

Scalar & Vector →

Scalar :-

(i) It has only magnitude & no direction, is required to measure, i.e. it just a numerical value.

(ii) ex :- a, b, c, 1, 2, 3 etc.

Vector :-

(i) It has both direction & magnitude & the direction shows the measuring path betⁿ the two points.

(ii) ex :- $\vec{A}, \vec{B}, \vec{a}, \vec{b}, \vec{F}$ etc.

Vector Representation :-

Generally a vector can be represented by using the variable of the coordinate system along with its unit vector.

So the vector can be represented as

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

Where, A_x, A_y, A_z are the components of x, y, z - coordinate system & $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors.

i.e. $\vec{A} = a\hat{i} + y\hat{j} + z\hat{k}$

Vector Classifications :-

1. Unit vector \longrightarrow

(i) This type of vector has only magnitude is one.

(ii) ex: $\hat{i}, \hat{j}, \hat{k}$ etc.
& $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.

2. Null vector Or zero vector \longrightarrow

(i) The vectors whose magnitude is zero is known as zero vector or null vector.

(ii) ex: $\vec{A}, \vec{C}, \vec{d}$.
 $|\vec{A}| = 0$

3. Parallel vector \longrightarrow

(i) Any two vectors are parallel vector if they have the same direction of propagation.

(ii) ex: \vec{A} \longrightarrow
 \vec{B} \longrightarrow

Relⁿ betⁿ vector & Unit vector :-

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Where \hat{A} = unit vector \hat{A}
 \vec{A} = vector.

Magnitude of a Vector :-

$$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{A}| = \sqrt{x^2 + y^2 + z^2}$$

It is the magnitude root of the sum of the squares of the coefficient of the unit vectors which is the magnitude of vector.

Algebra of vector :-

① Addition & Subtraction

If we take $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$

$$\vec{B} = \hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{A} + \vec{B} = 3\hat{i} + 6\hat{j} + \hat{k}$$

$$\vec{A} - \vec{B} = \hat{i} - 3\hat{k}$$

Product of Vector

(i) It is two types.

① Dot product (\cdot)

② Cross product (\times)

① Dot Product

$$(i) \hat{i} \cdot \hat{i} = 1 = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k}$$

Same unit vector is 1.

$$(ii) \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

Other unit vector is zero.

(iii) The any dot product result is scalar & qty.

$$\begin{aligned}
 &= -6(-\hat{j}) + 2\hat{k} - 3(-\hat{k}) + 9\hat{i} + \hat{j} - \hat{i} \\
 &= 6\hat{j} + 2\hat{k} + 3\hat{k} + 9\hat{i} + \hat{j} - \hat{i} \\
 &= 8\hat{i} + 7\hat{j} + 5\hat{k} \quad \text{which shows the} \\
 &\text{vector quality of cross product.}
 \end{aligned}$$

(iv) $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
 $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x) \quad \text{which is}$$

the

ex: - $\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$
 $\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= \hat{i}(-1+2) - \hat{j}(1-4) + \hat{k}(-3+2)$$

$$= \hat{i}(1) - \hat{j}(-3) + \hat{k}(-1) = \hat{i} + 3\hat{j} - \hat{k}$$

$$\hat{i} \cdot \hat{i} + \hat{j} \cdot \hat{j} - \hat{k} \cdot \hat{k} = 1 + 3 - 1 = 3$$

$$\hat{j} \cdot \hat{i} - \hat{j} \cdot \hat{j} + \hat{i} \cdot \hat{k} -$$

$$\hat{j} \cdot \hat{k} + \hat{i} \cdot \hat{i} - \hat{j} \cdot \hat{j} +$$

Angle betⁿ two vectors :

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\Rightarrow \cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{AB} \right)$$

$$\text{When } A = |\vec{A}| \text{ \& } B = |\vec{B}|$$

$$\text{Again } \vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

$$\Rightarrow |\vec{A} \times \vec{B}| = |AB \sin \theta \hat{n}|$$

$$\Rightarrow |\vec{A} \times \vec{B}| = AB \sin \theta |\hat{n}|$$

$$\Rightarrow \sin \theta = \frac{|\vec{A} \times \vec{B}|}{AB} \quad (\because |\hat{n}| = 1)$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{|\vec{A} \times \vec{B}|}{AB} \right)$$

