

# Physics

- formula
- Defl & statement
- Unit & dimension
- Problems.

## Oscillation! —

### Periodic Motion! —

The motion is said to be Periodic motion if it repeats itself in equal interval of time is called periodic motion.

### Simple Harmonic Motion (SHM)! —

If a body execute in SHM along with Periodic motion iff its restoring force is directly proportional to displacement and the direction of motion is opposite to the displacement.

### Derivation! —

Then F<sub>ext</sub>

$$\Rightarrow F = -kx \quad (i)$$

where  $x$  is the displacement &  $k$  is the Proportionality constant.

The negative sign indicates that the direction of motion is opposite to the displacement.

→ Against form Newton's 2nd Law,

$$F = ma$$
$$= m \frac{d^2x}{dt^2} \text{ also } v = \frac{dx}{dt}$$

$$\Rightarrow F = m \frac{d}{dt} \left( \frac{dx}{dt} \right)$$

$$\boxed{F = -\frac{m d^2x}{dt^2}} \quad \text{ii}$$

Comparing eqn (i) & (ii)

$$\rightarrow k_2 = \frac{md^2x}{dt^2}$$

$$\therefore \frac{md^2x}{dt^2} + k_2 = 0$$

$$\frac{d^2x}{dt^2} + \frac{k_2}{m} = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0} \quad \text{iii}$$

$$\text{where } \omega^2 = \frac{k_2}{m}$$

where  $\omega$  is the angular frequency.

Eqn (iii) is called the differential eqn of SHM.

Solution:-

Let the solution of the diff. eqn  $x = A e^{i\omega t}$

where  $A$  &  $\alpha$  are the constants.

$$\frac{dn}{dt} = A e^{\alpha t}, \alpha$$

$$\frac{d^2n}{dt^2} = A e^{\alpha t} \alpha^2$$

$$A \alpha^2 e^{\alpha t}$$

Putting in eqn (ii)

$$A \alpha^2 e^{\alpha t} + \omega^2 A e^{\alpha t} = 0$$

$$A e^{\alpha t} (\alpha^2 + \omega^2) = 0$$

$$\Rightarrow A \neq 0, A e^{\alpha t} \neq 0$$

$$\alpha^2 + \omega^2 = 0$$

$$\Rightarrow \alpha^2 = -\omega^2$$

$$\Rightarrow \alpha^2 = -\omega^2$$

$$\Rightarrow \alpha = \pm i\omega$$

So the solution is

$$n = A_1 e^{i\omega t} + A_2 e^{-i\omega t} \quad (iv)$$

where  $A_1$  &  $A_2$  are any constant.

$$e^{i\omega t} = \cos \omega t + i \sin \omega t$$

$$e^{-i\omega t} = \cos \omega t - i \sin \omega t$$

Putting in above eqn,

$$\begin{aligned} n &= A_1 (\cos \omega t + i \sin \omega t) + A_2 (\cos \omega t - i \sin \omega t) \\ &= \cos \omega t (A_1 + A_2) + i \sin \omega t (A_1 - A_2). \end{aligned}$$

Again putting,  $A_1 + A_2 = a \sin \phi$

$$i(A_1 - A_2) = a \cos \phi$$

$$\alpha = \cos\omega t \cdot a \sin \phi + \sin\omega t \cdot a \cos \phi$$

$$= a [\cos\omega t \cdot \sin \phi + \sin\omega t \cdot \cos \phi]$$

$$\boxed{\alpha = a \sin(\omega t + \phi)} \quad \checkmark$$

This is then ~~diff eq~~ of SHM soln.

### Special Cases!

(i) When the SHM starts from mean position, then  $\alpha = 0$  &  $t = 0$ .

$$\Rightarrow \boxed{0 = a \sin \phi}$$

$$\Rightarrow \sin \phi = 0$$

$$\Rightarrow \boxed{\phi = 0}$$

So the soln will be  $\boxed{\alpha = a \sin \omega t}$

(ii) If the SHM starts from extreme position, then  $\alpha = a$  &  $t = 0$ .

So putting  $\alpha = a \sin \phi$

$$\Rightarrow \sin \phi = 1$$

$$\Rightarrow \phi = 90^\circ - \frac{\pi}{2}$$

So the soln will be  $\boxed{\alpha = a \sin(\omega t + \frac{\pi}{2})}$

$$\Rightarrow \boxed{\alpha = a \cos \omega t}$$

## Amplitude

$a$  is defined as the max<sup>m</sup> displacement from the mean position for by a wave from the mean position is called Amplitude.

$a$  is denoted as 'a'.

## Time period

$T$  is the smallest interval of time to cover one oscillation.

$T$  is denoted as 'T'.

$$\text{we have } x = a \sin(\omega t + \phi)$$

$$\text{Let putting } t = t + \frac{2\pi}{\omega}$$

$$\text{so } x = a \sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \phi\right]$$

$$= a \sin\left[\omega t + 2\pi + \phi\right]$$

$$= a \sin(\omega t + \phi + 2\pi)$$

$$\boxed{x = a \sin(\omega t + \phi)}$$

As it taken repeat if self due to the

Value of  $\frac{2\pi}{\omega}$ , so we can define

the time period will be  $\boxed{T = \frac{2\pi}{\omega}}$ .

$$\text{Again } \omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow T = \frac{2\pi}{\sqrt{k/m}} = 2\pi \sqrt{\frac{m}{k}}$$

$$\boxed{T = 2\pi \sqrt{\frac{m}{k}}}$$

## frequency of SHM:

$\omega$  is defined as the reciprocal of time period.

$\omega$  is denoted as ' $\nu$ '.

Then  $\nu = \frac{1}{T}$

$$\therefore \nu = \frac{1}{2\pi T_{\text{per}}} = \frac{\omega}{2\pi}$$

$$\text{Also } \nu = \frac{1}{2\pi\sqrt{\frac{m}{k}}} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

## Velocity of SHM:

$$v = a \sin(\omega t + \phi)$$

$$v = \frac{dv}{dt} = \frac{d}{dt} [a \sin(\omega t + \phi)]$$

$$= a \cos(\omega t + \phi) \frac{d}{dt} (\omega t + \phi)$$

$$= a \cos(\omega t + \phi) \omega$$

$$= a \omega \cos(\omega t + \phi)$$

$$v = a \omega \cos(\omega t + \phi)$$

$$\text{Accel} = \frac{dv}{dt} = a$$

$$a = \frac{dv}{dt}$$

$$\Rightarrow a = \frac{d}{dt} [a \omega \cos(\omega t + \phi)]$$

$$\Rightarrow a = a \omega^2 \sin(\omega t + \phi) \frac{d}{dt} (\omega t + \phi)$$

$$\Rightarrow a = \omega^2 \{ -\sin(\omega t + \phi) \}$$

$$\Rightarrow [Acc = -\omega^2 \sin(\omega t + \phi)]$$

## Phase of SHM:-

- It is defined as the extra path covered by a wave when it is in motion.
- It is denoted as  $\phi$ .
- Here the displacement can be written as  $x = a \sin(\omega t + \phi)$ .
- Here  $(\omega t + \phi)$  is also known as Constant phase.  
at  $t=0$ , then  $\phi$  is also known as a constant phase or epoch.

