

# Kinematics and Dynamics of Machines ( K & DM )

Semester : 4TH

Branch : Mechanical Engineering

Module - I

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# 1.

# Mechanism

## Theory at a glance (IES, GATE & PSU)

### What is TOM

The subject theory of machine may be defined as that branch of engineering science which deals with the study of relative motion both the various parts of m/c and forces which act on them.

The theory of m/c may be sub divided into the following branches:

1. **Kinematics:** It deals with the relative motion between the various parts of the machine
2. **Dynamics:** It deals with the force and their effects, while acting upon the m/c part in motion.

**Resistance Body:** Resistant bodies are those which do not suffer appreciable distortion or change in physical form by the force acting on them e.g., spring, belt.

**Kinematic Link Element:** A resistant body which is a part of an m/c and has motion relative to the other connected parts is **term as link**.

A link may consist of one or more resistant bodies. Thus a link may consist of a number of parts connected in such away that they form one unit and have no relative motion to each other.

- **A link should have the following two characteristics:**
  1. It should have relative motion, and
  2. It must be a resistant body.

### Functions of Linkages

The function of a link mechanism is to produce rotating, oscillating, or reciprocating motion from the rotation of a crank or *vice versa*. Stated more specifically linkages may be used to convert:

1. Continuous rotation into continuous rotation, with a constant or variable angular velocity ratio.
2. Continuous rotation into oscillation or reciprocation (or the reverse), with a constant or variable velocity ratio.
3. Oscillation into oscillation, or reciprocation into reciprocation, with a constant or variable velocity ratio.

Linkages have many different functions, which can be classified according on the primary goal of the mechanism:

- **Function generation:** the relative motion between the links connected to the frame,
- **Path generation:** the path of a tracer point, or
- **Motion generation:** the motion of the coupler link.

## Types

1. **Rigid Link:** It is one which does not undergo any deformation while transmitting motion—C.R, etc.
2. **Flexible Link:** Partly deformed while transmitting motion—**spring, belts.**
3. **Fluid Link:** It formed by having the motion which is transmitted through the fluid by pressure. e. g, hydraulic press, hydraulic brakes.

## Kinematic Pair

Two element or links which are connected together in such a way that their relative motion is completely or successfully constrained form a kinematic pair. i.e. The term kinematic pairs actually refer to kinematic constraints between rigid bodies.

The kinematic pairs are divided into **lower pairs** and **higher pairs**, depending on how the two bodies are in contact.

- **Lower Pair:** When two elements have **surface contact** while in motion.
- **Higher Pair:** When two elements have **point or line of contact** while in motion.

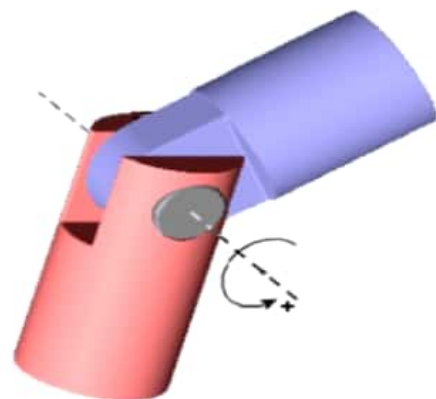
## Lower Pairs

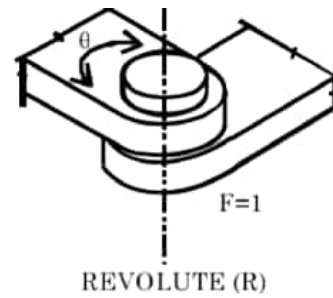
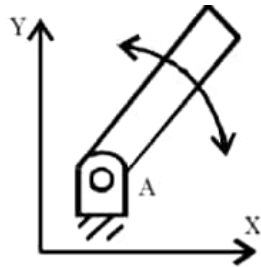
A pair is said to be a lower pair when the connection between two elements is through the area of contact. Its 6 types are:

- **Revolute Pair**
- **Prismatic Pair**
- **Screw Pair**
- **Cylindrical Pair**
- **Spherical Pair**
- **Planar Pair.**

## Revolute Pair

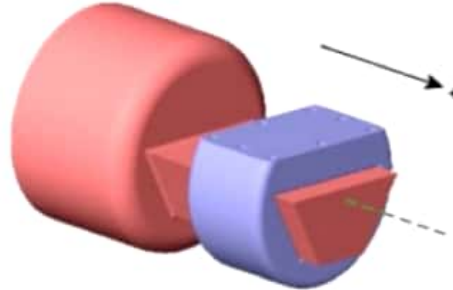
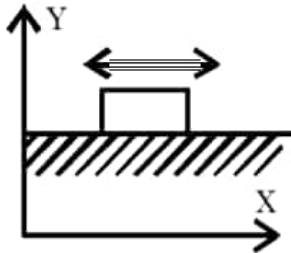
A revolute allows only a relative rotation between elements 1 and 2, which can be expressed by a single coordinate angle  $\theta$ . Thus a revolute pair has a single degree of freedom.





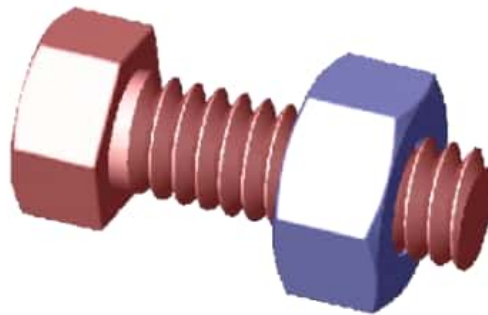
## Prismatic Pair

A prismatic pair allows only a relative translation between elements 1 and 2, which can be expressed by a single coordinate 'x'. Thus a prismatic pair has a single degree of freedom.



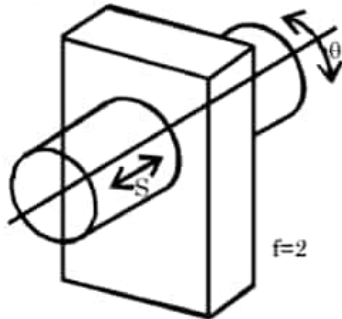
## Screw Pair

A screw pair allows only a relative movement between elements 1 and 2, which can be expressed by a single coordinate angle ' $\theta$ ' or 'x'. Thus a screw pair has a single degree of freedom. Example-lead screw and nut of lathe, **screw jack**.

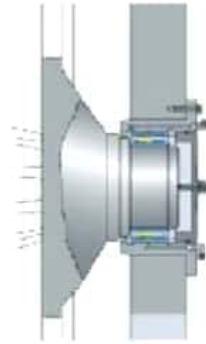


## Cylindrical Pair

A cylindrical pair allows both rotation and translation between elements 1 and 2, which can be expressed as two independent coordinate angle ' $\theta$ ' and ' $x$ '. Thus a cylindrical pair has two degrees of freedom.

