

I INTERFERENCE

The distribution of energy is uniform due to a single source but when there are two or more sources giving out waves of same amplitude and equal phase difference, then the distribution of energy is not uniform.

There are some points where the crest of one wave falls on the crest of other wave, resulting in increase of the amplitude of the resultant wave and thereby maximising the wave energy.

On the other hand, there are certain points where the crest of one wave falls on the trough of the other wave or vice versa, resulting in the decrease of the amplitude of the resultant wave and thereby minimising the wave energy.

Thus, in this phenomenon there is a redistribution of the energy.

The phenomenon of redistribution of energy in a medium due to the superposition of waves of same frequency, same amplitudes, travelling in a medium in the same direction and having a constant phase difference between them is known as interference.

The difficulty in observing the interference of light was due to the small value of the wavelength of light waves i.e. $\approx 10^{-3}$ mm.

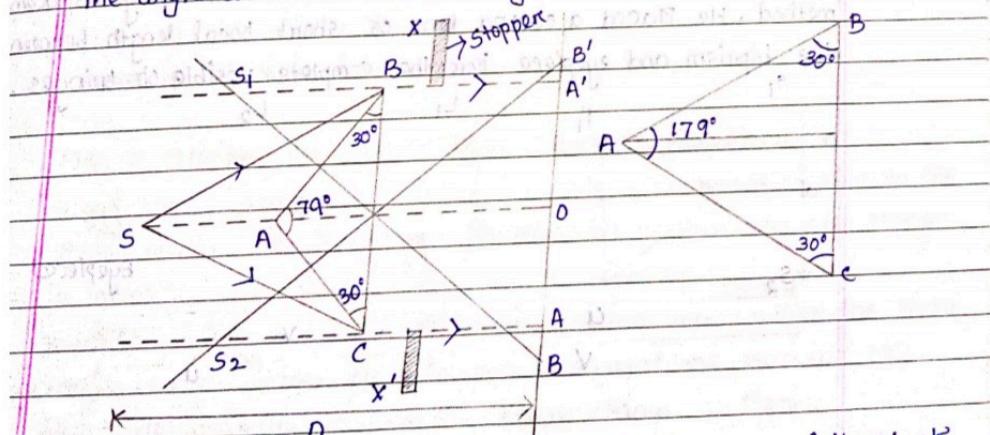
Thomas Young was the first physicist who observed the interference of light with two slits and a source of light.

FRESNEL'S BIPRISM :-

This is the special type of biprism which produces more clearly dark and bright fringes on the screen.

The Fresnel Biprism consists of two thin prisms joined at their base to produce isosceles triangle.

The angle at the prism is very small which is equal to 30° .



From the above fig 'S' is the monochromatic source in front of the biprism.

Now when the light falls from the source 'S' on the two portions AB and AC of the biprism it appears to form two virtual sources S_1 and S_2 as shown in fig, emit light waves which interfere and produce light and dark fringes.

Here D = Distance between source and screen.

d = Distance between two sources S_1 and S_2

λ = wavelength of light.

Scanned with CamScanner



Page No. 60
Date

The expression for fringe width will be

$$B = \frac{2D}{d}$$

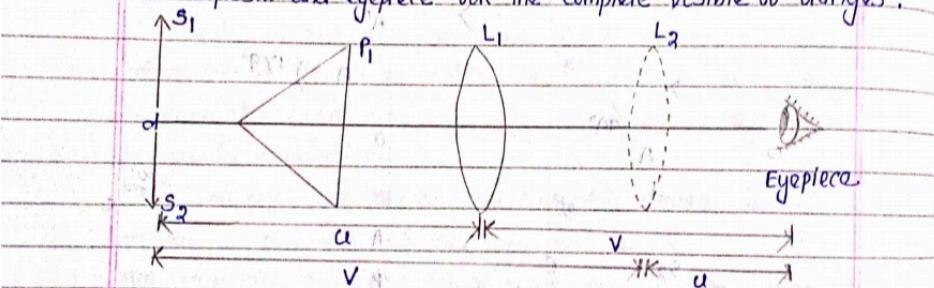
Since the two slits are purely virtual, then the problem arises that it becomes difficult to measure the distance between the slit d .

d is measured by placing a converging lens between the biprism and the screen, so that real images of the virtual slits are formed on the screen.

In this method we can determine d by using converging lenses.

DETERMINATION OF d' : -

Scientist Glazebrook measured the distance d' using displacement method. He placed a convex lens of short focal length between the biprism and eyepiece for the complete visible of fringes.



From the above fig ABC in the biprism

d = Distance between two slits

D = Distance between the source and screen

Consider a convex lens is placed very close to the biprism and the eyepiece is adjusted to be more than four times of the focal length of the convex lens. Then a magnified image of S_1 and S_2 produced as shown in fig.

Let d_1 is the distance between S_1 and S_2 when the convex lens is very close to the biprism and d_2 is the distance between S_1 and S_2 when the convex lens is far away

Scanned with CamScanner

Page No. 61

Date

Brom the biprism.