## DHM (Damped Harmonic Motion)

- In SHM the oscillation continues forever with constant amplitude but in actual case the oscillation do not continue with constant amplitude, rather than the amplitude goes on decreasing.
- ~~But~~ This decrease in amplitude is due to the resistive force or viscous force of the medium.
- So hence the harmonic motion which oscillates in the resistive medium then there will be the gradual

decrease in amplitude and known as  
Damped Harmonic Motion see DHM.

Let us consider a body whose mass is 'm', and has a restoring force  $F = -kx$ .  
Also there will be the resistive force which is  $F = -\gamma v$   
 $F = -\gamma \frac{dx}{dt}$

hence  $\gamma$  = Damping Coefficient  
and negative sign indicates that  
the motion is in  
opposite direction  
of velocity.

$$m \frac{d^2x}{dt^2} = -kx - \gamma \frac{dx}{dt}$$

$$\Rightarrow m \frac{d^2x}{dt^2} + kx + \gamma \frac{dx}{dt} = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m} x + \frac{\gamma}{m} \frac{dx}{dt} = 0$$

$$\text{Let } \frac{k}{m} = \omega^2$$

$$\Rightarrow \frac{\gamma}{m} = 2b, \text{ where } b^2 \text{ is constant.}$$

$$\text{Putting } \boxed{\frac{d^2x}{dt^2} + \omega^2 x + 2b \cdot \frac{dx}{dt} = 0}$$

This is the diff eqn of DHM.

①

Solution:-

Let the solution is  $n = A e^{\alpha t}$ .

$$\Rightarrow \frac{dn}{dt} = A e^{\alpha t} \cdot \alpha = A \alpha e^{\alpha t}$$

$$\Rightarrow \frac{d^2n}{dt^2} = A \alpha^2 e^{\alpha t}$$

Putting in eqn ①,

$$A \alpha^2 e^{\alpha t} + \omega^2 A e^{\alpha t} + 2b \cdot A e^{\alpha t} = 0$$

$$A e^{\alpha t} [\alpha^2 + \omega^2 + 2b\alpha] = 0$$

$$A e^{\alpha t} \neq 0 \Rightarrow \alpha^2 + \omega^2 + 2b\alpha = 0$$

~~Now  $\alpha = \omega \pm i\theta$~~   
So the gen is

$$\alpha = \frac{-2b \pm \sqrt{4b^2 - 4\omega^2}}{2 \cdot 1}$$

$$= \frac{-2b \pm \sqrt{4b^2 - 4\omega^2}}{2}$$

$$\alpha = -b \pm \sqrt{b^2 - \omega^2}$$

So the gen al sol<sup>g</sup>,

$$n = A_1 e^{(-b + \sqrt{b^2 - \omega^2})t} + A_2 e^{(-b - \sqrt{b^2 - \omega^2})t}$$

Evolution of  $A_1$  &  $A_2$

when  $n = a_0 \Rightarrow t = 0$

Putting in eqn ②

$$a_0 = A_1 + A_2 \quad \text{--- ③}$$

Again force max<sup>n</sup> displacement velocity zero.

$$\text{So } t=0 \quad \frac{d\eta}{dt} = 0$$

$$\frac{d\eta}{dt} = A_1 e^{(-b + \sqrt{b^2 - \omega^2})t} + (-b + \sqrt{b^2 - \omega^2}) A_2 e^{(b - \sqrt{b^2 - \omega^2})t}$$

force  $t=0 \quad \frac{d\eta}{dt} = 0$

$$\Rightarrow 0 = A_1 (-b + \sqrt{b^2 - \omega^2}) + A_2 (-b - \sqrt{b^2 - \omega^2})$$

$$\Rightarrow 0 = -b(A_1 + A_2) + \sqrt{b^2 - \omega^2}(A_1 - A_2)$$

$$\Rightarrow 0 = \omega - b\eta_0 + \sqrt{b^2 - \omega^2}(A_1 - A_2) \quad \omega$$

$$\Rightarrow b\eta_0 = \sqrt{b^2 - \omega^2}(A_1 - A_2)$$

$$\Rightarrow (A_1 - A_2) = \frac{b\eta_0}{\sqrt{b^2 - \omega^2}} \quad (4)$$

Adding eqn ③ & ④.

$$\eta_0(A_1 + A_2) + (A_1 - A_2) = \eta_0 + \frac{b\eta_0}{\sqrt{b^2 - \omega^2}}$$

$$\Rightarrow 2A_1 = \eta_0 \frac{\sqrt{b^2 - \omega^2}}{\sqrt{b^2 - \omega^2}} + b\eta_0$$

$$\Rightarrow 2A_1 = \eta_0 \cancel{(b + \sqrt{b^2 - \omega^2})}$$

$$\cancel{A_1 = \frac{\eta_0(b + \sqrt{b^2 - \omega^2})}{2\sqrt{b^2 - \omega^2}}}$$

$$\Rightarrow 2A_1 = \eta_0 \left(1 + \frac{b}{\sqrt{b^2 - \omega^2}}\right)$$

$$\Rightarrow A_1 = \frac{\eta_0}{2} \left(1 + \frac{b}{\sqrt{b^2 - \omega^2}}\right)$$

Substracting eqn ③ & ①

$$(A_1 + A_2) - (A_1 - A_2) = a_0 \cancel{+} \frac{ba_0}{\sqrt{b^2 - w^2}}$$

$$\Rightarrow A'_1 + A'_2 - A_1 + A_2 = a_0 \left(1 - \frac{b}{\sqrt{b^2 - w^2}}\right)$$

$$\Rightarrow 2A_2 = a_0 \left(1 - \frac{b}{\sqrt{b^2 - w^2}}\right)$$

$$\Rightarrow A_2 = \frac{a_0}{2} \left(1 - \frac{b}{\sqrt{b^2 - w^2}}\right)$$

So hence, the solution of the diff' eqn.

$$m = \frac{a_0}{2} \left(1 + \frac{b}{\sqrt{b^2 - w^2}}\right) e^{(-b + \sqrt{b^2 - w^2})t} + \frac{a_0}{2} \left(1 - \frac{b}{\sqrt{b^2 - w^2}}\right) e^{(-b - \sqrt{b^2 - w^2})t}$$