Ex^o $\vec{A} = 3\hat{i} - \hat{j} + \hat{k}$

$$\vec{B} = -2\hat{i} + \hat{j} - 3\hat{k}$$

Solⁿ $\vec{A} \cdot \vec{B} = (3\hat{i} - \hat{j} + \hat{k}) \cdot (-2\hat{i} + \hat{j} - 3\hat{k})$

$$\begin{aligned} &= -6\hat{i} \cdot \hat{i} + 3\hat{i} \cdot \hat{j} - 9\hat{i} \cdot \hat{k} + 2\hat{j} \cdot \hat{i} \\ &\quad - \hat{j} \cdot \hat{j} + 3\hat{j} \cdot \hat{k} - 2\hat{k} \cdot \hat{i} \\ &\quad + \hat{k} \cdot \hat{j} - 3\hat{k} \cdot \hat{k} \end{aligned}$$

$$\vec{A} + \vec{B} = \hat{i} - \hat{j} - 5\hat{k}$$

$$|\vec{A} + \vec{B}| = \sqrt{(1)^2 + (-1)^2 + (-5)^2} \\ = \sqrt{1+1+25} = \sqrt{27}$$

$$\vec{A} - \vec{B} = -7\hat{i} + 5\hat{j} - 3\hat{k}$$

$$|\vec{A} - \vec{B}| = \sqrt{(-7)^2 + (5)^2 + (-3)^2} \\ = \sqrt{49+25+9} = \sqrt{83}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{-3\hat{i} + 2\hat{j} - 4\hat{k}}{\sqrt{29}}$$

$$\hat{B} = \frac{\vec{B}}{|\vec{B}|} = \frac{4\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{26}} = \frac{4}{\sqrt{26}}\hat{i} - \frac{3}{\sqrt{26}}\hat{j} - \frac{1}{\sqrt{26}}\hat{k}$$

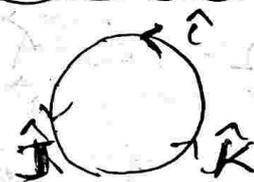
② CROSS PRODUCT \longrightarrow

(i)



$$\hat{i} \times \hat{j} = \hat{k} \\ \hat{j} \times \hat{k} = \hat{i} \\ \hat{k} \times \hat{i} = \hat{j}$$

anticlockwise



$$\hat{j} \times \hat{i} = -\hat{k} \\ \hat{i} \times \hat{k} = -\hat{j} \\ \hat{k} \times \hat{j} = -\hat{i}$$

(ii)

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

(iii) ex:

$$\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{B} = \hat{i} - 3\hat{k} + \hat{j}$$

$$\vec{A} \times \vec{B} = (2\hat{i} - 3\hat{j} + \hat{k}) \times (\hat{i} - 3\hat{k} + \hat{j})$$

$$= 2\hat{i} \cdot \hat{i} - 6\hat{i} \cdot \hat{k} + 2\hat{i} \cdot \hat{j} - 3\hat{j} \cdot \hat{i} + 9\hat{j} \cdot \hat{k} \\ - 3\hat{j} \cdot \hat{j} + \hat{k} \cdot \hat{i} - 3\hat{k} \cdot \hat{k} + \hat{k} \cdot \hat{j}$$

Resultant of Vectors \longrightarrow

If you take any two vectors in any two directions which making any angle θ with each other then the resultant vector can be written as using Parallelogram law of vector addⁿ is

$$|\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2AB \cos \theta}$$

$$\text{OR } R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

ex: $\vec{A} = -\hat{i} - 3\hat{k} + \hat{j}$

$\vec{B} = -2\hat{j} - \hat{k} + 4\hat{k} = -2\hat{j} + 3\hat{k}$

$\theta = 60^\circ$

$$|\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2AB \cos \theta}$$

$$|\vec{A}| = \sqrt{(-1)^2 + (-3)^2 + (1)^2} = \sqrt{11}$$

$$= \sqrt{1+9+1} = \sqrt{11}$$

$$|\vec{B}| = \sqrt{(-2)^2 + (3)^2} = \sqrt{13}$$

$$|\vec{R}| = \sqrt{(\sqrt{11})^2 + (\sqrt{13})^2 + 2\sqrt{11} \cdot \sqrt{13} \cos 60^\circ}$$

$$= \sqrt{11 + 13 + \sqrt{143}}$$
$$= 5.99 \text{ (Ans)}$$

(P.p.p)

