solution:

$m_1 = 80$ kg $\quad\quad m_2 = 50$ kg

$T_1 = 100^\circ = 373$ K $\quad\quad T_2 = 60^\circ C = 333$ K

$T_0 = 288$ K

Let final temperature $(T_f) = \dfrac{m_1T_1 + m_2T_2}{m_1 + m_2} = 357.62$ K

Availability decrease of 80 kg

$$A_{dec} = \int_{357.62}^{373} m\,c_p\,dT\left(1 - \frac{T_0}{T}\right)$$

$$= m\,c_p\left[(373 - 357.62) - 288\ln\left(\frac{373}{357.62}\right)\right]$$

$$= 1088.4 \text{ kJ}$$

Availability increase of 50 kg water

$$A_{in} = \int_{333}^{357.62} m\,c_p\left(1 - \frac{T_0}{T}\right)dT$$

$$= m\,c_p\left[(357.62 - 333) - 288\ln\left(\frac{357.62}{333}\right)\right]$$

$$= 853.6 \text{ kJ}$$

$\therefore$ Availability loss due to mixing

$= (1088.4 - 853.6)$ kJ

$= 234.8$ kJ

Q8.7 **A lead storage battery used in an automobile is able to deliver 5.2 MJ of electrical energy. This energy is available for starting the car.**

Let compressed air be considered for doing an equivalent amount of work in starting the car. The compressed air is to be stored at 7 MPa, 25°C. What is the volume of the tank that would be required to let the compressed air have an availability of 5.2 MJ? For air, $pv = 0.287\,T$, where T is in K, p in kPa, and v in m³/kg.

(Ans. 0.228 m³)

Solution: Electrical Energy is high Grade Energy so full energy is available

$\therefore \quad A_{electric} = 5.2$ MJ $= 5200$ kJ

Availability of compressed air

$$A_{air} = u_1 - u_0 - T_0(s_1 - s_0)$$

$$= m\,c_v\,(T_1 - T_0) - T_0\,(s_1 - s_0)$$

$$(s_1 - s_0) = c_v\ln\frac{p_1}{p_0} + c_p\ln\frac{v_1}{v_0} \quad\quad = c_p\ln\frac{T_1}{T_0} - R\ln\frac{p_1}{p_0}$$

$$\Delta W = T_0\,R\ln\frac{p_1}{p_0}$$

$= 298 \times 0.287 \times \ln\left(\dfrac{7000}{100}\right)$ $\quad\quad$ Here $T_1 = T_0 = 25^\circ C = 298$ K

$= 363.36$ kJ/kg $\quad\quad$ Let atm $\quad p_r = 1$ bar $= 100$ kPa

Given $p_1 = 7$ MPa $= 7000$ kPa

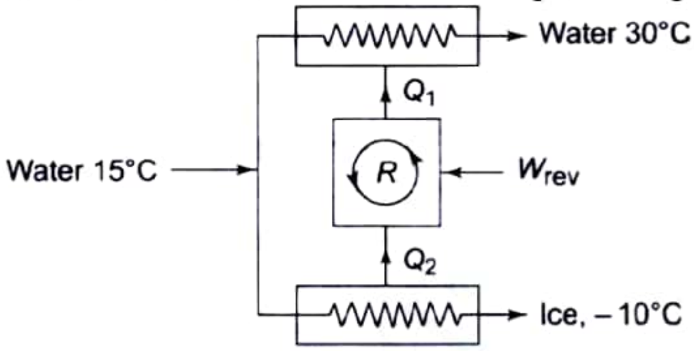
$\therefore$ Required mass of air = $\dfrac{5200}{363.36}$ kg = 14.311 kg

Specific volume of air at 7 MPa, 25°C then

$$v = \frac{RT}{p} = \frac{0.287 \times 298}{7000}\ m^3/kg = 0.012218\ m^3/kg$$

$\therefore$ Required storage volume (V) = 0.17485 m^3

Q8.8 **Ice is to be made from water supplied at 15°C by the process shown in Figure. The final temperature of the ice is – 10°C, and the final temperature of the water that is used as cooling water in the condenser is 30°C. Determine the minimum work required to produce 1000 kg of ice.**



Take c_p for water = 4.187 kJ/kg K, c_p for ice = 2.093 kJ/kg K, and latent heat of fusion of ice = 334 kJ/kg.

(**Ans.** 33.37 MJ)

Solution: Let us assume that heat rejection temperature is (T_0)

(i) Then for 15°C water to 0° C water if we need W_R work minimum.

Then $(COP) = \dfrac{Q_2}{W_R} = \dfrac{T_2}{T_0 - T_2}$

or $W_R = Q_2 \dfrac{(T_0 - T_2)}{T_2}$

$$= Q_2\left(\frac{T_0}{T_2} - 1\right)$$

When temperature of water is

T if change is dT

Then $\cancel{d}Q_2 = -mc_p\ dT$

(heat rejection so –ve)

$$\therefore \quad \cancel{d}W_R = -m\,c_p\,dT\left(\frac{T_0}{T} - 1\right)$$

$$\therefore \quad W_{R_1} = -m\,c_p\int_{288}^{273}\left(\frac{T_0}{T} - 1\right)dT$$

$$= m\,c_p\left[T_0 \ln\frac{288}{273} - (288 - 273)\right]$$

$$= 4187\left[T_0 \ln\frac{288}{273} - 15\right] kJ$$

(ii) W_R required for 0° C water to 0 ° C ice

$$W_{R_{II}} = Q_2\left(\frac{T_0}{T_2} - 1\right)$$

$$= mL\left(\frac{T_0}{T_2} - 1\right)$$

$$= 1000 \times 335\left(\frac{T_0}{273} - 1\right)$$

$$= 335000\left(\frac{T_0}{273} - 1\right) \text{kJ}$$

(iii) W_R required for 0° C ice to –10 ° C ice.

When temperature is T if dT temperature decreases

$\therefore \quad \cancel{d}Q_2 = -mc_{p\ ice}\ dT$

$\therefore \quad \cancel{d}W_R = -m\,c_{p\,ice}\,dT\left(\frac{T_0}{T} - 1\right)$

$\therefore \quad W_{R_{II}} = m\,c_{p\,ice}\int_{263}^{273}\left(\frac{T_0}{T} - 1\right)dT = m\,c_{p\,ice}\left[T_0 \ln\frac{273}{263} - (273 - 263)\right]$

Let $\quad c_{p,ice} = \frac{1}{2}\ c_{p,water} = \frac{4.187}{2}$ kJ/kg

$$= 1000 \times \frac{4.187}{2}\left[T_0 \ln\frac{273}{263} - 10\right]$$

$$= 2093.5\left[T_0 \ln\frac{273}{263} - 10\right] \text{kJ}$$

$\therefore \quad$ Total work required

$W_R = (i) + (ii) + (iii)$

$= [1529.2\ T_0 - 418740]$ kJ

$\therefore \quad W_R$ and T_0 has linear relationship

$\therefore \quad T_0 = \frac{15 + 30}{2}\,°C = 22.5°C = 295.5$ K

$\therefore \quad W_R = 33138.6$ kJ = 33.139 MJ