..... ⑤

\* Putting  $\sqrt{b^2 - w^2} = \beta$

$$m = \frac{a_0}{2} \left[ \left(1 + \frac{b}{\beta}\right) e^{(-b + \beta)t} + \left(1 - \frac{b}{\beta}\right) e^{(-b - \beta)t} \right]$$

Case - I

when  $b > w$ , then  $b^2 > w^2$

so  $b^2 - w^2$  is nearly equal to  $b^2$

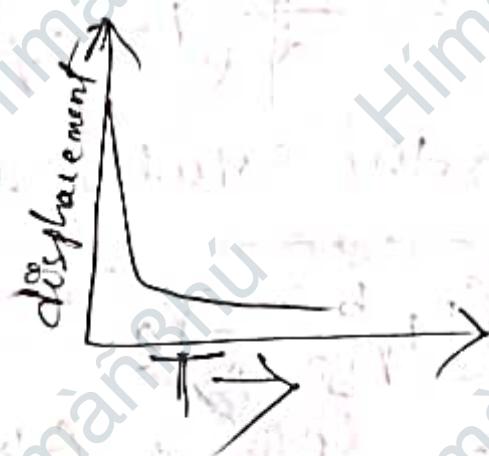
$$\boxed{b^2 - w^2 \approx b^2}$$

Substituting in eqn ⑤

$$m = a_0$$

This is the case of Over Damping or heavy damping. so the motion is called over damp.

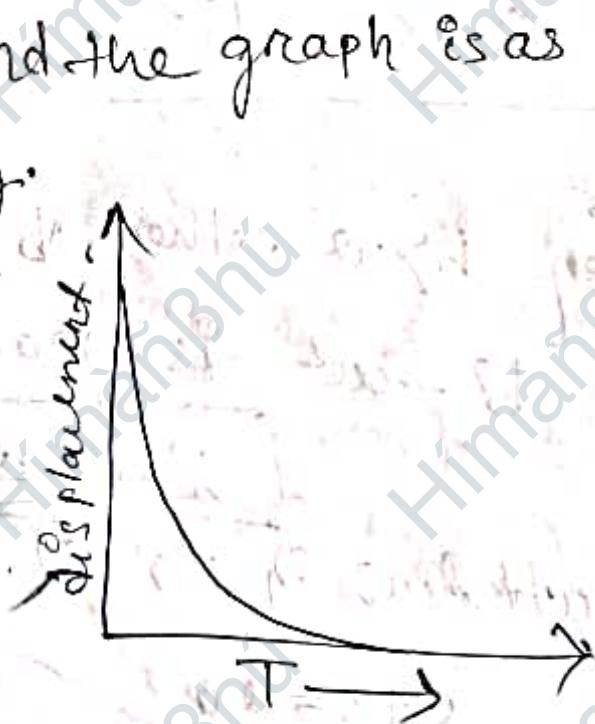
A periodic & deadbeat oscillation & the graph is as shown in fig.



### Case - 2

When  $b = \omega$  then  $b^2 = \omega^2$   
so  $\alpha_1 = \omega$  defined.

This is the case of critical damping and the graph is as shown in fig.



Case - 3

when  $b < \omega$  then  $b^2 < \omega^2$ ,  
 $\sqrt{b^2 - \omega^2} \leq \sqrt{-\omega^2} = \sqrt{-1} \sqrt{\omega}$

so  $\eta = \frac{a_0}{2} \left( 1 + \frac{b}{i\omega} \right) e^{(-b+i\omega)t} = \frac{a_0}{2} \left( 1 + \frac{b}{i\omega} \right) e^{(-b+i\omega)t} + \frac{a_0}{2} \left( 1 + \frac{b}{i\omega} \right) e^{(-b-i\omega)t}$

$$= \frac{2a_0}{2} \left[ \left( 1 + \frac{b}{i\omega} \right) e^{(-b+i\omega)t} + a_0 \left( 1 + \frac{b}{i\omega} \right) e^{(-b-i\omega)t} \right]$$

This is the case of Underdamping or small damping.

### Time Period

It is the time taken to complete one oscillation. It is denoted as  $T$ .  
 and the expression for the time period in the DHM can be written as

$$\text{i.e } T = \frac{2\pi}{\beta}$$

Again we know,

$$T = \frac{2\pi}{\sqrt{b^2 - \omega^2}} \quad [ \because \beta = \sqrt{b^2 - \omega^2} ]$$

### Frequency

$\omega$  is the reciprocal of time period.  
 $\omega$  is the reciprocal of time period.  
 & it can be denoted as ' $\eta$ '.

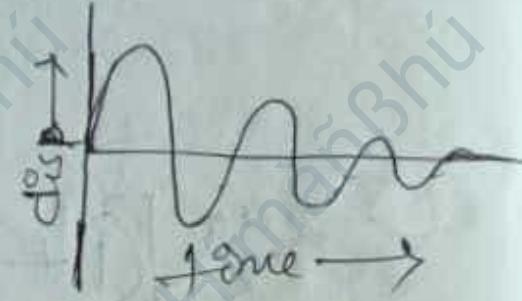
$$\eta = \frac{1}{T} = \frac{1}{\frac{2\pi}{\sqrt{b^2 - \omega^2}}} = \frac{\sqrt{b^2 - \omega^2}}{2\pi}$$

$\Rightarrow$  magnitude form,

$$\eta = \frac{\sqrt{w^2 - b^2}}{2\pi}$$

### Amplitude:

In this case, the amplitude goes on decreasing with time.



### Quality Factor:

It is defined as the ratio due to total energy stored in the system to the energy loss per period.

Mathematically,

$$Q = 2\pi \times \frac{\text{Total energy in the system}}{\text{energy loss per period}}$$

$$= 2\pi \times \frac{E}{P T}$$

where,  $E$  is the total

where  $E$  is the total energy of the system.

$P$  is the Power Dissipation.

$\& P = \frac{E}{T}$ .  $E$  is the Time period

$$P = \frac{E}{T}$$

Then putting  $\varphi = 2\pi \cdot \frac{E}{\hbar/\tau T}$

$$\boxed{\varphi = \frac{2\pi c}{T}}$$

whereas  $\tau$  is the relaxation time.

and now,  $\varphi = \beta \tau$  where  $(\beta = \frac{2\pi}{T})$

in case of DHM,

$$\boxed{\varphi = \omega \tau}$$

Again,  $\omega^2 = \frac{k}{m}$

$$\Rightarrow \omega = \sqrt{\frac{k}{m}}$$

then Putting,

$$\boxed{\varphi = \sqrt{\frac{k}{m}} \tau}$$

### Relaxation Time! ( $\tau$ )

It is defined as the time in which the total mechanical energy becomes  $1/e$  of its initial value.

by the expression for energy, i.e  
B and initial energy  $E_0$ .