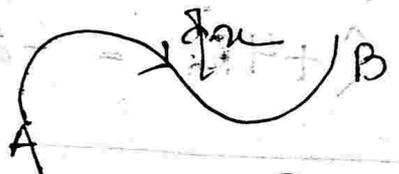


Vector Integral :-

The integration over any vector field which can define the orientation of the vector is known as vector integral. It is three types.

① Line Integral →

The integration over the line in the vector field is defined as line integral.



Mathematically we can write

$$\int_C \mathbf{v} \cdot d\mathbf{r} = \int_C (v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) \cdot d\mathbf{r}$$

② Surface Integral →

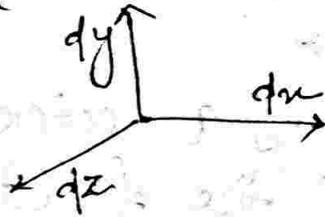
The integration over the surface in the vector field is called a surface integral.

$$\iint_S \mathbf{v} \cdot d\mathbf{S} = \iint_S (v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) \cdot d\mathbf{S}$$



⑤ Volume Integral →

The integration over the volume in the vector field is called volume integral.



$$\iiint_V \mathbf{v} \cdot d\mathbf{v} = \iiint_V (v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) dx \cdot dy \cdot dz$$

Gradient of a Vector Field →

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

The gradient of a vector field is defined as the vector applying on the scalar field which forming a vector field in the any direction.



* Particular direction *
* Gradient of field *

In mathematically, the gradient of the vector field written as

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

∴ The gradient is a vector operation which operates on a scalar funcⁿ to produce a vector whose magnitude is the max^m rate of change of the funcⁿ.

of \vec{A} is any vector field whose scalar
 funcⁿ is A
 So $\vec{\nabla} A = \hat{i} \frac{\partial A}{\partial x} + \hat{j} \frac{\partial A}{\partial y} + \hat{k} \frac{\partial A}{\partial z}$

Divergence \rightarrow

(i) The divergence of a vector field is the dot product of the field which gives a result (scalar) quantity.

It tells how much flux is entering for leaving a small volume per unit volume.

(ii) OR It is the dot product of gradient of a vector with any vector field in any direction.

(iii) let $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$$\text{Div. } \vec{A} = \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Curl of a Vector Field :

(i) The curl is a vector operator that describe the infinitesimal (random rotation) rotation of a vector field in a 3 dimensional space.

OR

It is the cross product of gradient of a vector field with any vector field.

(ii) In Mathematically,

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{\nabla} = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y} (A_z) - \frac{\partial}{\partial z} (A_y) \right] \hat{i} + \left[\frac{\partial}{\partial z} (A_x) - \frac{\partial}{\partial x} (A_z) \right] \hat{j} + \left[\frac{\partial}{\partial x} (A_y) - \frac{\partial}{\partial y} (A_x) \right] \hat{k}$$

Q.1 of the vector field

$$\vec{r} = 3x^2 \hat{i} - y^3 \hat{j} + 2z \hat{k} \text{ then find } \text{div. } \vec{r}$$

& curl of \vec{r} & grad of \vec{r} ?

Solⁿ: - Given that $\vec{r} = 3x^2 \hat{i} - y^3 \hat{j} + 2z \hat{k}$

$$\text{Div. } \vec{r} = \frac{\partial}{\partial x} (3x^2) + \frac{\partial}{\partial y} (-y^3) + \frac{\partial}{\partial z} (2z)$$

$$= 6x - 3y^2 + 2$$

$$\text{grad. } \vec{r} = \frac{\partial}{\partial x} (3x^2) \hat{i} + \frac{\partial}{\partial y} (-y^3) \hat{j} + \frac{\partial}{\partial z} (2z) \hat{k}$$

$$= 6x \hat{i} - 3y^2 \hat{j} + 2 \hat{k}$$

$$\text{curl. } \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2 & -y^3 & 2z \end{vmatrix}$$

$$= \hat{i} \left[\frac{\partial}{\partial y} (2z) + \frac{\partial}{\partial z} (y^3) \right] - \hat{j} \left[\frac{\partial}{\partial x} (2z) - \frac{\partial}{\partial z} (3x^2) \right] + \hat{k} \left[\frac{\partial}{\partial x} (-y^3) - \frac{\partial}{\partial y} (3x^2) \right]$$

$$= \hat{i}[0] - \hat{j}[0] + \hat{k}[0] = 0 \quad (\text{Ans})$$

Rules :-

(i) In case of solenoid the divergence of vector field must be zero.