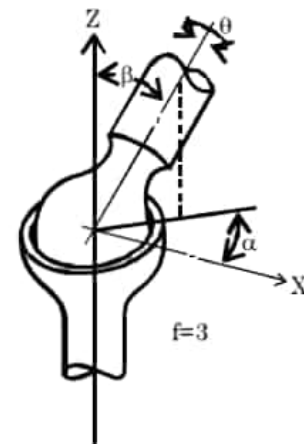
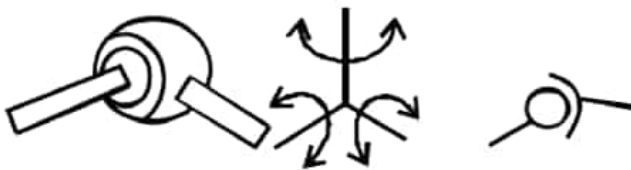
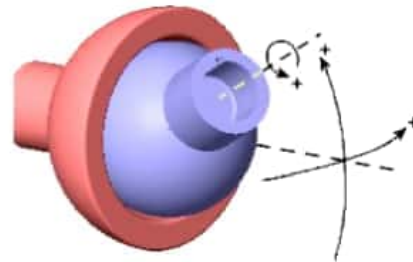


CYLINDRICAL (C)



## Spherical Pair

A spherical pair allows three degrees of freedom since the complete description of relative movement between the connected elements needs three independent coordinates. Two of the coordinates ' $\alpha$ ' and ' $\beta$ ' are required to specify the position of the axis OA and the third coordinate ' $\theta$ ' describes the rotation about the axis OA. e.g. – **Mirror attachment of motor cycle**.

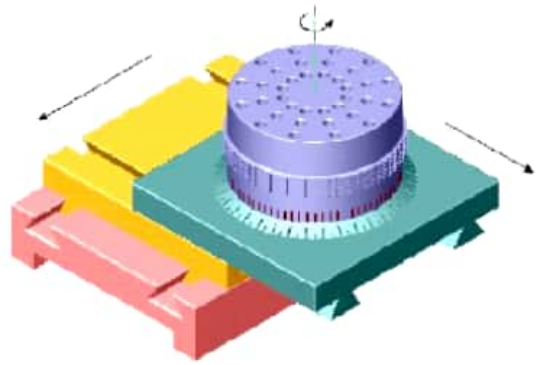
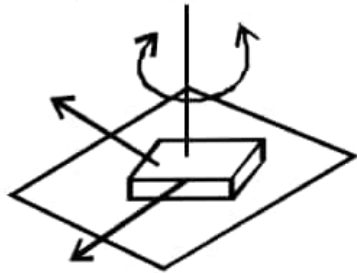


SPHERICAL (G)

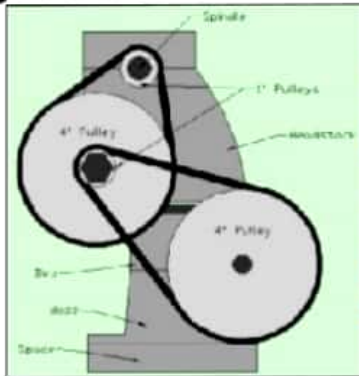
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## Planar Pair

A planar pair allows three degrees of freedom. Two coordinates  $x$  and  $y$  describe the relative translation in the  $xy$ -plane and the third ' $\theta$ ' describes the relative rotation about the  $z$ -axis.



## Higher Pairs



A higher pair is defined as one in which the connection between two elements has only a **point or line of contact**. A cylinder and a hole of equal radius and with axis parallel make contact along a surface. Two cylinders with unequal radius and with axis parallel make contact along a line. A point contact takes place when spheres rest on plane or curved surfaces (ball bearings) or between teeth of a skew-helical gears. In roller bearings, between teeth of most of the gears and in cam-follower motion. The degree of freedom of a kinetic pair is given by the number independent coordinates required to completely specify the relative movement.

## Wrapping Pairs

Wrapping Pairs comprise belts, chains, and other such devices

## Sliding Pair

When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show that a sliding pair has a completely constrained motion.

## Turning pair

When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe

spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of a turning pair. A turning pair also has a completely constrained motion.

**Rolling pair:** When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.

**According to mechanical constraint between the elements:**

1. **Closed Pair:** When two elements of a pair are held together mechanically. e.g., all lower pair and some of higher pair.
2. **Unclosed Pair (Open Pair):** When two elements of a pair are not held together mechanically. e.g., cam and follower.

## Kinematic Constraints

Two or more rigid bodies in space are collectively called a *rigid body system*. We can hinder the motion of these independent rigid bodies with **kinematic constraints**. *Kinematic constraints* are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.

## Types of Constrained Motions

Following are the three types of constrained motions:

1. **Completely constrained motion;** When motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion. For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank, as shown in Fig.5.1

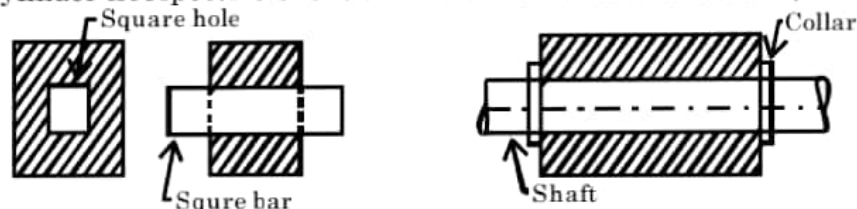


Fig. 5.2 Square bar in a square hole    Fig. 5.3 Shaft with collar in a circular hole.

The motion of a square bar in a square hole, as shown in Fig.5.2, and the motion of a shaft with collars at each in a circular hole, as shown in Fig. 5.3, are also examples of completely constrained motion.

2. **Incompletely constrained motion:** When the motion between a pair can take place in more than one direction, then the motion is called an incomplete constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig. 5.4., is an incompletely constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other.

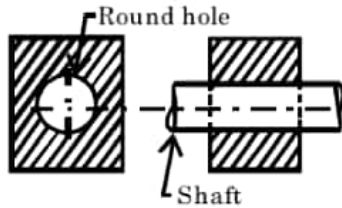


Fig. 5.4 Shaft in a circular hole.

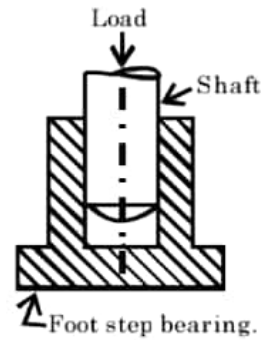


Fig. 5.4 Shaft in a foot step bearing.

**3. Successfully constrained motion:** When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a **foot-step bearing** as shown in Fig 5.5. The shaft may rotate in a bearing or it may rotate in a bearing or it may upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion. But if the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.

## Kinematic chain

A kinematic chain is a **series of links** connected by kinematic pairs. The chain is said to be closed chain if every link is connected to at least two other links, otherwise it is called an open chain. A link which is connected to only one other link is known as singular link. If it is connected to two other links, it is called binary link. If it is connected to three other links, it is called ternary link, and so on. A chain which consists of only binary links is called simple chain.

If each link is assumed to form two pairs with two adjacent links, then relation between the No. of pairs (p) forming a kinematic chain and the number of links (l) may be expressed in the form of an equation:

$$l = 2p - 4$$

Since in a kinematic chain each link forms a part of two pairs, therefore there will be as many links as the number of pairs.