According to defn,  $\boxed{E = \frac{1}{e} E_0}$

But we know  $\boxed{E = E_0 e^{-2b\tau}}$

$$\Rightarrow \frac{E_0}{e} = E_0 e^{-2b\tau}$$

$$\Rightarrow e^{-1} = e^{-2b\tau}$$

$$\Rightarrow 2b\tau = 1 \Rightarrow \boxed{\tau = \frac{1}{2b}}$$

## 6 Logarithmic decrement!

We have the expression for displacement  $x = a_0 e^{-bt} \cos(\beta t + \phi)$

Here the amplitude ( $a_0 e^{-bt}$ ) goes on decreasing. So in general form we can write as  $a_n = a_0 e^{-bt} \cancel{e^{bt}}$

$$\therefore a_{n+1} = a_0 e^{-b(t+T_{1/2})}$$

$$\begin{aligned} \text{Now } \frac{a_n}{a_{n+1}} &= \frac{a_0 e^{-bt}}{a_0 e^{-b(t+T_{1/2})}} \\ &= \frac{e^{-bt}}{e^{-bt} \cdot e^{-bT_{1/2}}} \\ &= e^{-bT_{1/2}} \end{aligned}$$

$$\boxed{\frac{a_n}{a_{n+1}} = e^{-bT_{1/2}} = d}$$

Where,  $d$  is called the logarithm decrement and can be written as  $d = e^{-bT_{1/2}}$ .

$$\boxed{\log d = -\frac{bT_{1/2}}{2}}$$

This is the expression for logarithmic decrement.

## F.H.M (Forced Harmonic Motion) on D.H.O (Driven Harmonic Oscillation)

In DHM the amplitude goes on decreasing after some interval of time due to the resistive medium. To maintain the oscillation in DHM, we have to apply an external periodic force whose frequency greater than the natural frequency.

So hence, if the oscillation of the particle continue over a time with an external driving force or Periodic.

Force then it is called Forced Harmonic Oscillation or Driven Harmonic oscillation.

Let us consider a particle of mass  $m$ , oscillating under a Driven Force ( $f_0 \sin pt$ )

in the direction of motion. So we can write

$$m \frac{d^2y}{dt^2} = -Ky - r \frac{dy}{dt} + f_0 \sin pt$$

Where, Restoring force =  $-Ky$

Resistive force =  $-r \frac{dy}{dt}$

$$\Rightarrow m \frac{d^2y}{dt^2} + Ky + r \frac{dy}{dt} = f_0 \sin pt$$

$$\Rightarrow \frac{d^2y}{dt^2} + \frac{k}{m} y + \frac{r}{m} \frac{dy}{dt} = \frac{f_0}{m} \sin pt$$

Now,  
Putting  $\frac{K}{m} = \omega^2$

$$\frac{2}{m} = 2b$$

$\frac{f_0}{m} = f_0$  (Amplitude Factor)

$$\Rightarrow \boxed{\frac{d^2\eta}{dt^2} + \omega^2 \eta + 2b \frac{d\eta}{dt} = f_0 \sin pt}$$

This is the diff. eqn of FHM.

Sol

The equation of FHM can be written  
i.e.  $\eta(t) = u_1(t) + u_2$ .

where  $u_1$  is the complementary func  
&  $u_2$  is the Particular func.

In general the soln of complementary  
func can be written as

$$u_1 = a_0 e^{-bt} \sin(\beta t + \phi) \quad (11)$$

Similarly if Particular func the  
general eqn can be written as

$$u_2 = A \sin(pt - \phi)$$

(14)

Differentiate both sides,

$$\frac{d\omega_2}{dt} = A \cos(\omega t - \phi) P$$

Again  $\frac{d^2\omega_2}{dt^2} = AP - \sin(\omega t - \phi) \cdot P$   
 $= -AP^2 \sin(\omega t - \phi)$

Putting these values in the differentiated  
eqn we get

$$-AP^2 \sin(\omega t - \phi) + \omega A \sin(\omega t - \phi) + 2bAP \cos(\omega t - \phi) = f_0 \sin[(\omega t - \phi) + \phi]$$

$$\Rightarrow -AP^2 \sin(\omega t - \phi) + \omega^2 \sin(\omega t - \phi) + 2bAP \cos(\omega t - \phi) =$$

$$f_0 [\sin(\omega t - \phi) \cdot \cos \phi + \cos(\omega t - \phi) \cdot \sin \phi]$$

$$\Rightarrow A \sin(\omega t - \phi) [\omega^2 - P^2] + 2bAP \cos(\omega t - \phi) =$$

$$f_0 \sin(\omega t - \phi) \cos \phi + f_0 \cos(\omega t - \phi) \sin \phi$$

Equating the co-efficients of  $\sin(\omega t - \phi)$  &  
also  $\cos(\omega t - \phi)$  for both sides of the  
eqn we get:

$$A(\omega^2 - P^2) = f_0 \cos \phi \quad \text{--- (a)}$$

$$2bAP = f_0 \sin \phi \quad \text{--- (b)}$$

Squaring & Adding eqn (a) & (b) we get

$$A^2 (\omega^2 - P^2)^2 + 4b^2 A^2 P^2 = f_0^2 \cos^2 \phi + f_0^2 \sin^2 \phi$$

$$\Rightarrow A^2 [(\omega^2 - P^2)^2 + 4b^2 P^2] = f_0^2 (\cos^2 \phi + \sin^2 \phi)$$

$$\Rightarrow A^2 [(\omega^2 - P^2)^2 + 4b^2 P^2] = f_0^2$$

$$\Rightarrow A^2 = \frac{F_0^2}{(\omega^2 - P^2)^2 + 4b^2P^2}$$

$$\Rightarrow A = \frac{F_0}{\sqrt{(\omega^2 - P^2)^2 + 4b^2P^2}}$$

This is the expression for the amplitude of the FHM.

### Resonance $\Rightarrow$ Amplitude Resonance

It is defined as the amplitude of the FHM goes on decreasing after some time interval then it is called resonance.

Or the time interval for which the amplitude goes on decreasing due to some external forced is called so.

The phenomenon for which the amplitude of the forced oscillation is max<sup>m</sup> is called so.

As the amplitude is max<sup>m</sup> due to the driving force so it is also called amplitude resonance.