i.e. $\text{div } \vec{V} = 0$

(ii) In case of irrotational vector field the curl of the vector field must be zero. i.e. $\text{curl } \vec{V} = 0$

Gauss Divergence Theorem :-

(i) This theorem transfers the volume integration to the surface integration, & it states that the volume integral of div. of the vector field \vec{V} taken over any volume (V) is equal to the surface integral of the vector field around any closed curve in the (any) field.

(ii) Mathematically we can write as -

$$\iiint_V \text{div } \vec{V} \, dV = \iint_S \vec{V} \cdot \vec{n} \, dS$$

Stoke's Theorem :-

(i) This theorem transforms the line integral to the surface integral & it states that the line integral of a vector field (\vec{l}) around any closed curve is equal to the surface integral of curl of the vector field around the closed curve in the field.

(ii) Mathematically it can be written as -

$$\oint \vec{l} \cdot d\vec{l} = \oint_S (\text{curl } \vec{l}) \cdot d\vec{s}$$

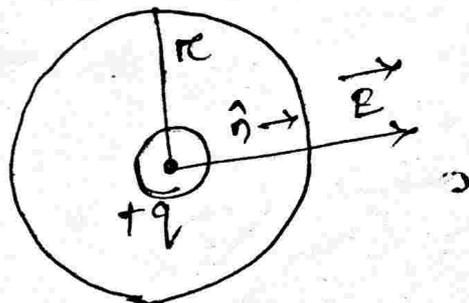
$$= \oint_S (\nabla \times \vec{l}) \cdot d\vec{s}$$

Gauss Law of Electrostatic in Free Space :-

(i) Gauss law of electrostatic tells about the total electric flux passing through the closed surface or for particular volume linking with the electric field in the surface.

(ii) It states that the electric flux (Φ) through a closed surface is equal to the ~~one~~ $1/\epsilon_0$ times the total charge of the body.

i.e. $\Phi = \frac{q}{\epsilon_0}$



(iii) In case of Electric field whose surface vector is \vec{s} then it can be written as

$$d\phi = E ds \cos\theta$$

$$\Rightarrow \phi = \oint_c \vec{E} \cdot d\vec{s}$$

where: E = Electric field.
 $d\vec{s}$ = surface vector.

This is the integral form of Gauss law.

(iv) In case of any medium, we have

we know that $\epsilon = \epsilon_r \epsilon_0$

$$\text{So } \phi = \frac{q}{\epsilon_r \epsilon_0}$$

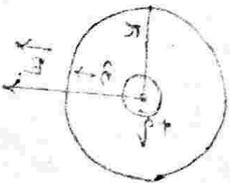
$$\Rightarrow \phi = \frac{q \epsilon_r \epsilon_0}{\epsilon}$$

ϵ_0 = Absolute Permeability

ϵ_r = Relative Permeability

ϵ = Permeability in the medium.

(ϕ) is the total electric flux through a closed surface. The total charge of the surface is equal to the flux of the electric field through the surface.



$$\frac{q}{\epsilon} = \phi$$

Faraday's Laws of Electromagnetic Induction

- * single cell the current flowing - emf
- * current flowing in total ckt - P.D.

1st Law:

Whenever the magnetic flux link with a closed ckt changes an emf which is induced in the ckt by the magnetic effect in the ckt. This phenomenon is known as electro magnetic induction.

This law tells about the total magnetic flux passing through the closed surface will changes an emf induced in it.

2nd Law:

The magnitude of induced emf is equal to the rate of change of flux linked with the closed ckt.