For the first position of lens L_1 , we have

$$\frac{v}{u} = \frac{\text{size of image}}{\text{size of object}} = \frac{d_1}{d} \quad \text{①}$$

For the second position of lens L_2 , we have

$$\frac{v}{u} = \frac{d_2}{d} \quad \text{②}$$

Multiplying eqn ① and eqn ②, we get

$$\frac{v}{u} \times \frac{v}{u} = \frac{d_1}{d} \times \frac{d_2}{d}$$

$$\Rightarrow d^2 = d_1 d_2$$

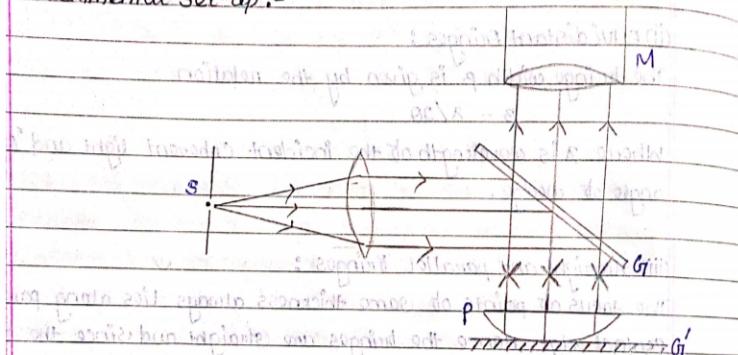
$$\Rightarrow d = \sqrt{d_1 d_2}$$

where d = distance between two sources.

NEWTON'S RINGS :-

A phenomenon in which an interference pattern is obtained by the reflection of light between two surfaces, a spherical surface and an adjacent touching flat surface is known as Newton's ring.

Experimental set-up :-



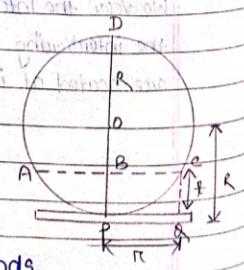
The light source from source 'S' form a parallel beam after passing through convex lens 'L'. Then the parallel beam of light is incident on a glass plate 'G' which make an angle 45° to the beam. After reflection, the beam is observed using a microscope 'M'.

The interference pattern is observed using a microscope 'M'. The cause of interference is the reflection of the beam from the lower surface of the lens and upper surface of the plate.

THEORY : Consider any point 'P' on the plate. Let the radius of curvature of the curved surface of the lens be 'R'.

Let us take a point 'Q' on the plate such that the thickness 't' of the film at it be 't'. All the points which corresponds

Scanned with CamScanner



to thickness 't' will lie on a circle with 'P' as its center and radius 'r'. As their centre and radius 'r'.

$$AB \times BC = PAB \times BD = PB(PD - PB)$$

$$AB = BC = R$$

$$PB = t \text{ and } OP = R$$

$$r \times r = t(2R - t)$$

If $R \gg t$, then on neglecting the thickness 't' we get

$$r^2 = 2Rt$$

$$t = \frac{r^2}{2R}$$

Newton's Ring by Reflected light :-

We know that the dark fringes are obtained in the interference pattern produced by the reflected rays from normal incidence as per the condition.

$$2nt = na \text{ where } n=0, 1, 2, 3, \dots \text{ cm radius}$$

And for the bright fringes the condition is

$$2nt = (2n+1) \frac{\lambda}{2}$$

If R_n be the radius of the n^{th} order interference fringe, then the

We know that the dark fringes are obtained in the interference pattern produced by the reflected light for normal incidence as per the condition.

$2nt = na$ where $n=0, 1, 2, 3, \dots$

And for the bright fringes the condition is

$$2nt = (2n+1) \frac{\lambda}{2}$$

If R_n be the radius of the n^{th} order interference fringe, then the thickness 't' corresponding to radius r_n becomes

$$t = \frac{r_n}{2R}$$

Substituting the above value of 't' in the condition for dark fringes

$$\frac{2nr_n^2}{2R} = na$$

$$\Rightarrow r_n = \sqrt{n\lambda R}$$

For air refractive index $M=1$ so

$$r_n = \sqrt{n\lambda R}$$

Scanned with CamScanner

Now the diameter 'D' of the n^{th} dark fringes

$$D_n = 2r_n = 2\sqrt{n\lambda R}$$

Similarly the diameter 'D' of the n^{th} bright fringe can be calculated as

$$D_n = 2\sqrt{(n+1)\lambda R}$$

MEASUREMENT OF WAVELENGTH:

For the dark fringes, the diameter 'D' of n^{th} given by

$$D_n = 2\sqrt{n\lambda R} \quad (\text{i})$$

For the m^{th} dark rings

$$D_m = 2\sqrt{m\lambda R} \quad (\text{ii})$$

Squaring and subtracting eqn (i) from eqn (ii)

$$D_m^2 - D_n^2 = 4(m-n)\lambda R$$

$$\Rightarrow \lambda = \frac{D_m^2 - D_n^2}{4(m-n)R}$$

Newton's rings are observed by the experimental setup.

The rings are focussed on the cross wire of the eyepiece.

The cross wire is made to focus on any dark ring and the reading is taken from the microscope.

Again the microscope is shifted to the right side and reading of the microscope is again taken for each circular dark rings.

The diameter of the dark rings are calculated and the graph is plotted between D^2 along Y-axis and the no. of rings on X-axis.

The slope of the straight line represents the value $\frac{D_m^2 - D_n^2}{(m-n)R}$. Hence the wavelength λ of sodium light is determined.

Scanned with CamScanner

The slope of the straight line represents the value $\frac{D_m^2 - D_n^2}{(m-n)}$.
Hence the wavelength λ of sodium light is determined!