As the amplitude is max<sup>m</sup>.

$$A = \frac{F_0}{\sqrt{(\omega^2 - P^2)^2 + 4b^2P^2}}$$

that means  $\frac{d}{dp}[(\omega^2 - p^2)^2 + 4b^2p^2] = 0$

$$\Rightarrow 2(\omega^2 - p^2)(-2p) + 4b^2 \cdot 2p = 0$$

$$\Rightarrow 4p[(\omega^2 - p^2)(-1) + 2b^2] = 0$$

$$\Rightarrow -(\omega^2 - p^2) = -2b^2$$

$$\Rightarrow p^2 = \omega^2 - 2b^2$$

$$\Rightarrow p = p_r = \sqrt{\omega^2 - 2b^2}$$

so the resonance frequency is

$$\frac{p_r}{2\pi} = \frac{\sqrt{\omega^2 - 2b^2}}{2\pi}$$

### Sharpness of Resonance —

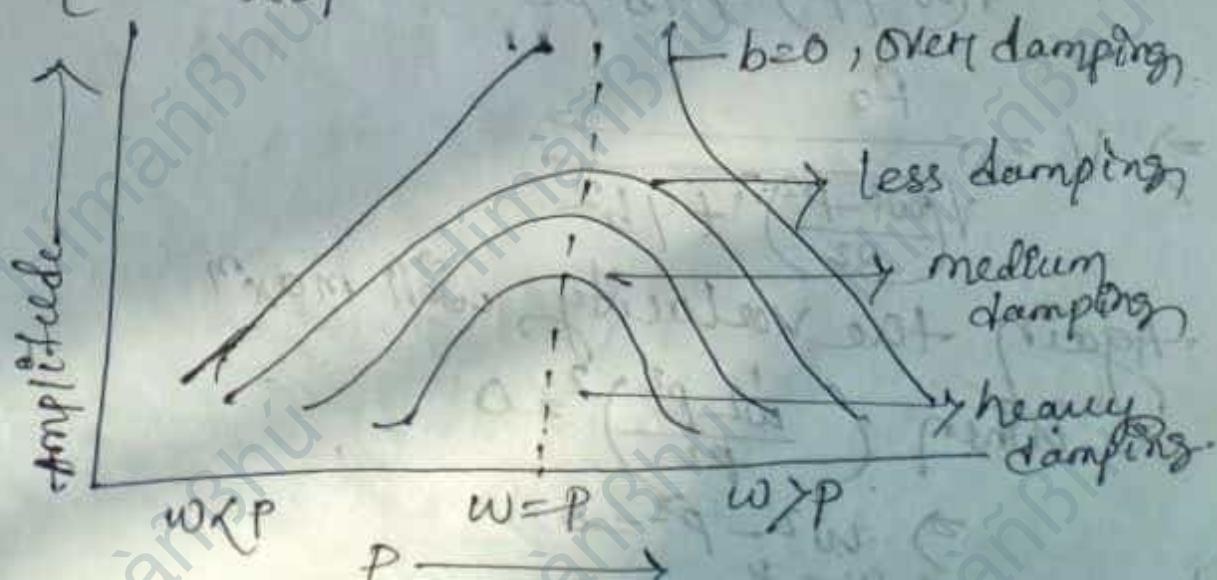
It is the measure of the fall of amplitude with the external driving force.

We have the expression for amplitude

$$A = \frac{F_0}{\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2}}$$

when  $\omega = p$

$$\text{then } A = \frac{F_0}{2bp}$$



When  $b=0$

$$\Rightarrow A = \frac{f_0}{2bP} \rightarrow \infty$$

So the amplitude will be max<sup>m</sup> or  $\infty$ ,  
which is actually impossible.

## Velocity Resonance!

The displacement in harmonic motion  
is  $x = A \sin (Pt - \phi)$

$$\text{Now } V = \frac{dx}{dt} = AP \cos (Pt - \phi)$$

When velocity will be max<sup>m</sup>:

$$\cos (Pt - \phi) = 1$$

$$\text{So } V_0 = AP$$

So max<sup>m</sup> velocity  $V = V_0 = AP$

$$\text{Again } V = AP \cos (Pt - \phi)$$

Putting the value of  $A$  in above

$$V = \frac{f_0}{\sqrt{(w^2 + P^2)^2 + 4b^2P^2}} \times P \cos (Pt - \phi)$$

$$\Rightarrow V = \frac{f_0}{\sqrt{\left(\frac{w^2 - P^2}{P^2}\right)^2 + 4b^2}}$$

Again the velocity will max<sup>m</sup>  
when  $\left(\frac{w^2 - P^2}{P^2}\right)^2 = 0$

$$\Rightarrow w^2 - P^2 = 0$$

$$\Rightarrow w = P$$

This is the condition for which the velocity is maximum in force oscillation & this is called velocity resonance.

### Quality Factor:

It is defined as 2π times, the ratio of total energy stored in the system to the energy lost per period.

$$Q = 2\pi \frac{E_{avg}}{P_{avg} T}$$

$$\text{Now, } E = KE + PE$$

$$= \frac{1}{2} m v^2 + \frac{1}{2} m \omega^2 A^2$$

$$= \frac{1}{2} m [A P \cos(\phi) + \dot{A} P \sin(\phi)]^2$$

$$+ \frac{1}{2} m \omega^2 [A P \sin(\phi) + \dot{A} P \cos(\phi)]^2$$

$$\Rightarrow E = \frac{1}{2} m A^2 P^2 \cos^2(\phi) + \frac{1}{2} m \omega^2 A^2 \sin^2(\phi)$$

we have the average value of

$\cos^2(\phi)$  &  $\sin^2(\phi)$  over a time period is half.

$$\text{So, } E = \frac{1}{4} m A^2 P^2 + \frac{1}{4} m \omega^2 A^2$$

$$\Rightarrow E = \frac{1}{4} m A^2 (P^2 + \omega^2)$$

The power loss,

$$P = \frac{m A^2 P^2}{2\pi} \times T = \frac{2\pi}{P}$$

$$Q = 2\pi \times \frac{\frac{1}{4} m A^2 (P^2 + \omega^2)}{\frac{m A^2 P^2}{2\pi} \times 2\pi/P}$$

$$\Rightarrow Q = 2\pi \frac{V_0 m A^2 (P^2 + \omega^2) \cdot 2\pi P}{m A^2 \omega^2 \cdot 2\pi}$$

$$\Rightarrow Q = \frac{2\pi \times V_0 C P^2 (\omega^2 + \omega^2) Z}{P \pi}$$

$$\Rightarrow Q = \frac{1}{2} \left( \frac{P^2 + \omega^2}{P} \right) Z P$$

$$\Rightarrow Q = \frac{1}{2} \left( 1 + \frac{\omega^2}{P^2} \right) Z P$$

as  $\omega \approx P$

$$\text{so } Q = \frac{1}{2} (1 + 1) Z P$$

$$\boxed{\Rightarrow Q = Z P}$$

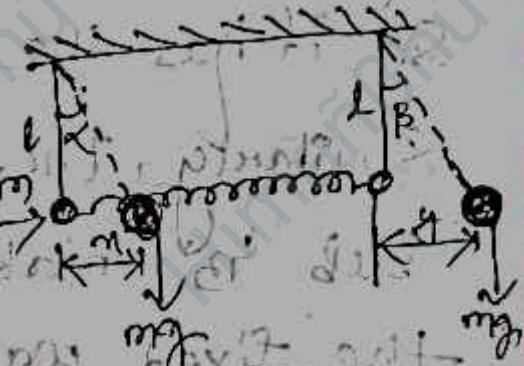
This is the expression for quality factor.