Mathematically we can write as

$$|\mathcal{E}| = \left| \frac{d\phi}{dt} \right|$$

Displacement Current

i) It is the eqv amount of current i.e. flow (in the cell in a combination ckt.)

It is the special type of current which appears in the region in which the electric flux is changing with time.

(ii) In mathematical we can write

$$I_d = \epsilon_0 \frac{dq}{dt}$$

Using Gauss Theorem

$$\text{i.e. } \phi = \frac{q}{\epsilon_0}$$

$$\Rightarrow I_d = \epsilon_0 \frac{d(\phi/\epsilon_0)}{dt}$$

$$\Rightarrow I_d = \epsilon_0 \times \frac{1}{\epsilon_0} \times \frac{dq}{dt}$$

$$\Rightarrow \boxed{I_d = \frac{dq}{dt}}$$

(iii) when the capacitor is fully charged

then $\frac{dq}{dt} = 0$ or $\frac{d\phi}{dt} = 0$

$$\text{So } \boxed{I_d = 0}$$

✓ Ampere's Circuital Law \rightarrow

(i) Gauss law in an electrostatic field is the alternative form of Coulomb's law of electrostatic field.

Similarly the ampere's circuital law is the alternative form of Biot-Savart's law in the magnetostatic field.

(ii) The ampere's circuital law keep the relationship between magnetic field (B) with the current (I) flowing through the ckt.

(iii) So ampere's circuital law states that the line integral of magnetic volume around any closed ckt is equal to the μ_0 times of the total current flowing through the ckt.

So we can write in mathematical,

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I}$$

Again, if we take the magnetic field of intensity

$$\oint \frac{\vec{B}}{\mu_0} \cdot d\vec{l} = I$$

$$\Rightarrow \boxed{\oint \vec{H} \cdot d\vec{l} = I}$$

It is the another form of ampere's circuital form.

In simplifying form we can write -

$$\boxed{BL = \mu_0 I}$$

where L = total length of the closed ckt.

In general form the ampere's circuital law can be written as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I + I_d)$$

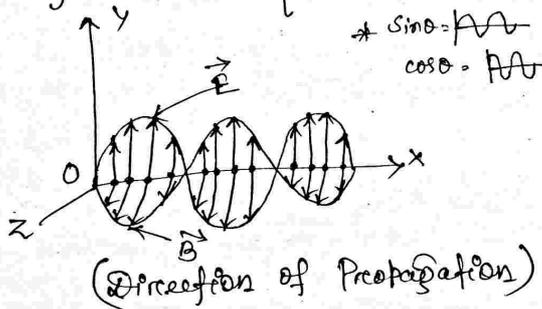
$$\Rightarrow \boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{dq}{dt} \right)}$$

Electromagnetic Wave :-

According to Faraday's law of electromagnetic induction, that a wave of electric & magnetic field that propagated through the medium with making disturbance in it.

This disturbance (can be) observed by Maxwell & he (show) that the electric field & magnetic field are mutually perpendicular to each other in the direction of propagation. These waves are called as electromagnetic wave.

Hence, the electromagnetic waves consists of a sinusoidal wave, in which the electric field & magnetic field are must be right angle to each other in the direction of field (propagation).



Generally, the electromagnetic wave are the transverse wave. The speed of the electromagnetic wave can be written as

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

where, μ_0 = Permeability
 ϵ_0 = Permeability in the medium.

Now putting the value μ_0 & ϵ_0 in the above we get

$$c = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}}$$

$$\Rightarrow c = 3 \times 10^8 \text{ m/sec}$$

which is the general velocity of light. So light is also electromagnetic wave. Then, in general form the speed of electromagnetic wave we can write as

$$c = \frac{1}{\sqrt{\mu \epsilon}}$$

where μ = Absolute Permeability
 ϵ = Absolute Permeability of the medium.