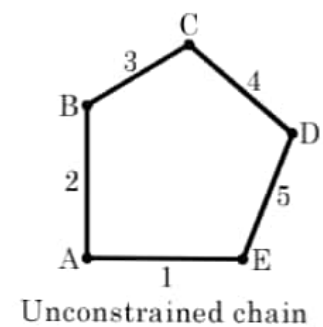
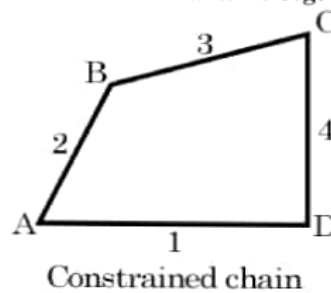
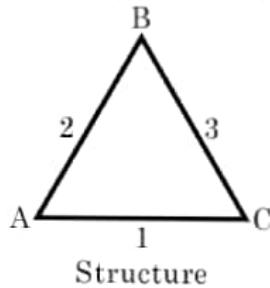
Another relation between the number of links (l) and the number of joints (j) which constitute a kinematic chain is given by the expression:

$$j = \frac{3}{2}l - 2$$

Where,  
l = no. of links

$p$  = no. of pairs  
 $j$  = no. of binary joints.

If  $L.H.S. > R.H.S. \Rightarrow$  Structure  
 $L.H.S. = R.H.S. \Rightarrow$  Constrained chain  
 $L.H.S. < R.H.S. \Rightarrow$  Unconstrained chain. e.g.



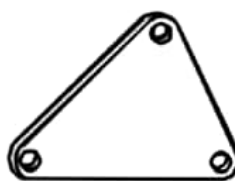
## Types of Joints

1. Binary Joint
2. Ternary Joint
3. Quaternary Joint.

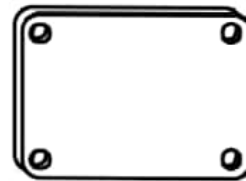
### Links, Joints and Kinematic Chains



Binary Link



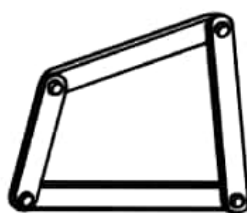
Ternary Link



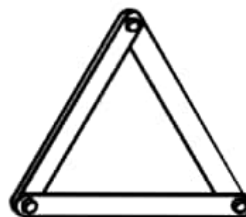
Quaternary Link

Every link must have nodes. The number of nodes defines the type of link.

- **Binary link** - One link with two nodes
- **Ternary link** - One link with three nodes
- **Quaternary link** - One link with four nodes



DOF=+1



DOF=0



DOF=-1

Now that we have defined degrees of freedom, We can look at the illustration above and determine the degrees of freedom of each.

<u>Joint Type</u>	<u>DOF</u>	<u>Description</u>
<u>A</u> First order pin joint	1	two binary links joined at a common point
<u>B</u> Second order pin joint	2	three binary links joined at a common point
<u>C</u> Half Joint	1 or 2	Rolling or sliding or both

## Mechanism

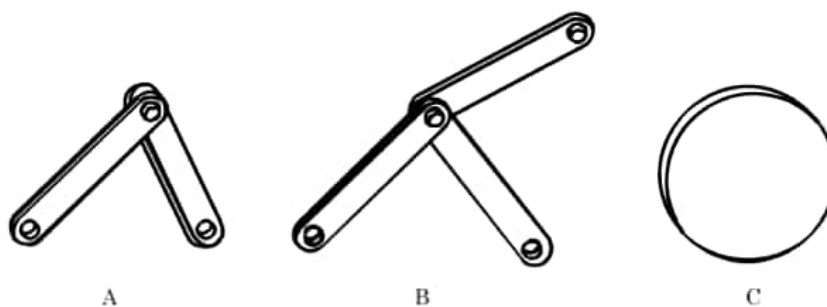
**Mechanism:** When one of the link of a kinematic chain is fixed, it will be a mechanism. If the different link of the same kinematic chain is fixed, the result is a different mechanism. The primary function of a mechanism is to transmit or modify motion.

**Machine:** When a mechanism is required to transmit power or to do some particular kind of work it is known as a machine.

**Structure:** An assemblage of resistant bodies having no relative motion between them and meant for carrying load having straining action called structure.

**Inversions:** Mechanism is one in which one of the link of kinematic chain is fixed. Different mechanism are formed by fixing different link of the same kinematic chain are known as inversions of each other.

## Mechanisms and Structures



- A **mechanism** is defined by the number of **positive** degrees of freedom. If the assembly has zero or negative degrees of freedom it is a structure.
- A **structure** is an assembly that has **zero** degrees of freedom. An assembly with **negative** degrees of freedom is a structure with residual stresses.

## Degrees of freedom

**Degree of Freedom:** It is the **number of independent variables** that must be specified to define completely the condition of the system.

A kinematic chain is said to be movable when its d.o.f.  $\geq 1$  otherwise it will be locked. If the d.o.f. is 1 the chain is said to be constrained.

Figure4-1 shows a rigid body in a plane. To determine the DOF this body we must consider how many distinct ways the bar can be moved. In a two dimensional plane such as this

computer screen, there are 3 DOF. The bar can be translated along the x axis, translated along the y axis, and rotated about its central.

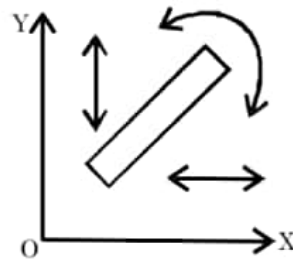
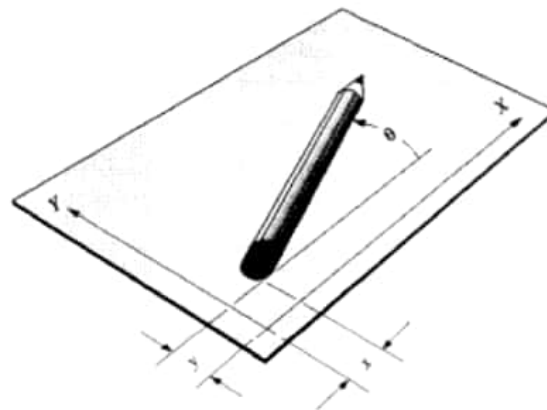


Figure 4-1 Degrees of freedom of a rigid body in a plane

Consider a pencil on a table. If the corner of the table was used as a reference point, two independent variables will be required to fully define its position. Either an X-Y coordinate of an endpoint and an angle or two X coordinates *or* two Y coordinates. No single variable by itself can never fully define its position. Therefore the system has two degrees of freedom.



**Fig.** Degree of freedom of a Rigid Body in Space

An unrestrained rigid body in space has six degrees of freedom: three translating motions Along the x, y and z axes and three rotary motions around the x, y and z axes respectively.

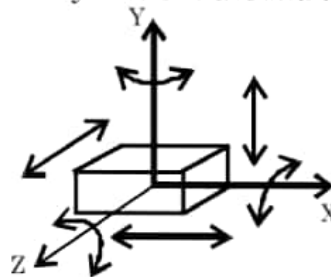


Figure 4-2 Degrees of freedom of a rigid body in space

Unconstrained rigid body in space possesses 6 d.o.f.