Scanned with CamScanner

2011-2012
Date:

Page No. 111
Date:

MEASUREMENT OF REFRACTIVE INDEX:-

Any transparent liquid whose refractive index 'M' is to be measured is inserted between the lens lens and the glass plate of Newton's rings set up. The diameter of the n^{th} dark rings is measured using the relation

$$D_n^2 = \frac{4n\lambda R}{M} \quad (i)$$

Similarly for m^{th} dark rings

$$D_m^2 = \frac{4m\lambda R}{M} \quad (ii)$$

Subtracting eqn (i) from eqn (ii)

$$(D_m^2 - D_n^2)_{\text{liquid}} = \frac{4(m-n)\lambda R}{M} \quad (iii)$$

Now repeat the experiment again with liquid being replaced by air in between the lens and glass plate of the Newton's rings set-up. For air film $M=1$, so that

$$(D_m^2 - D_n^2)_{\text{air}} = 4(m-n)\lambda R \quad (iv)$$

Dividing eqn (iv) by eqn (iii), we have

$$\frac{M}{(D_m^2 - D_n^2)_{\text{air}}} = \frac{(D_m^2 - D_n^2)_{\text{liquid}}}{(D_m^2 - D_n^2)_{\text{air}}}$$

Hence the refractive index of the transparent liquid can be measured, once we know the diameter of Newton's rings with liquid film and then using air film.

$$R = C/R + D \Rightarrow R = C/R + D$$

ELECTROMAGNETIC NATURE OF LIGHT :-

The electromagnetic theory of light was first given by Maxwell. According to this theory light travels in the form of electromagnetic waves in which electric and magnetic field vectors are mutually perpendicular to each other and both of them are perpendicular to the direction of propagation of the wave.

No medium is required for these waves and the velocity of these waves depends upon the electric and magnetic properties of medium.

The expression for the velocity of these waves can be written as

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Where, μ_0 = magnetic permeability of free space

ϵ_0 = Magnetic Susceptibility of free space.

For any medium the expression for velocity can be written as

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$

Where, μ and ϵ are called magnetic and electric Permittivity of any medium.

* The electromagnetic theory of light was successful in explaining the optical phenomena like reflection, refraction, diffraction and polarisation but fail to explain the phenomena like photoelectric emission, Compton effect, scattering phenomena, absorption, emission of radiation etc.

After that Huygen and some other scientists given the proper explanation about the wave characteristics of light.

WAVE FRONT:-

According to wave theory of light a source of light sends out disturbance in all the direction in a homogeneous medium. The disturbance reaches all those particles of medium such that the phase of disturbance must same. Hence at any instant all such particles must be vibrating in phase with each other.

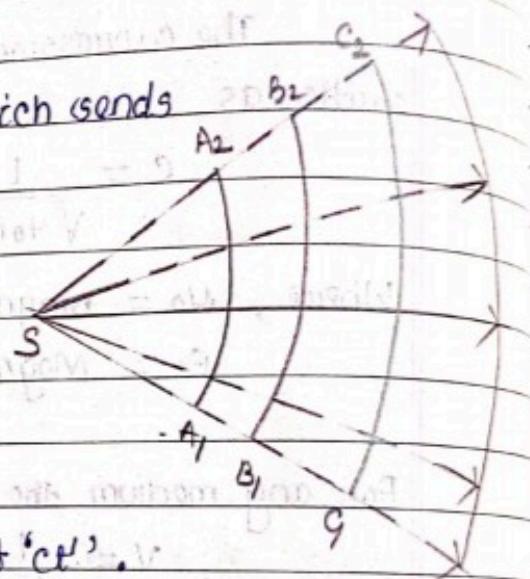
So the locus of all the particles of the medium such that they are vibrating the same phase is called the wave front.

Here 'S' is the source of light which sends out waves in all directions in a homogeneous isotropic medium.

Here the velocity of light is c .

After time 't' the wave reach the point at a distance of ct .

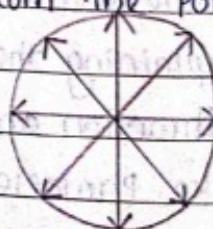
Similarly after time ' t' ' the wave reaches the point at a distance of ct' .



TYPES OF WAVE FRONT:-

1) SPHERICAL WAVE FRONT:-

A spherical wave front is produced by a point source of light because the locus of such points are equidistance from the point source as shown in fig.



2) CYLINDRICAL WAVE FRONT:-

A cylindrical wave front is produced by the linear source of light because the locus of all such points are equidistance from a linear source of light.

3) PLANE WAVE FRONT:-

A small part of a spherical or cylindrical wave front such that the locus of all the points must exist in the plane is known as Plane wave front.

* NOTE :-

The wave front representing a parallel beam is a Plane wavefront while the wavefront representing a converging and diverging beam of light is a spherical wave front.

HUYGEN'S PRINCIPLE:-

According to Huygen's wave theory of light each point source is a centre of disturbance from which the waves are spread out in all direction setting the other particles vibration.

Here the locus of all the other particles in all the directions having the same phase is called wavefront.

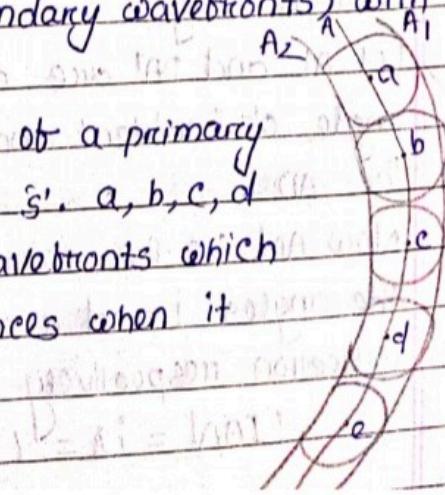
According to Huygen's principle

1) Every point in a given wavefront act as a source of a new disturbance called secondary wavelets.

2) The secondary wavelets spread out in all direction in medium with the velocity of light.

3) The envelope of these wavelets in the former direction give the new wavefront, (secondary wavefronts) with that instant.

Hence 'AB' represents the section of a primary wavefront due to the point source 'S'. a, b, c, d are the points of the primary wavefronts which would be the secondary disturbances when it travels with a velocity 'c'.