Plane Electromagnetic wave
 let's consider a electromagnetic wave propagate in x-axis as shown in fig. Here, the electric field & magnetic field are mutually perpendicular to each other. So the wave eqⁿ in general form are in

component forms we can write as

$$\vec{E} = E_0 \cos(kx - \omega t)$$

$$\vec{B} = B_0 \cos(kx - \omega t)$$

where E_0 & B_0 are the maxⁿ value of the field & $\omega =$ angular frequency



$$= 2\pi N$$

$$2\pi k = \frac{2\pi}{\lambda}$$

Now $\frac{\omega}{k} = \frac{2\pi N}{\frac{2\pi}{\lambda}} = N\lambda = c$

$$\frac{E}{B} = \frac{E_0}{B_0} = c$$

* where $c = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T} = \lambda \times \frac{1}{T} = \lambda N$

In case of electromagnetic wave the ratio of maxⁿ value of electric field to the maxⁿ value of magnetic field is a constant which is equal to velocity of light.

Electromagnetic Energy

The electromagnetic wave carries energy & momentum when they propagate in the medium. So the total electromagnetic energy will be the sum of electric energy & magnetic energy.

Now the electric energy per unit volume can be written as

$$U_E = \frac{1}{2} \vec{E} \cdot \vec{D}$$

$$\Rightarrow U_E = \frac{1}{2} \epsilon E^2$$

Similarly, the magnetic energy can be written as $U_B = \frac{1}{2} \vec{B} \cdot \vec{H}$

$$\Rightarrow U_B = \frac{1}{2} \mu H^2$$

So the total electromagnetic energy $U_E + U_B = \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 = \frac{1}{2} (\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H})$

$$\Rightarrow U_{EM} = (U_E + U_B) = \frac{1}{2} \epsilon (E^2 + \mu H^2) = \frac{1}{2} (\epsilon E^2 + \mu H^2)$$

Poynting Theorem

(i) It states that the rate of energy transfer from a region of space equal to the rate of work done plus the energy flux leaving that region.

(ii) So in mathematical form the differential form can be written as

$$-\frac{\partial U}{\partial t} = \nabla \cdot \vec{S} + \vec{J} \cdot \vec{E}$$

✓ Maxwell Electromagnetic Eq^s →

There are four eq^s which can be regarded as the basis of electric & magnetic phenomena.

These eq^s are known as Maxwell eq^s & can be written as →

① $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$
(Gauss law of Electrostatic)

② $\oint \vec{B} \cdot d\vec{s} = 0$
(i.e. Gauss law for magnetostatic)

③ $e = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$
(Faraday's law of Electromagnetic Induction)

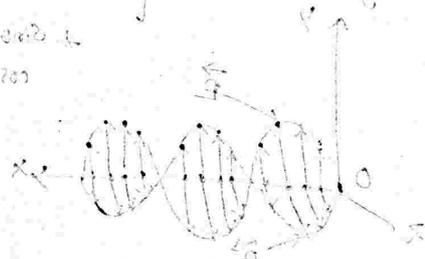
④ $\oint \vec{B} \cdot d\vec{l} = \mu_0 (I + \epsilon_0 \frac{d\phi}{dt})$
(Modified Ampere's Circuital law)

① → It states that the electric flux (ϕ) through a closed surface is equal to the $\frac{1}{\epsilon_0}$ times the total charge of the body.
i.e. $\phi = \frac{q}{\epsilon_0}$

② → It states that the magnetic field B has divergence equal to zero. In other words, that it is a solenoidal vector field. No magnetic monopoles exists.

③ → It states that the total magnetic flux passing through the closed surface will changes an emf induced in it.

④ → It states that the line integral of magnetic volume around any closed ckt is equal to the μ_0 times the total current flowing through the ckt.



(containing is current)

Poynting Vector \rightarrow

(i) In case of electromagnetic wave the cross product of vector field (\vec{E}) & vector field strength (\vec{H}) gives a vector (\vec{P}) which is called Poynting vector.

(ii) Mathematically it can be written as

$$\vec{P} = \vec{E} \times \vec{H}$$

(iii) Since \vec{E} & \vec{H} are mutual perpendicular to each other in the direction of propagation. So if we consider a wave along x -axis then we can write

$$\vec{E} = \hat{j} E_y$$

$$\vec{H} = \hat{k} H_z$$

$$\vec{P} = \hat{j} E_y \times \hat{k} H_z$$

$$\Rightarrow \vec{P} = (\hat{j} \times \hat{k}) E_y H_z$$

$$\Rightarrow \vec{P} = \hat{i} E_y H_z$$

This indicates the Poynting vector must be along x -axis.

$$\vec{P} = \vec{E} \times \vec{H}$$