Joint/Pair	D.O.F.	Variable
Pin Joint	1	$\theta$
Sliding Joint	1	S
Screw Pair	1	$\theta$ or S
Cost. Pair	2	$\theta, S$
Spherical Pair	3	$\theta, j, \psi$
Planar Pair	3	xy $\theta$

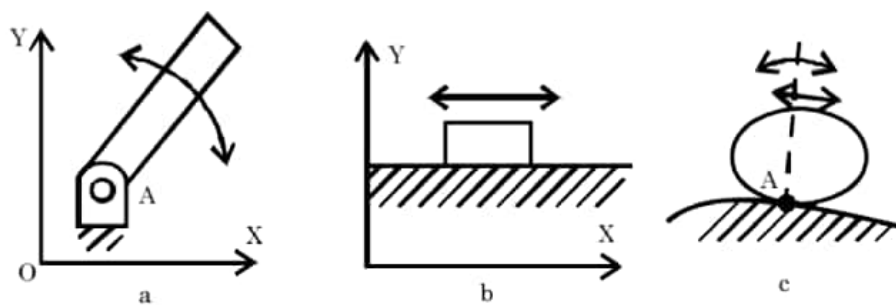
→ Revolute Pair

## Kutzbach criterion

The number of degrees of freedom of a mechanism is also called the mobility of the device. The **mobility** is the number of input parameters (usually pair variables) that must be independently controlled to bring the device into a particular position. The **Kutzbach criterion** calculates the mobility.

In order to control a mechanism, the number of independent input motions must equal the number of degrees of freedom of the mechanism.

For example, Figure shows several cases of a rigid body constrained by different kinds of pairs.



**Figure.** Rigid bodies constrained by different kinds of planar pairs

In Figure-a, a rigid body is constrained by a **revolute pair** which allows only rotational movement around an axis. It has one degree of freedom, turning around point A. The two lost degrees of freedom are translational movements along the  $x$  and  $y$  axes. The only way the rigid body can move is to rotate about the fixed point A.

In Figure -b, a rigid body is constrained by a **prismatic pair** which allows only translational motion. In two dimensions, it has one degree of freedom, translating along the  $x$  axis. In this example, the body has lost the ability to rotate about any axis, and it cannot move along the  $y$  axis.

In Figure-c, a rigid body is constrained by a **higher pair**. It has two degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

Now let us consider a plane mechanism with / number of links. Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be  $(l-1)$  and thus the total

number of degrees of freedom will be 3 (l-1) before they are connected to any other link. In general, a mechanism with l number of links connected by j number of binary joints or lower pairs (i.e. single degree of freedom pairs) and h number of higher pairs (i.e. two degree of freedom pairs), then the number of degrees of freedom of a mechanism is given by

$$n = 3(l - 1) - 2j - h$$

This equation is called Kutzbach criterion for the movability of a mechanism having plane motion.

If there are no two degree of freedom pairs (i.e. higher pairs), then  $h = 0$ . Substituting  $h = 0$  in equation (i), we have

$$n = 3(l - 1) - 2j$$

Where,

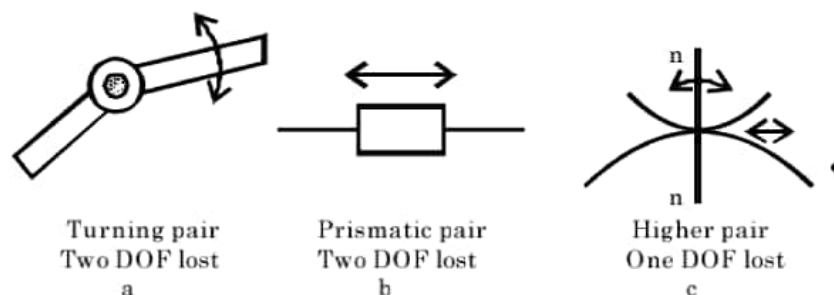
$n$  = degree of freedom

$l$  = no. of link

$j$  = no. of joints/no. of lower pair

$h$  = no. of higher pair.

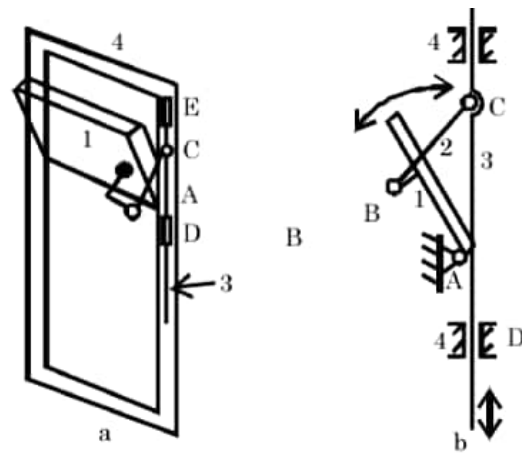
In general, a rigid body in a plane has three degrees of freedom. Kinematic pairs are constraints on rigid bodies that reduce the degrees of freedom of a mechanism. Figure 4-11 shows the three kinds of pairs in planar mechanisms. These pairs reduce the number of the degrees of freedom. If we create a lower pair (Figure 4-11a, b), the degrees of freedom are reduced to 2. Similarly, if we create a higher pair (Figure 4-11c), the degrees of freedom are reduced to 1.



**Figure.** Kinematic Pairs in Planar Mechanisms

## Example 1

Look at the transom above the door in Figure 4-13a. The opening and closing mechanism is shown in Figure 4-13b. Let's calculate its degree of freedom.



**Figure.** Transom mechanism

$n = 4$  (link 1, 3, 3 and frame 4),  $l = 4$  (at A, B, C, D),  $h = 0$

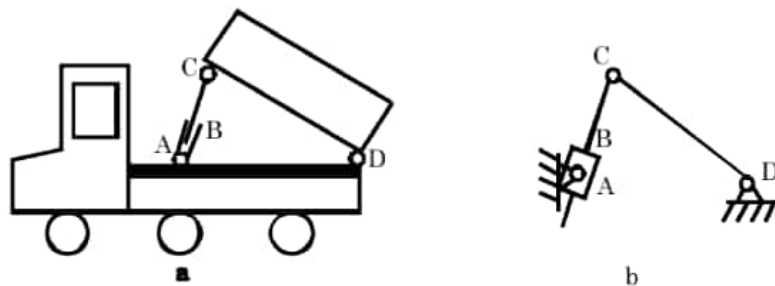
$$F = 3(4 - 1) - 2 \times 4 - 1 \times 0 = 1$$

(4-2)

Note: D and E function as a same prismatic pair, so they only count as one lower pair.

## Example 2

Calculate the degrees of freedom of the mechanisms shown in Figure 4-14b. Figure 4-14a is an application of the mechanism.