FRESNEL DIFFRACTION

The Phenomenon of bending of light around the corners of an obstacle and enrichment of light into the regions of geometrical shadow is known as diffraction.

Fresnel made the following assumptions to explain the phenomenon of diffraction:

- 1) A wave front can be split up into a large number strips called Fresnel strips. These strips or zones are of very small area. The area is so small such that each point on the wavefront acts as a fresh source of secondary waves in all direction.
- 2) The secondary waves emanate from each element of wavefront continuously.
- 3) The resultant amplitude at any point is obtained by the superposition of all the secondary waves reaching from various Fresnel strips or zones.
- 4) The effect at any point due to a particular Fresnel zone depends on :
 - a) The distance of the point from the zone
 - b) The inclination of the with reference to the concerned zone
 - c) Area of the zone.

FRESNEL'S HALF PERIOD ZONES FOR PLANE WAVE:-

Consider a plane wave ABCD perpendicular to the direction of propagation wave.

Let 'S' be the source emitting plane wave front and wavelength λ . Let 'P' be the observational point, where the waves have phase difference due to following reasons -

- (i) Obliqueness of the waves reaching the point 'P' from various points of the wavefront has different values.
- (ii) Different distances are travelled by waves from different points.

To determine the resultant effect at point 'P' due to entire wavefront, which is at a distance 'd' horizontally away from the wavefront.

The entire wavefront is divided into elementary regions called zones with point 'O' as the centre.

Concentric imaginary spheres on the wavefront with point 'O' as centre and having radii of various values.

$OM_1 = d + \frac{\lambda}{2}$, $OM_2 = (d + \lambda)$, $OM_3 = (d + \frac{3\lambda}{2})$ and so on.

so that wavefront is cut into concentric circles of radii OM_1, OM_2, OM_3 and so on.

The area between two spheres is called half period zone. The point 'O' is called the pole of the wavefront with respect to Point 'P'.

Since the waves from the corresponding points of the adjacent areas differ in path by half wavelength, the annular rings in the wavefront are called half period zones or Fresnel zones.

In Fresnel zones the waves reach the points with a time delay of half time period.

The point 'O' is known as the pole of the wavefront with respect to the point 'P'.

RADIUS OF HALF PERIOD ZONE:

The radius of the first half period zone

$$OM_1 = \sqrt{PM_1^2 - PO^2} = \sqrt{(d + \frac{\lambda}{2})^2 - d^2}$$

$$= \sqrt{d^2 + \frac{\lambda^2}{4} + 2d\frac{\lambda}{2} - d^2} = \sqrt{\frac{\lambda^2}{4} + \lambda^2} = \sqrt{\frac{5\lambda^2}{4}} = \frac{\sqrt{5}\lambda}{2}$$

$$= \frac{\sqrt{5}\lambda}{2} = \sqrt{5}\lambda \quad (\because \frac{\lambda^2}{4} \text{ is negligibly very small})$$

Scanned with CamScanner

$$\begin{aligned} &= \left[d^2 + \frac{\lambda^2}{4} + 2d\frac{\lambda}{2} - d^2 \right]^{\frac{1}{2}} \\ &= \left[d^2 + \frac{\lambda^2}{4} + \lambda^2 - d^2 \right]^{\frac{1}{2}} \\ &= \left[\frac{\lambda^2}{4} + \lambda^2 \right]^{\frac{1}{2}} \\ &= \left[\frac{5\lambda^2}{4} \right]^{\frac{1}{2}} = \frac{\sqrt{5}\lambda}{2} \end{aligned}$$

similarly the radius of second half period zone is given by

$$\begin{aligned} OM_2 &= \sqrt{(d + \lambda)^2 - d^2} \\ &= \sqrt{d^2 + \lambda^2 + 2d\lambda - d^2} \\ &= \sqrt{\lambda^2 + 2d\lambda} \\ &= \sqrt{2d\lambda} \end{aligned}$$

The radius of n^{th} half period zone

$$OM_n = \sqrt{(d + n\lambda)^2 - d^2}$$

$$= \sqrt{n\lambda}$$

Hence, the radius of half Period zone is Proportional to the square roots of natural numbers.

$$= \sqrt{d^2 + \lambda^2 + 2d\lambda - d^2}$$

$$= \sqrt{\lambda^2 + 2d\lambda}$$

$$= \sqrt{2d\lambda}$$

The radius of n^{th} half Period zone

$$OM_n = \sqrt{\left(\frac{d + n\lambda}{2}\right)^2 - d^2}$$

$$= \sqrt{n\lambda\lambda}$$

Hence, the radius of half Period zone is proportional to the square roots of natural numbers.

AREA OF HALF PERIOD ZONES :-

The area of n^{th} zone is given by

$$\text{Area of } n^{th} \text{ zone} = \text{Area of first } n \text{ zones} - \text{Area of } (n-1) \text{ zones}$$

$$= \pi(OM_n)^2 - \pi(OM_{n-1})^2$$

$$= \pi[d\lambda] - \pi[(n-1)\lambda]$$

$$= \pi[n\lambda] - \pi[(n-1)\lambda]$$

$$= \pi\lambda$$

Hence the area of n^{th} zone is independent of the number of zone and so the area of each and every zone is constant.

Scanned with CamScanner

DISTANCE OF POINT 'P' FROM HALF PERIOD ZONE :-

The distance of point 'P' from half period zone, is given by

$$\frac{(d + n\lambda) + (d + (n-1)\lambda)}{2}$$

$$= \frac{d + n\lambda + d + (n-1)\lambda}{2} = \frac{2d + 2n\lambda}{2}$$

$$= \frac{2d + 2n\lambda - \lambda}{2}$$

$$= \frac{2d + n\lambda}{2}$$

$$= \frac{4d + 2n\lambda - \lambda}{4}$$

$$= d + \frac{(2n-1)\lambda}{4}$$

AMPLITUDE OF THE WAVELET AT POINT P :-

The amplitude at point 'P' due to one zone depends on the following factors:

(i) is directly proportional to the area of zone.