## Couple Oscillation Or two coupled oscillation

In these type of oscillation we have to connect two simple pendulums by a spring whose mass is very small. If the two pendulums are suspended from the fixed wall & are connected with a massless spring then the two oscillators is called two couple oscillation.

Let us consider, two simple pendulums whose mass of bob are same and are suspended from the fixed wall. For that, the length of the simple pendulum. Both the bob of the simple pendulum are connected with the massless spring whose spring constant or force constant is "K".

The normal length of the spring is equal to the distance betw the bobs when they are in their eqm position.



Hence the restoring force of the bob 'A' due to spring is  $-k(x-y)$ .

Similarly, the restoring force of the bob 'B' due to spring is  $-k(y-z)$ .

~~Ans~~ As the displacement 'x' of the bob 'A' makes an angle  $\alpha$  with the fixed wall due to spring.

So it has the component  $ie \sin\alpha$ .

Similarly, the displacement 'y' of the bob 'B' makes an angle  $\beta$  with the fixed wall due to spring.

So it has the component  $ie \sin\beta$ .

Due to the mass of bob, so it has two components of the restoring force that are  $(-mg \sin\alpha)$  &  $(-mg \sin\beta)$ .

So for bob A, the restoring force is  $-mg \sin\alpha = ie \sin\alpha$  or  $(-\frac{mg}{k})$ .

Similarly, for bob B, the restoring force is  $-mg \sin\beta$ , i.e. will be  $-\frac{mg}{k}$ .

24 So the eqn of motion can be written as

$$m \frac{d^2x}{dt^2} = -k(x-y) - mg \frac{y}{l}$$

$$\& m \frac{d^2y}{dt^2} = -k(y-x) - mg \frac{x}{l}$$

$$\Rightarrow m \frac{d^2x}{dt^2} + k(x-y) + mg \frac{y}{l} = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}(x-y) + \frac{g}{l}y = 0 \quad \text{--- (1)}$$

Similarly,  $\frac{d^2y}{dt^2} + \frac{k}{m}(y-x) + \frac{g}{l}x = 0 \quad \text{--- (2)}$

Then these two eqns. are known as  
coupled eqns.

Again putting  $\omega_1^2 = g/l$

So the eqns can be written as,

$$\frac{d^2x}{dt^2} + \frac{k}{m}(x-y) + \omega_1^2 x = 0 \quad \text{--- (3)}$$

Similarly,  $\frac{d^2y}{dt^2} + \frac{k}{m}(y-x) + \omega_1^2 y = 0 \quad \text{--- (4)}$

for finding the solution of the

two differential eqn.

We have to adding & subtracting

eqn (3) & (4) respectively.

$$\Rightarrow \frac{d^2}{dt^2}(x+y) + \omega_1^2(x+y) = 0 \quad \text{--- (5)}$$

$$\Rightarrow \frac{d^2}{dt^2}(x-y) + \frac{2k}{m}(x-y) + \omega_1^2(x-y) = 0 \quad \text{--- (6)}$$

$$\Rightarrow \frac{d^2}{dt^2}(m\ddot{y}) + \left(\frac{2k}{m} + \omega_1^2\right)(m\ddot{y}) = 0$$

Again replacing  $(m-y) = Q_2$   
 $m(Q_2) = Q_1$

Putting in above eqn ⑤ & ⑥, then  
 the eqns are,

$$\frac{d^2}{dt^2} Q_1 + \omega_1^2 Q_1 = 0 \quad \text{--- ⑦}$$

Similarly  $\frac{d^2}{dt^2} Q_2 + \frac{2k}{m} Q_2 + \omega_2^2 Q_2 = 0 \quad \text{--- ⑧}$

$$\Rightarrow \frac{d^2}{dt^2} Q_2 + \left(\frac{2k}{m} + \omega_2^2\right) Q_2 = 0$$

$$\Rightarrow \frac{d^2}{dt^2} Q_2 + \omega_2^2 Q_2 = 0 \quad \text{--- ⑨}$$

$$\Rightarrow \text{where } (\omega_2^2 = \frac{2k}{m} + \omega_1^2)$$

These eqns are called De-coupled eqns  
 and  $Q_1$  &  $Q_2$  are called De-coupled constant.

### Normal Co-ordinates! —

The co-ordinates ~~Q<sub>1</sub> & Q<sub>2</sub>~~ and ~~Q<sub>1</sub> + Q<sub>2</sub>~~

$Q_1 = m\ddot{y}$  &  $Q_2 = m\ddot{y}$   
 are called normal co-ordinate of  
 the coupled system.

The normal co-ordinate are the linear  
 combination of the variable ~~m~~  $m\ddot{y}$

so hence the oscillation which describe  
in terms of the normal co-ordinates  
are independent & are called  
Normal modes of oscillation.

### Normal Modes of Frequency:

$$\omega_1^2 = g/l$$

$$\Rightarrow \omega_1 = \sqrt{g/l}$$

$$g\omega_2^2 = \omega_r^2 + \frac{2K}{m}$$

$$= g/l + \frac{2K}{m}$$

$$\Rightarrow \omega_2 = \sqrt{g/l + 2K/m}$$

Now we have frequencies are

$$\boxed{\omega_1 = \frac{\omega_1}{2\pi} = \frac{\sqrt{g/l}}{2\pi}}$$

$$\boxed{\omega_2 = \frac{\omega_2}{2\pi} = \frac{1}{2\pi} \sqrt{g/l + \frac{2K}{m}}}$$

### Normal modes of oscillation:

The system oscillating with normal mode  
of frequency which have the normal  
co-ordinates that are

$$Q_1 = Mx + y$$

$$Q_2 = My - x$$

having the external periodic force act  
on the system then this is the phenomena  
of normal modes of oscillation.