**Figure.** Dump truck

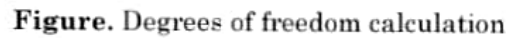
$n = 4$ ,  $l = 4$  (at A, B, C, D),  $h = 0$

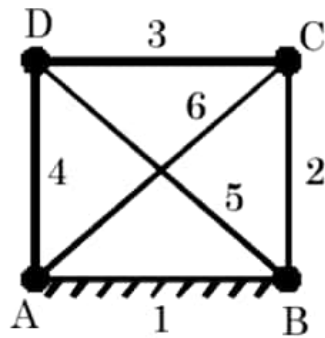
$$F = 3(4 - 1) - 2 \times 4 - 1 \times 0 = 1$$

(4-3)

## Example 3

Calculate the degrees of freedom of the mechanisms shown in Figure.


$$F = 3(6 - 1) - 2 \times 7 - 1 \times 0 = 1$$
$$F = 3(4 - 1) - 2 \times 4 - 1 \times 2 = 1$$

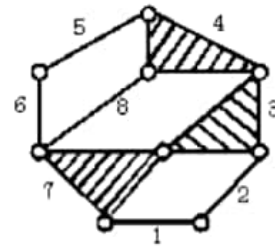


(e) Six bar mechanism

Now, consider the kinematic chain shown in Fig. It has 8 links but only three ternary links. However, the links 6, 7 and 8 constitute a double pair so that the total number of pairs is again 10. The degree of such a linkage will be

$$F = 3(8 - 1) - 2 \times 10$$

$$= 1$$



$$N_b = 4; N_t = 4; N_0 = 0; N = 8; L = 4$$

$$P_1 = 11 \text{ by counting}$$

$$\text{or } P_1 = (N + L + 1) = 11$$

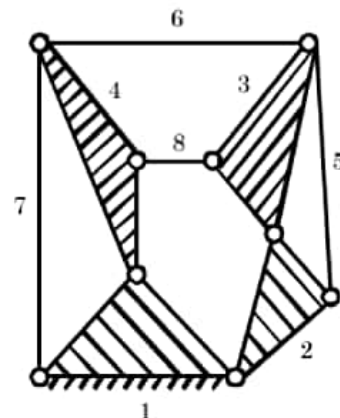
$$F = 3(N - 1) - 2P_1$$

$$= 3(8 - 1) - 2 \times 11 = -1$$

$$\text{or } F = N - (2L + 1)$$

$$= 8 - (2 \times 4 + 1) = -1$$

The linkage has negative degree of freedom and thus is a SUPER STRUCTURE.



$$N_b = 4; N_t = 4; N_0 = 0; N = 8; L = 3$$

$$P_1 = 10 \text{ (by counting)}$$

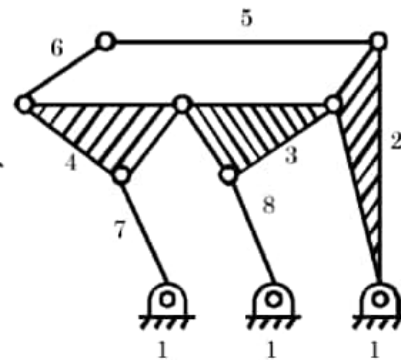
$$\text{or } P_1 = (N + L - 1) = 10$$

$$F = N - (2L + 1) = 8 - (2 \times 3 + 1) = 1$$

$$\text{or } F = 3(N - 1) - 2P_1$$

$$= 3(8 - 1) - 2 \times 10 = 1$$

i.e., the linkage has a constrained motion when one of the seven moving links is driven by an external source.

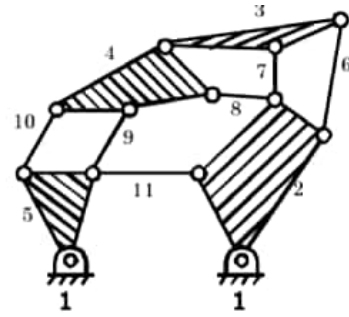


$$N_b = 7; N_t = 2; N_0 = 2; N = 11$$

$$L = 5; P_1 = 15$$

$$F = N - (2L + 1) = 11 - (2 \times 5 + 1) = 0$$

Therefore, the linkage is a structure.



The linkage has 4 loops and 11 links. Referring Table 1.2, it has 2 degrees of freedom. With 4 loops and 1 degree of freedom, the number of joints 13. Three excess joints can be formed by

6 ternary links or

4 ternary links and 1 Quaternary link or

2 ternary links, or

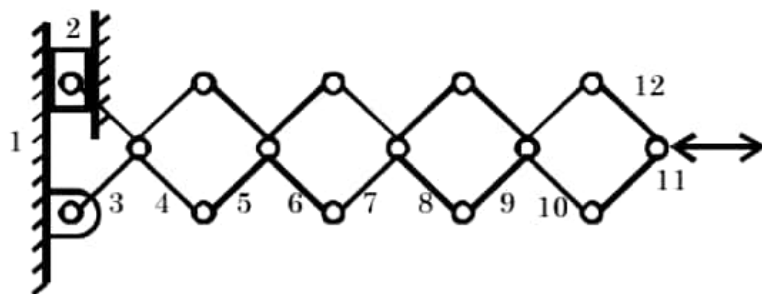
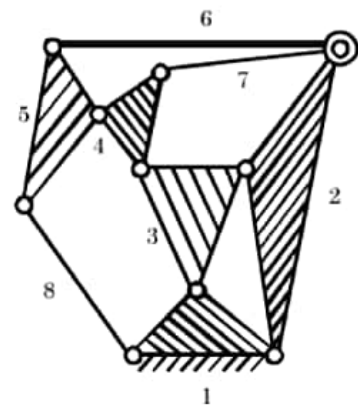
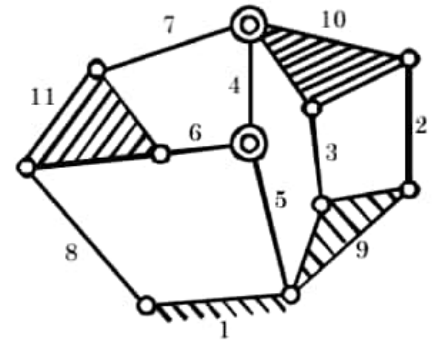
a combination of ternary and quaternary links with double joints.

Figure 1.20 (b) shows one of the possible solutions.

There are 4 loops and 8 links.

$$F = N - (2L + 1) = 8 - (4 \times 2 + 1) = -1$$

It is a superstructure. With 4 loops, the number of links must be 10 to obtain one degree of freedom. As the number of links is not to be increased by more than one, the number of loops has to be decreased. With 3 loops, 8 links and 10 joints, the required linkage can be designed. One of the many solutions is shown in Fig.



It has 5 loops and 12 links.

It has 1 degree of freedom and thus is a mechanism.

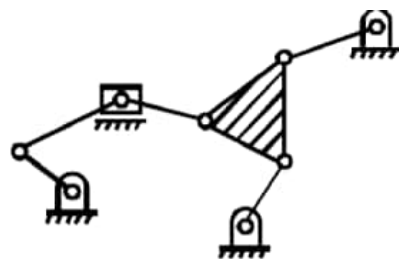
The mechanism has a sliding pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 8 (Fig.)  
=10

(At the slider, one sliding pair and two turning pairs)

$$F = 3(N - 1) - 2P_1 - P_2$$

$$= 3(8 - 1) - 2 \times 10 - 0 = 1$$



The mechanism has a cam pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 7 (Fig.)

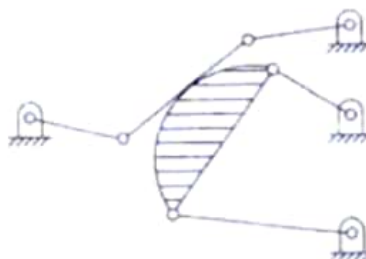
Number of pairs with 1 degree of freedom = 8

Number of pairs with 2 degrees of freedom = 1

$$F = 3(N - 1) - 2P_1 - P_2$$

$$= 3(7 - 1) - 2 \times 8 - 1 = 1$$

Thus, it is a mechanism with one degree of freedom.