(ii) the amplitude falls off inversely as the square of the distance of the zone from Point 'P'.

(iii) The amplitude depends upon the obliquity of each zone.

obliquity is the angle between the normal to the wavefront and the line joining the Point 'P' to the zone.

Mathematically, amplitude is measured as directly proportional to $\sqrt{2}(1 + \cos\theta)$.

If R_n is the amplitude at Point 'P' due to the zone, then

amplitude in $R_n = \text{Area of zone} \times \text{obliquity factor}$

distance of point P from zone

$$= \frac{\pi\lambda d \times \frac{1}{2}(1 + \cos\theta)}{d + \frac{(2n-1)\lambda}{4}}$$

Scanned with CamScanner

$$= \frac{1}{2} \pi R \lambda (1 + \cos \theta)$$

$R_n = \pi R (1 + \cos \frac{\theta}{n})$

With increase in the value of n , the factor $\cos \frac{\theta}{n}$ will decrease so amplitude R_n will decrease.

RESULTANT AMPLITUDE OF POINT 'P' DUE TO ENTIRE WAVELENGTH :-

Since the path difference between two consecutive zones is $\lambda/2$, so light from half period zones differ by an angle of π .

If R_1, R_2, R_3 and so on be the amplitude at point 'P' from various zones the resultant amplitude at point 'P' is given by

$$R = R_1 - R_2 + R_3 - R_4 + \dots = (-1)^{n-1} R_n$$

$$R_2 = R_1 + R_3; R_4 = \frac{R_3 + R_5}{2}$$

$$\text{Hence } R = \frac{R_1 + R_3}{2} + \frac{R_3 + R_5}{2} \dots \text{ (for odd values of } n\text{)}$$

$$\text{and } R = \frac{R_1 + R_3}{2} - \frac{R_3 + R_5}{2} \dots \text{ (for even values of } n\text{)}$$

For large values of n , both R_{n-1} and $\frac{R_n}{2}$ is neglected.

Hence resultant amplitude at point 'P' due to entire wavelength.

$$R = \frac{R_1}{2} - \frac{R_3}{2} \dots \text{ (Amplitude of first half period zone)}$$

INTENSITY AT POINT 'P' DUE TO ENTIRE WAVE FRONT :-

Since the intensity 'I' is directly proportional to the square of the amplitude, hence

$$I \propto R^2$$

$$I = R^2$$

$$\text{Hence } I = \left(\frac{R_1}{2}\right)^2 = \frac{I_1}{4}$$

Thus the intensity at point 'P' due to is equal to $\frac{1}{4}$ of intensity due to half period zones.

Scanned with CamScanner

THEORY OF A ZONE PLATE :-

An optical device that works on the principle of Fresnel's zone is called zone plate. It obstructs the light from the alternate half period zones. Although the role of zone plates is to focus light by using diffraction phenomenon.

PRINCIPLE :-

As per the theory of Fresnel, the radii of various half period zones are directly proportional to the square root of natural numbers i.e. $\sqrt{1}, \sqrt{2}, \sqrt{3}$ etc.

CONSTRUCTION :-

A Plane thin plate of glass forms a zone plate. Concentric circles having radii proportional to the square root of $k\sqrt{1}, k\sqrt{2}, k\sqrt{3}$ and so on are drawn on the plate.

Here 'k' is any arbitrary constant equal to $\sqrt{\lambda d}$, where 'd' is the distance from the centre and λ is the wavelength of light.

The space enclosed between the circles corresponds to Fresnel's half period zones.

The spaces or zones are made opaque alternately i.e. either even or odd. The method to construct a zone plate generally

numbers i.e. $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$ etc.

CONSTRUCTION :- Going to sketching construction with pencil

A Plane thin plate of glass forms a zone plate. Concentric circles having radii proportional to the square root of $k\sqrt{1}$, $k\sqrt{2}$, $k\sqrt{3}$ and so on are drawn on the plate.

Here 'k' is any arbitrary constant equal to $\sqrt{n}\lambda$, where 'n' is the distance from the centre and λ is the wavelength of light.

The space enclosed between the circles corresponds to Fresnel's half period zones.

The spaces or zones are made opaque alternately i.e. either even or odd. The method to construct a zone plate generally involves the following two ways:

(i) A number of concentric circles are drawn on a piece of stiff white card board with the condition that their radii are proportional to the square root of natural numbers. Then, we blacken the alternate zones and the photograph is taken.

(ii) second method to construct a zone plate is to photograph Newton's rings in normal reflected light. In this type of zone plate, the area of dark and bright rings are equal when the incident light is parallel and is coming from a

Scanned with CamScanner

monoChromatic zones. source.

POSITIVE ZONE PLATE :-

A positive zone plate is one in which the central zone is transparent.

This plate contains all odd zones as transparent and all even zones as opaque.

The resultant amplitude at a point to which these zones act as half period zones will be the algebraic sum of amplitudes R_1, R_2, R_3 i.e. $R_1 + R_2 + R_3$.

NEGATIVE ZONE PLATE :-

A negative zone plate is one in which the central zone is opaque.

This plate contains all even zones are transparent and all odd zones are opaque.