If the system is initially disturbed in such a way that only one normal mode is excited then the oscillation takes place for the certain interval of time with a particular mode of frequency, then this is the case of normal modes of oscillation.

### $\varphi_1$ mode or In-Phase mode:

By taking any one of the normal co-ordinates then we have to write it as  $m_y = 0$ , so we get  $m = y$ .

So this is the condition for which the initial displacement & the final displacement must be same & are called as in-phase mode.

### $\varphi_2$ mode or Out phase mode:

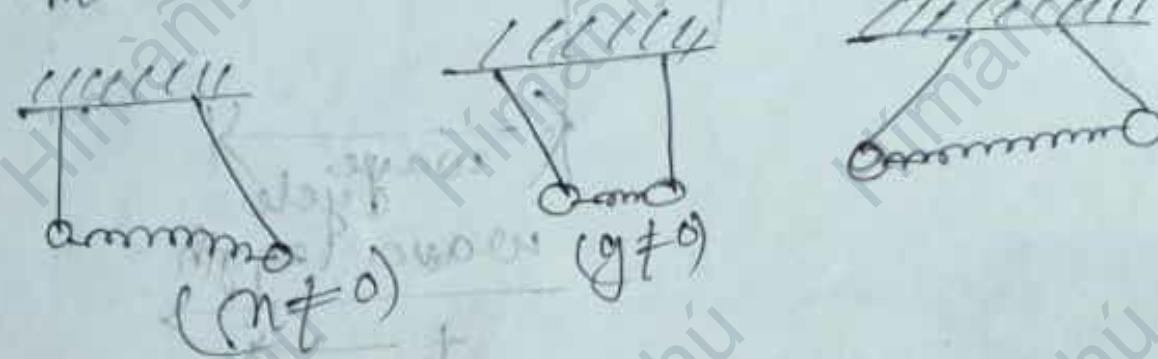
By taking any one of the normal co-ordinates then we have to write,  $m_y \neq 0$  so we get  $m = -y$ .

So this is the condition for which the displacement of the bar A must be different in

magnitude & direction w.r.t. the displacement of the other bob.

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This is the condition for which the oscillation takes place in discriminant manner and are called as out phase mode.

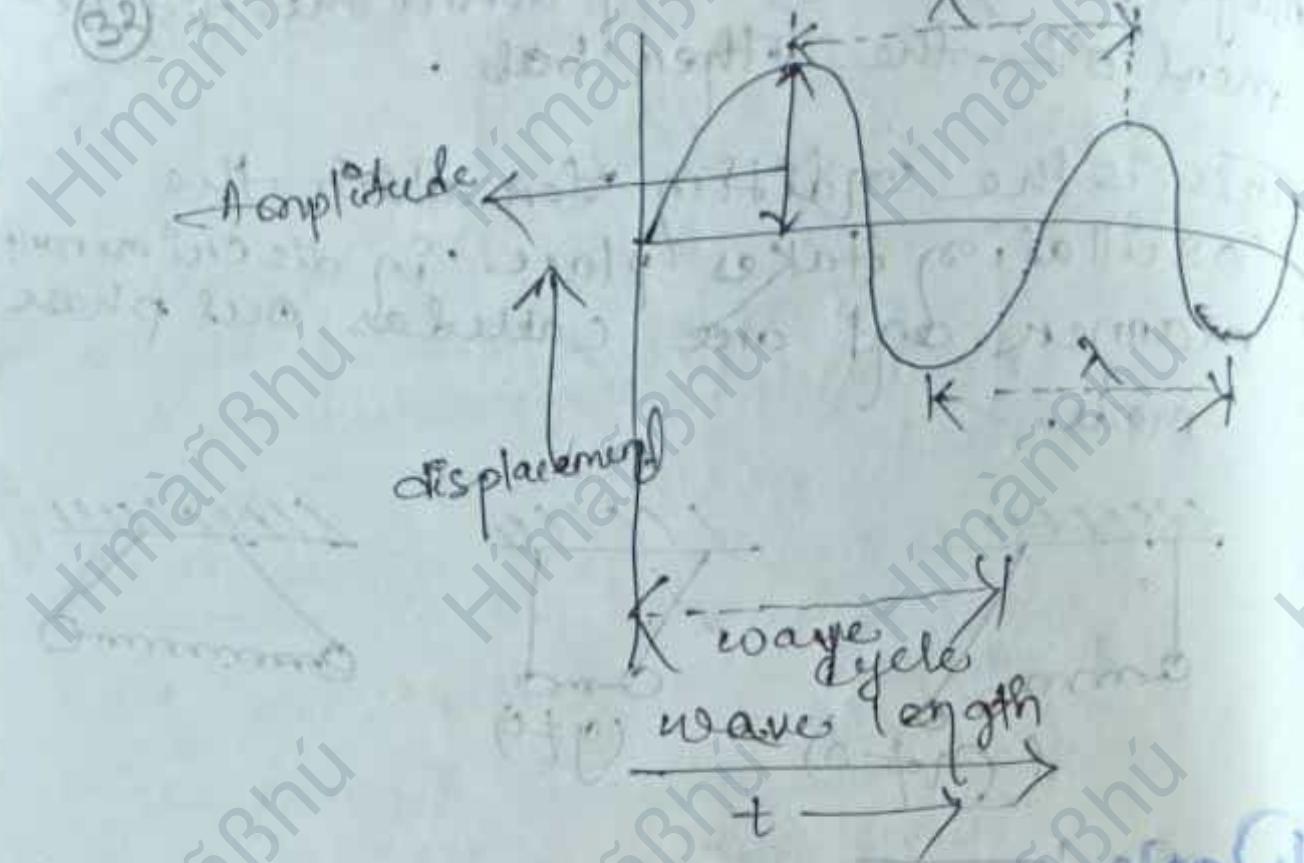


## Wave:

It is the characteristic property of the particle which can help to change the medium vibration to transfer the energy from one medium to the another medium.

or  
It is force due to the disturbance of the particle in the medium.

- \* It involves the transfer of energy from 1 point to another but not from 1 point to the system.
- \* Transfer the matter of the way are of two types, either it makes compression or rarefaction or it makes crest or trough in its path.



### Wavelength!

It is the length b/w two consecutive crest or trough.

### Amplitude!

It is the maxm displacement from the mean axis to its peak point in a wave.

### Wave cycle!

It is the complete way which can complete one crest & trough during the propagation of the wave.

### Time period.

It is defined as the time taken to complete one oscillation.

frequency!

(33)

It is defined as the no. of oscillations in one sec. or It is defined as reciprocal of time period.

$$n = \frac{1}{T}$$

Its unit is Hz.

\*  $10\text{Hz} = 10$  oscillations/sec.