## Grubler Criterion

### Grubler's Criterion for Plane Mechanisms

The Grubler's criterion applies to mechanisms with only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting  $n = 1$  and  $h = 0$  in Kutzbach equation, we have

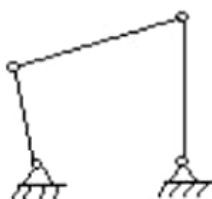
$$1 = 3(l - 1) - 2j \quad \text{or} \quad 3l - 2j - 4 = 0$$

This equation is known as the Grubler's criterion for plane mechanisms with constrained motion.

A little consideration will show that a plane mechanism with a movability of 1 and only single degree of freedom joints can not have odd number of links. The simplest possible mechanisms of this type are a four bar mechanism and a slider – crank mechanism in which  $l = 4$  and  $j = 4$ .

## Grashof 's law

In the range of planar mechanisms, the simplest groups of lower pair mechanisms are four bar linkages. A **four bar linkage** comprises four bar-shaped links and four turning pairs as shown in Figure 5-8.



The link opposite the frame is called the **coupler link**, and the links which are hinged to the frame are called **side links**. A link which is free to rotate through 360 degree with respect to a second link will be said to **revolve** relative to the second link (not necessarily a frame). If it is possible for all four bars to become simultaneously aligned, such a state is called a **change point**.

Some important concepts in link mechanisms are:

1. **Crank:** A side link which revolves relative to the frame is called a *crank*.
2. **Rocker:** Any link which does not revolve is called a *rocker*.
3. **Crank-rocker mechanism:** In a four bar linkage, if the shorter side link revolves and the other one rocks (*i.e.*, oscillates), it is called a *crank-rocker mechanism*.
4. **Double-crank mechanism:** In a four bar linkage, if both of the side links revolve, it is called a *double-crank mechanism*.
5. **Double-rocker mechanism:** In a four bar linkage, if both of the side links rock, it is called a *double-rocker mechanism*.

## Classification

Before classifying four-bar linkages, we need to introduce some basic nomenclature.

In a four-bar linkage, we refer to the *line segment between hinges* on a given link as a **bar** where:

- $s$  = length of shortest bar
- $l$  = length of longest bar
- $p, q$  = lengths of intermediate bar

**Grashof's theorem** states that a four-bar mechanism has *at least* one revolving link if

$$s + l \leq p + q$$

and all three mobile links will rock if

$$s + l > p + q$$

The inequality 5-1 is **Grashof's criterion**.

All four-bar mechanisms fall into one of the four categories listed in Table 5-1:

Case	$l + s$ vers. $p + q$	Shortest Bar	Type
1	<	Frame	Double-crank
2	<	Side	Rocker-crank
3	<	Coupler	Double rocker
4	=	Any	Change point

5	>	Any	Double-rocker
Table: Classification of Four-Bar Mechanisms			

From Table we can see that for a mechanism to have a crank, the sum of the length of its shortest and longest links must be less than or equal to the sum of the length of the other two links. However, this condition is necessary but not sufficient. Mechanisms satisfying this condition fall into the following three categories:

1. When the shortest link is a side link, the mechanism is a crank-rocker mechanism. The shortest link is the crank in the mechanism.
2. When the shortest link is the frame of the mechanism, the mechanism is a double-crank mechanism.
3. When the shortest link is the coupler link, the mechanism is a double-rocker mechanism.

## Inversion of Mechanism

Method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as inversion of the mechanism.

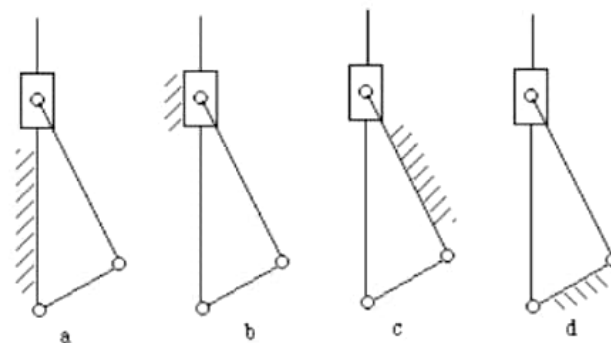
**Inversion** is a term used in kinematics for a reversal or interchanges of form or function as applied to kinematic chains and mechanisms.

## Types of Kinematic Chains

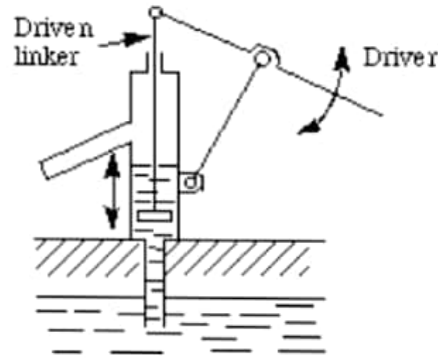
The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view:

1. Four bar chain or quadric cyclic chain,
2. Single slider crank chain, and
3. Double slider crank chain.

For example, taking a different link as the fixed link, the slider-crank mechanism shown in Figure 5-14a can be inverted into the mechanisms shown in Figure 5-14b, c, and d. Different examples can be found in the application of these mechanisms. For example, the mechanism of the pump device in Figure 5-15 is the same as that in Figure 5-14b.



**Figure: Inversions of the crank-slide mechanism**

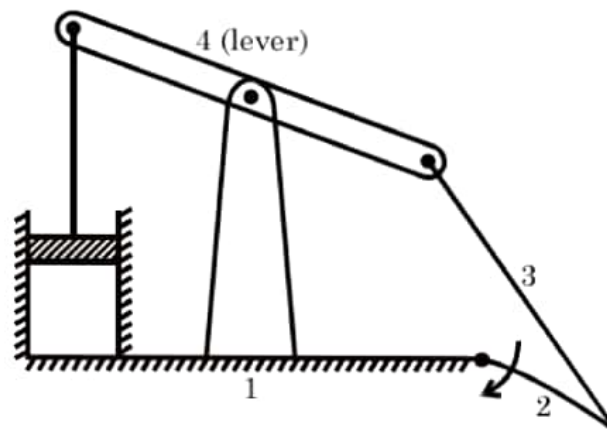


**Figure. A pump device**

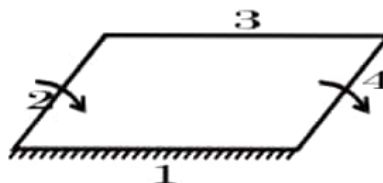
- Keep in mind that the inversion of a mechanism does not change the motions of its links relative to each other but does change their absolute motions.
- Inversion of a kinematic chain has no effect on the relative motion of its links.
- The motion of links in a kinematic chain relative to some other links is a property of the chain and is not that of the mechanism.
- For  $L$  number of links in a mechanism, the number of possible inversions is equal to  $L$ .

## 1. Inversion of four bar chain

(a) Crank and lever mechanism/Beam engine (1<sup>st</sup> inversion).



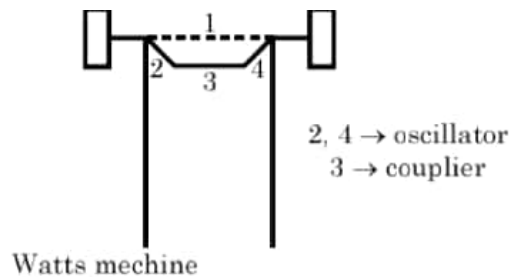
(b) Double crank mechanism (Locomotive mechanism) 2<sup>nd</sup> inversion.