The resultant amplitude at point will be the algebraic difference of amplitudes $R_1, R_2, R_3 \dots$ i.e. $R_1 - R_2 - R_3 - R_4 \dots$

ANALYTICAL TREATMENT :-

Suppose there is no zone plate and all the zones of the plate are transparent, then resultant amplitude 'R' of the light reaching at a certain point is given by $R = R_1 - R_2 + R_3 - R_4 + \dots + R_n$ where R_1, R_2, \dots, R_n are amplitudes contributed by first, second, third \dots n^{th} half period zones.

For large value of n

Resultant amplitude $R = R_1/2$

since the intensity 'I' is directly proportional to the square of the amplitude,



For a positive zone plate, the resultant amplitude 'R' is given by

$$R = R_1 + R_2 + R_3 + \dots + R_n$$

$$= n \cdot (R_1)$$

(Where n is total no. of zones)

The intensity 'I' of positive zone plate becomes

$$I' \propto n^2 (R_1)^2$$

Hence the intensity I' of positive zone plate is n^2 times the intensity I .

Similarly for negative zone plates the resultant intensity will be n^2 times the intensity when all the zones are exposed to light.

WHEN INCIDENT WAVEFRONT IS PLANE:

The incident wavefront is plane when the parallel rays from distant source are allowed to be made incident.

Various zones acting as half period zones corresponding to the point 'P' which is placed at a distance 'b' from it, the radii of various zones are

$$r_1 = \sqrt{na^2}$$

$$r_2 = \sqrt{(n+1)a^2}$$

$$r_3 = \sqrt{3}na^2$$

$$r_n = \sqrt{(n-1/2)a^2}$$

a is the wavelength of the light.

$$a = \frac{\pi n^2}{n-1}$$

This distance a is in back the focal length (f) of zone plate, so

Scanned with CamScanner

Page No.	73
Date	

$$b_n = a = \frac{n^2 - 1}{n-1} a$$

Here in this case the area of all the zones remain same. It is equal to πa^2 .

WHEN INCIDENT WAVEFRONT IS SPHERICAL:

Let a zone plate XY - Placed perpendicular to the plane of paper.

'S' is a monochromatic source of light at a distance 'u' which sends spherical waves of wavelength ' λ '.

Point 'P' is at a distance 'v' from 'S'.

From point 'O' at zone plate.

$$r_1 = OM_1$$

$$r_2 = OM_2$$

$$r_3 = OM_3$$

$$r_n = OM_n$$

$$OM_1 + M_1 P = SO + OP + \lambda/2 = u + v + \lambda/2$$

a distance ' u ' which sends spherical waves of wavelength ' λ '.

Point 'P' is at a distance ' v ' from point 'O' at zone plate.

$$r_1 = OM_1$$

$$r_2 = OM_2$$

$$r_3 = OM_3$$

$$r_n = OM_n$$

$$SM_1 + M_1 P = SO + OP + \lambda/2 = u + v + \lambda/2$$

$$SM_2 + M_2 P = SO + OP + 2\lambda/2 = u + v + 2\lambda/2$$

$$SM_3 + M_3 P = SO + OP + 3\lambda/2 = u + v + 3\lambda/2$$

$$\text{Similarly } SM_n + M_n P = SO + OP + n\lambda/2 = u + v + n\lambda/2 \quad (i)$$

In right angle triangle SOX

$$SM_n = [SO^2 + OM_n^2]^{1/2} = [u^2 + r_n^2]^{1/2}$$

$$= [u^2 + r_n^2]^{1/2} = u [1 + \frac{r_n^2}{u^2}]^{1/2}$$

using binomial theorem

$$(1+\alpha)^n = 1+n\alpha + \dots$$

$$SM_n = u \left[1 + \frac{r_n^2}{u^2} \right]$$

Scanned with CamScanner

$$\text{So we have } SM_n = u + \frac{r_n^2}{2u^2}$$

$$\text{Similarly } M_n P = v + \frac{r_n^2}{2v^2}$$

Substituting the values of SM_n and $M_n P$ in eqn ①

$$u + \frac{r_n^2}{2u^2} + v + \frac{r_n^2}{2v^2} = u + v + \frac{n\lambda}{2}$$

$$\Rightarrow r_n^2 \left[\frac{1}{u^2} + \frac{1}{v^2} \right] = n\lambda$$

$$\Rightarrow \frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{r_n^2}$$

Comparing the above equation with the lens eqn

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{f} = \frac{n\lambda}{r_n^2}$$

$$\Rightarrow f = \frac{r_n^2}{n\lambda}$$

\therefore zone plate is acting as a converging or convergent lens.

Again

$$r_n^2 = f n \lambda \Rightarrow f = \frac{r_n^2}{n\lambda}$$

Since f and λ are constant $\Rightarrow r_n^2 \propto n$

$$\Rightarrow r_n \propto \sqrt{n}$$

i.e. radii of zones are proportional to the square root of natural numbers.

AREA OF n th ZONE :-

$$\text{Area} = \pi [r_n^2 - r_{n-1}^2]$$

$$= \pi [(n\lambda)^2 - (n-1)\lambda^2]$$

$$= \pi [n\lambda^2 - (n-1)\lambda^2]$$

$$= \pi [n\alpha f - (n-1)\alpha f]$$

Page No. 75
Date

$$= \pi [n\alpha f - n\alpha f + \alpha f]$$

∴ (Area)_{nth} zone = $\pi \alpha f$

$$\frac{1}{V} = \frac{1}{U} + \frac{1}{V}$$

$$\Rightarrow \frac{1}{f} = \frac{UV}{(U-V)}$$

∴ Area of n^{th} zone = (Area)_{nth} zone = $\frac{\pi \alpha f UV}{(U-V)}$

Area of n^{th} zone is independent of n .