## Types of Waves:

Generally, there are two types of wave:-

① Longitudinal wave

② Transverse wave.

### ① Longitudinal wave:

If the particle vibration is parallel to the direction of propagation then this type of wave is longitudinal wave.

Eg:- sound wave.

### ② Transverse wave:

If the particle vibration is perpendicular to the direction of propagation then this type of wave is Transverse wave.

Transverse wave.

Eg - Light wave & electromagnetic wave.

## Velocity of wave

Velocity =  $\frac{\text{displacement}}{\text{time}}$

In longitudinal wave

In the net,  $[v = \sqrt{\frac{k}{\rho}}]$

where  $v$  = Longitudinal velocity of longitudinal wave

$k$  = bulk modulus of the medium

$\rho$  = Density of the medium.

In transverse wave

The velocity,  $[v = \sqrt{\frac{k}{\rho}}]$

$v$  = Velocity of transverse wave

$k$  = Rigidity of the medium.

$\rho$  = Density of the wave.

## Wave eqn! —

We have the expression for the wave can be written as,

$$\Psi(x, t) = A \sin(kx - \omega t + \phi) \quad (1)$$

Where  $A$  = Amplitude factor.

$k$  = Propagation vector.

$\omega$  = angular frequency.

$T$  = Time period / Time varif.  
 $\phi$  = constant or additional constant.

Now differentiate the eqn w.r.t  $x$ ,

$$\frac{d\psi}{dx} = \frac{d}{dx} A \sin(Kx - wt + \alpha)$$

$$= A \cos(Kx - wt + \alpha) \cdot K.$$

Again differentiate the eqn w.r.t  $t$ .

$$\frac{d\psi}{dt} = A \cos(Kx - wt + \alpha) \cdot (-\omega)$$

$$= -A \omega \cos(Kx - wt + \alpha).$$

Similarly the 2nd derivative in both the cases can be written as

$$\frac{d^2\psi}{dx^2} = \frac{d}{dx} [ -A \cos(Kx - wt + \alpha) \cdot K ]$$

$$= -A^2 \sin(Kx - wt + \alpha) \cdot K$$

$$= -K^2 \psi(x, t) \quad \text{--- (ii)}$$

$$\frac{d^2\psi}{dt^2} = A \omega^2 \sin(Kx - wt + \alpha)$$

$$= -\omega^2 \psi(x, t) \quad \text{--- (iii)}$$

Now taking the ratio of eqn (ii),

$$\frac{\frac{d^2\psi}{dx^2}}{\frac{d^2\psi}{dt^2}} = \frac{-K^2 \sin(Kx - wt + \alpha) \cdot \psi(x, t)}{-\omega^2 \psi(x, t)}$$

$$\Rightarrow \left[ \frac{d^2\psi}{dx^2} = \frac{K^2}{\omega^2} \frac{d^2\psi}{dt^2} \right]$$

This is the differential eqn of the wave form and which can be written as, (in reverse form)

$$\frac{d^2\varphi}{dt^2} = \frac{\omega^2}{K^2} \frac{d^2\varphi}{dx^2}$$

we have,

$$\begin{aligned} \frac{\omega^2}{K^2} &= v^2 \\ \Rightarrow \frac{d^2\varphi}{dt^2} &= v^2 \frac{d^2\varphi}{dx^2} \end{aligned}$$

Reflection & Transmission Co-efficiency on Dunes!

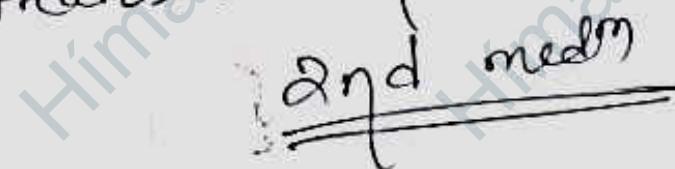
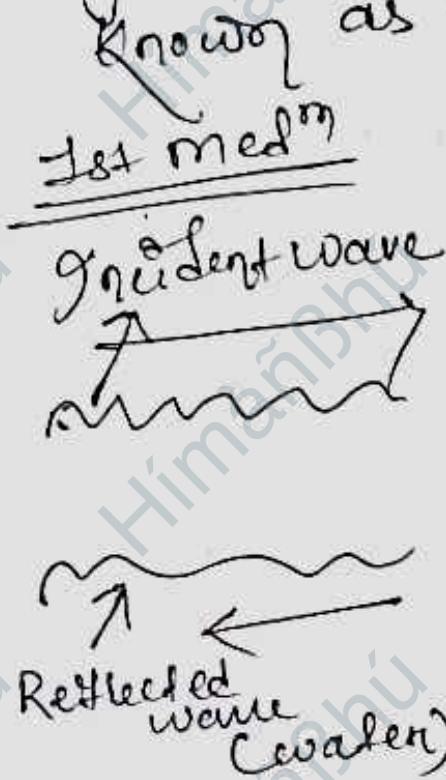
Reflected Wave

When the initial wave travelling from one medium to the another medium and if the wave refract from the transmission layer or interface layer. Then the amplitude of the wave will reduce or decreases from the initial wave then this wave is called "Reflection Wave".

## Transmissing Wave!

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when the initial wave travelling from one medium to another medium and if the wave passes to the another medium by the transmission layer then that type of wave is known as transmission wave.



where a wave travelling through a given medium to the another medium, then a part is transmitted to the second medium & the part is reflected to the 1st medium.

→ Some of the characteristics of reflected or transmitted wave are different from those incident wave. So the waves are different from those incident wave which reflected

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back is known as reflected wave & the wave which transmitted in another medium / second medium is known as transmitted wave.

The following features which observed from the reflected & transmitted wave are as follows! —

- (1) The amplitude of both the reflected & transmitted wave are less than that of the amplitude of incident wave.
- (2) The speed of the reflected wave is same as that of the incident wave but the speed of the transmitted wave is slightly greater than or

less than  $\sqrt{\frac{T}{m}}$  from the incident wave depending on the medium.

(3) The expression for velocity for the reflected wave & transmitted wave can be written as  $V = \sqrt{T/m}$ .

$T$  = Tension

$m$  = mass of the body.

(4) The frequency of reflected and the transmitted wave are same that of the incident wave.

(5) The expression for frequency is  $\nu = V/\lambda$

$V$  = Velocity of wave

$\lambda$  = wavelength.

(6) The wavelength of the reflected wave is same that of the incident wave but the wave length of the transmitted wave is reduced from the incident wave.

(7) The reflected wave poles is inverted with respect to the incident wave poles. but the transmission wave can not be inverted.

### Reflection Co-efficient

$R$  is defined as the ratio of intensity of reflected wave to the intensity of incident wave.