(c) Double lever mechanism (Ackermann steering) 3<sup>rd</sup> inversion.

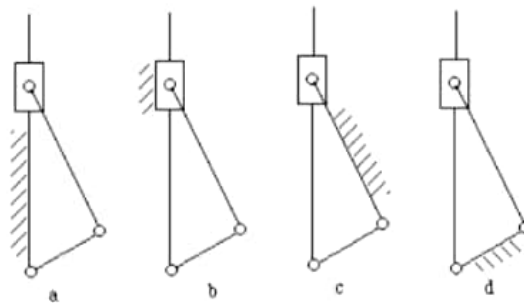
2, 4 → Oscillator

3 → Coupler

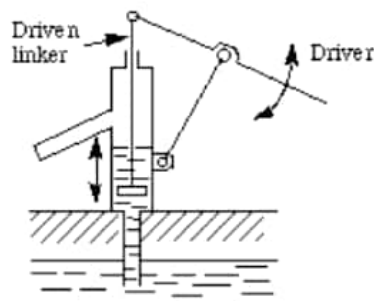


## 2. Inversion of the Slider-Crank Mechanism

Inversion is a term used in kinematics for a reversal or interchanges of form or function as applied to kinematic chains and mechanisms. For example, taking a different link as the fixed link, the slider-crank mechanism shown in Figure (a) can be inverted into the mechanisms shown in Figure (b), (c), and d. Different examples can be found in the application of these mechanisms. For example, the mechanism of the pump device in Figure is the same as that in Figure (b).



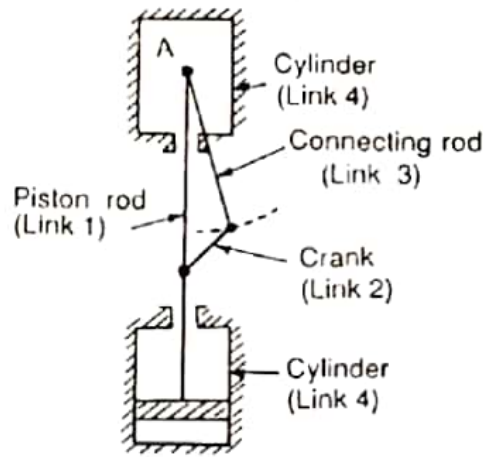
**Figure.** Inversions of the crank-slide mechanism



**Figure.** A pump device

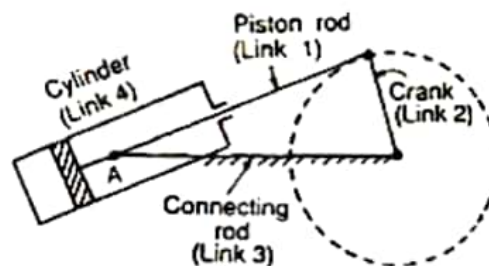
Keep in mind that the inversion of a mechanism does not change the motions of its links relative to each other but does change their absolute motions.

**1. Pendulum pump or Bull engine:** In this mechanism the inversion is obtained by fixing cylinder or link4 (i.e. sliding pair), as shown in figure below.



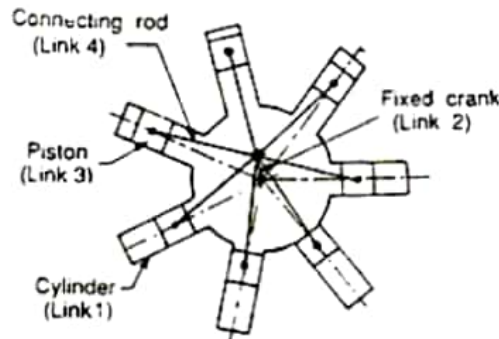
The **duplex** pump which is used to supply feed water to boilers uses this mechanism.

**2. Oscillating cylinder engine:** It is used to convert reciprocating motion into rotary motion.



**Fig. Oscillating cylinder engine**

**3. Rotary internal combustion engine or Gnome engine:**

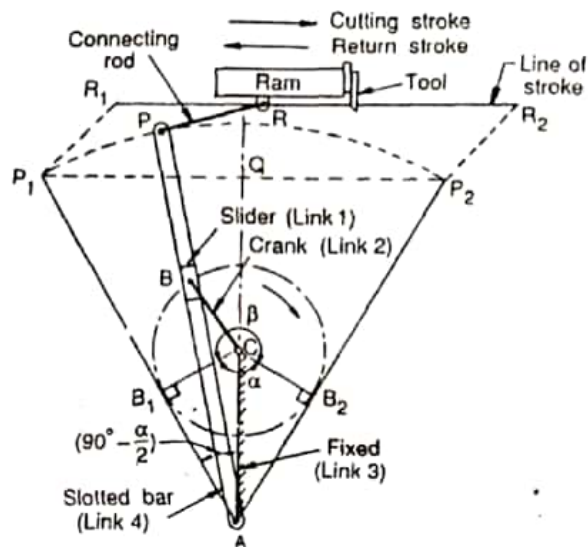


**Fig. Rotary internal combustion engine**

Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place.

## Quick return motion mechanism

**4. Crank and slotted lever quick return motion mechanism:** This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.



$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \text{ or } \frac{360^\circ - \alpha}{\alpha}$$

Length of stroke

$$= 2 AP \times \frac{CB}{AC}$$

**Note:** We see that the angle  $\beta$  made by the forward or cutting stroke is greater than the angle  $\alpha$  described by the return stroke. Since the crank rotates with uniform angular speed, therefore the return stroke is completed within shorter time. Thus it is called quick return motion mechanism.

### 5. Whitworth quick return motion mechanism:

This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link  $CD$  (link 2) forming the turning pair is fixed, as shown in Figure. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank  $CA$  (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at  $A$  slides along the slotted bar  $PA$  (link 1) which oscillates at a pivoted point  $D$ . The connecting rod  $PR$  carries the ram at  $R$  to which a cutting tool is fixed. The motion of the tool is constrained along the line  $RD$  produced, i.e. along a line passing through  $D$  perpendicular to  $CD$ .