MULTIPLE FOCI OF ZONE PLATE :-

Zone plate has many focal points or foci.

The focal length is given by the expression

$$f_n = \frac{n\alpha^2}{n\lambda}$$

If $n=1$, then, first focal length is given by f_1 as $f_1 = \frac{\alpha^2}{\lambda}$

If $n=2$, then, second focal length is given by f_2 as $f_2 = \frac{\alpha^2}{2\lambda}$

The intensity of second focal length is slightly lesser in comparison to first focal length.

A series of foci exists in which the intensity diminishes as we shift along the axis of zone plate.

The third order focal length is one-third of first focal length.

$$f_3 = \frac{\alpha^2}{3\lambda} = \frac{1}{3} f_1$$

Similarly m^{th} order focal length is one m^{th} of the first focal length.

In general

Page No. 76
Date

$$f_{(2m+1)} = \frac{\alpha^2}{(2m+1)\lambda}, \text{ where } m=0, 1, 2, 3, \dots$$

If the source of light is placed at infinity i.e. $U \rightarrow \infty$, then the focal length of zone plate is given by

$$f_n = V$$

Hence the focal length of a zone plate is defined as the distance from the pole where intensity observed is maximum and the source is placed at infinity.

FRESNEL DIFFRACTION AT A STRAIGHT EDGE :-

Consider a straight edge XY placed at a distance a from a source of

light 'S' which is usually an illuminated slit parallel to the edge.

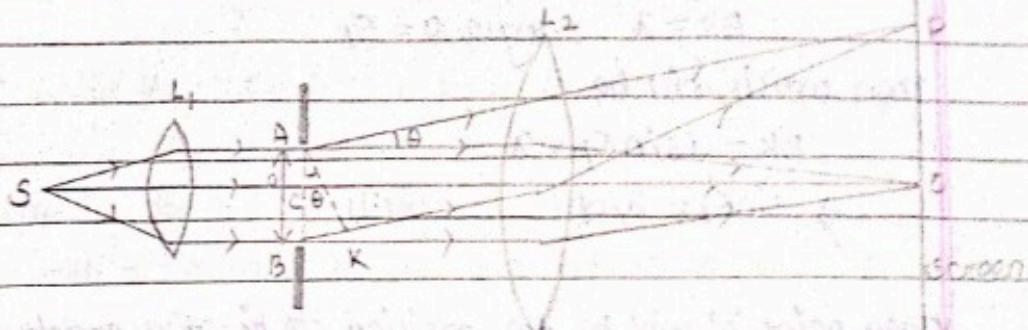
Let us consider a point 'P' at a distance V from the edge XY and at a distance b from the source 'S'.

Let θ be the angle between the line SP and the normal to the edge XY at point 'Q'.

Let ϕ be the angle between the line QP and the normal to the edge XY at point 'Q'.

UNIT - 04

FRAUNHOFER DIFFRACTION AT A SINGLE SLIT :-



Consider a monochromatic source of light S , emitting light waves of wavelength λ , placed at the convex L_1 .

Let the diffracted light is focussed by convex lens L_2 on screen XY .

The point 'p' is of maximum or minimum intensity depending on the path difference between the secondary waves.

Let width of slit $AB = d$

CENTRAL MAXIMUM:

The wavelets are initially in phase and their optical paths between the slit and point 'o' are also equal. Therefore the wavelets reinforce each other and give rise to central maximum at point 'o'.

POSITIONS AND WIDTHS OF SECONDARY MAXIMA AND MINIMA:

Let point 'p' is on the screen at which the wavelets travelling at an angle ' θ '.

The secondary waves originating from points A and B in direction ' θ ' is equal to BK .

From ΔABK

$$BK = (AB) \sin \theta$$

$$\text{path difference} \Rightarrow BK = d \sin \theta \quad (i)$$

If the path difference,

$$\Delta k = \lambda, \text{ angle } \theta = \theta_1$$

Then eqn(i) will be

$$\Delta k = ds \sin \theta_1 = \lambda$$

$$\Rightarrow \sin \theta_1 = \frac{\lambda}{d} \quad (\text{ii})$$

Then point 'p' will be the position of first secondary minimum.

Similarly if the path difference

$$\Delta k = 2\lambda \text{ and } \theta = \theta_2 \text{ then eqn(ii) can be}$$

$$2\lambda = ds \sin \theta_2$$

$$\Rightarrow \sin \theta_2 = \frac{2\lambda}{d}$$

such a point on the screen will be position of second secondary minimum.

In general, for n^{th} minimum

$$\sin \theta_n = \frac{n\lambda}{d} \text{ where } n \text{ is an integer.}$$

If the path difference Δk is odd integral multiple of $\lambda/2$,

then the directions of various secondary maximum can be obtained.

Thus, for secondary maxima

$$\Delta k = ds \sin \theta_n = (n+1) \frac{\lambda}{2}$$

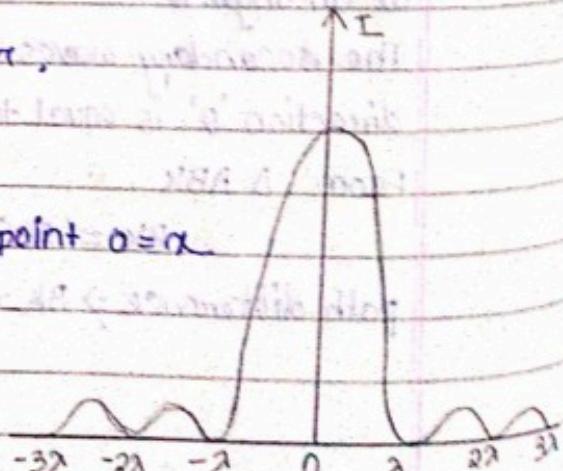
where $n = 1, 2, 3, \dots$ an integer.

If focal length of lens $L_2 = f$

Distance of first minimum from point o = x

Then

$$\sin \theta = \frac{x}{f} = \frac{\lambda}{d}$$



$$\Rightarrow d = \frac{b\lambda}{d}$$

Hence width of central maximum $2a = \frac{2b\lambda}{d}$ (iv)

Resultant intensity at Point 'P' :-

since path difference between wavelets from point A and B
of aperture in direction 'θ'

$$BK = ds \sin \theta$$

The corresponding phase difference $= \frac{2\pi}{\lambda} \times \text{path difference}$
 $= \frac{2\pi}{\lambda} (ds \sin \theta)$

Let the width of the slit be divided into 'n' equal parts.

The amplitude of variation at 'P' due to the waves from each part will be same.

Then phase difference between the waves from any two consecutive part is

$$\phi = \frac{1}{n} \left(\frac{2\pi}{\lambda} ds \sin \theta \right)$$

Hence the resultant amplitude at 'P' is given by

$$R = \frac{a \sin(n\phi/2)}{\sin(\phi/2)}$$

$$\Rightarrow R = a \sin \left[\frac{n}{2} \left(\frac{1}{n} \frac{2\pi}{\lambda} ds \sin \theta \right) \right]$$

$$\sin \left[\frac{\pi ds \sin \theta}{n\lambda} \right]$$

$$\Rightarrow R = \frac{a \sin \left[\frac{\pi ds \sin \theta}{\lambda} \right]}{\sin \left(\frac{\pi ds \sin \theta}{n\lambda} \right)}$$

Let $\pi d \sin \theta = \alpha$ (v)

$$\text{Then } R = \frac{a \sin \alpha}{\sin(\alpha/n)}$$

since α/n is small, therefore $\sin \frac{\alpha}{n} \approx \frac{\alpha}{n}$

$$\text{Hence } R = \frac{a \sin \alpha}{(\alpha/n)} = \frac{n a \sin \alpha}{\alpha}$$

As $n \rightarrow \infty$, $\alpha \rightarrow 0$ but the product na is finite. Let $na = A$

$$\text{Then } R = A \frac{\sin \alpha}{\alpha} \quad (\text{vi})$$

Therefore resultant intensity at point 'P' being proportional to square of amplitude.

$$I \propto R^2$$

~~at point P~~ $I = KR^2$ where K is constant of proportionality

$$I = K \left(\frac{A \sin \alpha}{\alpha} \right)^2$$

$$I = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \quad (\text{vii})$$

where constant 'K' is taken as unity.

DIRECTIONS OF MAXIMUM AND MINIMUM :-

Principle Maximum :

From eqn (vi) resultant amplitude R is given by

$$R = \frac{A \sin \alpha}{\alpha} = \frac{A}{\alpha} \left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots \right]$$

$$R = A \left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots \right]$$

R will be maximum when $\alpha = \pi d \sin \theta = 0$

$$\therefore \theta = 0 \quad \therefore R_{\text{max}} = A$$

Hence maximum intensity I_0 is proportional to A^2 .

This will occur when $\theta = 0$ or at point 'O' on the screen.

Secondary Minima:

From eqn (vii) intensity is minimum when $\sin \alpha = 0$

$$\alpha = \pm \pi, \pm 2\pi, \pm 3\pi \dots \text{etc} = \pm n\pi$$

$$\alpha = \frac{\pi d \sin \theta}{\lambda} = \pm n\pi$$

$\Rightarrow d \sin \theta = \pm n\lambda$ where $n = 1, 2, 3, \dots$ etc gives the direction of first, second, third etc. minimum.

SECONDARY MAXIMUM :-

To find the direction of secondary maximum intensity, differentiate eqn (vii) w.r.t α and equate to zero.

$$\frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha} \left[A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \right] = 0$$

$$A^2 \left(\frac{2 \sin \alpha}{\alpha} \right) \left[\frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} \right] = 0$$

$$\Rightarrow \frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0 \Rightarrow \alpha \cos \alpha - \sin \alpha = 0$$

$$\Rightarrow \alpha \cos \alpha = \sin \alpha$$

$$\Rightarrow \alpha = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha \quad (\text{viii})$$

This equation is solved graphically by plotting curves

$$y = \alpha \text{ and } y = \tan \alpha$$

The first curve is a straight line and the second curve is a discontinuous curve.

These values are approximately

$$\alpha = 0, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

Substituting these values of α in (VII)

(i) For central maximum $\alpha = 0$

$$I = I_0 = A^2$$

(ii) For first secondary maximum

$$\alpha = \frac{3\pi}{2}$$

$$I_1 = A^2 \left[\frac{\sin \frac{3\pi}{2}}{\frac{3\pi}{2}} \right]^2$$

$$= -4 \frac{A^2}{9\pi^2} (-1)^2 = \frac{4A^2}{9\pi^2} \approx \frac{A^2}{22} = \frac{I_0}{22}$$

(iii) For second secondary maximum

$$I_2 = A^2 \left[\frac{\sin \frac{5\pi}{2}}{\frac{5\pi}{2}} \right]^2$$

$$= \frac{4A^2}{25\pi^2} (1)^2 = \frac{4A^2}{25\pi^2} \approx \frac{I_0}{61} \text{ and so on.}$$

Hence intensities of the successive maxima are in the ratio

$$1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} : \dots$$

The intensity distribution in Fraunhofer diffraction from a single slit is shown in fig.

The minima lie at $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$

The principle maxima at $\alpha = 0$ and secondary maxima at $\frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots$