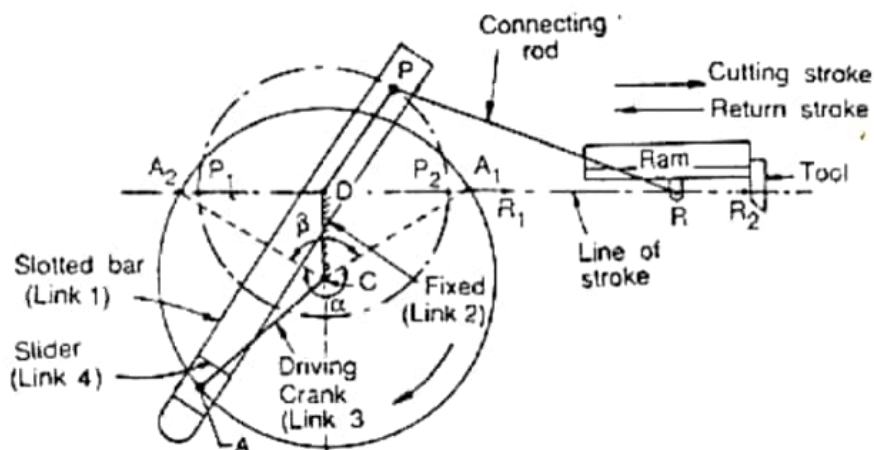


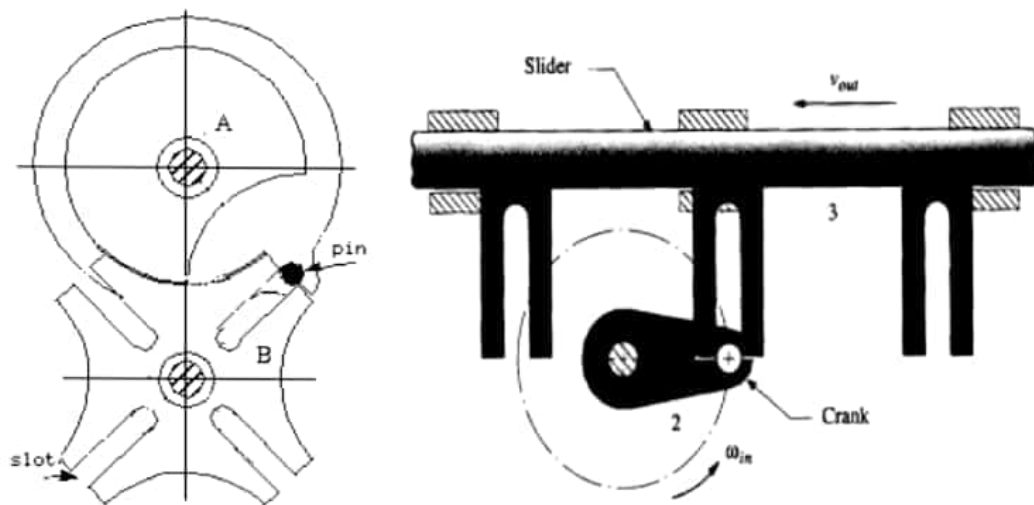
Fig. Whitworth quick return motion mechanism

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \text{ or } \frac{360^\circ - \beta}{\beta}$$

**Note:** In order to find the length of effective stroke  $R_1 R_2$ , mark  $P_1 R_1 = P_2 P_2 = PR$ . The length of effective stroke is also equal to  $2 PD$

## The Geneva Wheel

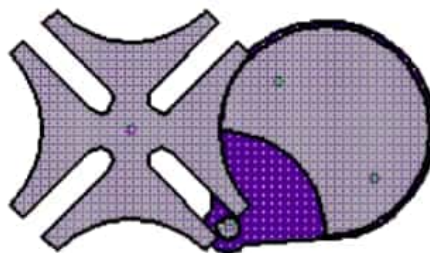
An interesting example of intermittent gearing is the **Geneva Wheel** shown in Figure 8-4. In this case the **driven wheel**,  $B$ , makes one fourth of a turn for one turn of the **driver**,  $A$ , the **pin**,  $a$ , working in the **slots**,  $b$ , causing the motion of  $B$ . The circular portion of the driver, coming in contact with the corresponding hollow circular parts of the driven wheel, retains it in position when the pin or tooth  $a$  is out of action. The wheel  $A$  is cut away near the pin  $a$  as shown, to provide clearance for wheel  $B$  in its motion.



**Figure.** Geneva wheel

If one of the slots is closed,  $A$  can only move through part of the revolution in either direction before pin  $a$  strikes the closed slot and thus stop the motion. The device in this modified form was used in watches, music boxes, *etc.*, to prevent over winding. From this application it received the name Geneva stop. Arranged as a stop, wheel  $A$  is secured to the spring shaft, and  $B$  turns on the axis of the spring barrel. The number of slots or interval units in  $B$  depends upon the desired number of turns for the spring shaft.

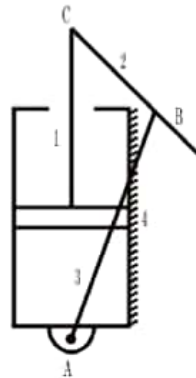
An example of this mechanism has been made in Sim Design, as in the following picture.



**BIJAN KUMAR GIRI**

- Geneva mechanism is used to **transfer components** from one station to the other in a rotary transfer machine
- Geneva mechanism produces intermittent rotary motion from continuous rotary motion.

**Hand Pump:** Here also the slotted link shape is given to the slider and vice-versa, in order to get the desired motion.



Here the slider (link 4) is fixed and hence, it is possible for link 1 to reciprocate along a vertical straight line. At the same time link 2 will rotate and link 3 will oscillate about the pin.

## Inversion of Double slider crank chain

It has four binary links, two revolute pairs, two sliding pairs.

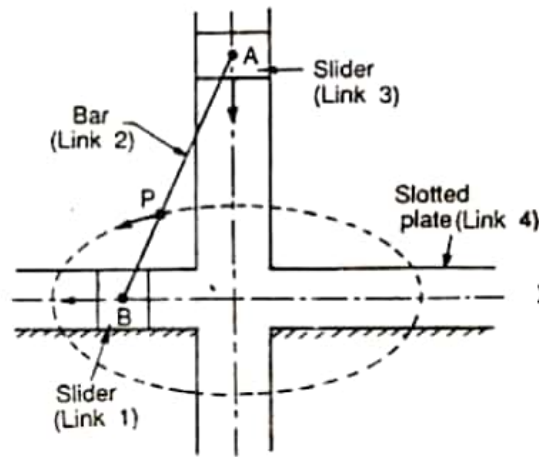
Its various types are:

- Elliptical Trammel
- Scotch Yoke mechanism
- Oldham's coupling.

## Elliptical trammels

It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Figure. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3, are known as sliders and form sliding pairs with link 4. The link  $AB$  (link 2) is a bar which forms turning pair with links 1 and 3.

When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as  $P$  traces out an ellipse on the surface of link 4, as shown in Figure (a). A little consideration will show that  $AP$  and  $BP$  are the semi-major axis and semi-major axis of the ellipse respectively. This can be proved as follows:



**Note:** If  $P$  is the mid-point of link  $BA$ , then  $AP = BP$ . Hence if  $P$  is the midpoint of link  $BA$ , it will trace a circle.

## Scotch yoke mechanism

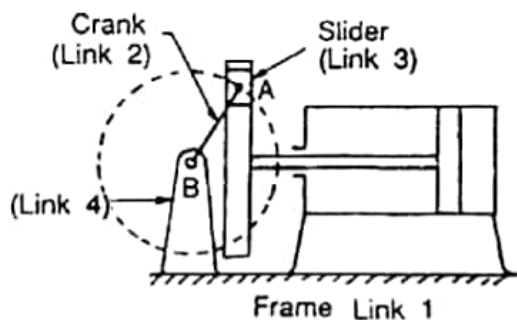
Here the constant rotation of the crank produces harmonic translation of the yoke. Its four binary links are:

1. Fixed Link
2. Crank
3. Sliding Block
4. Yoke

The four kinematic pairs are:

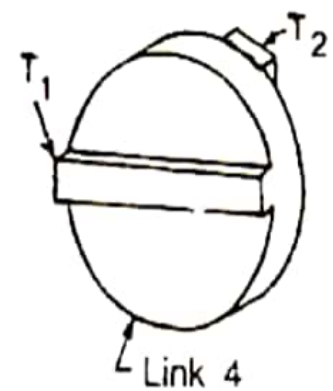
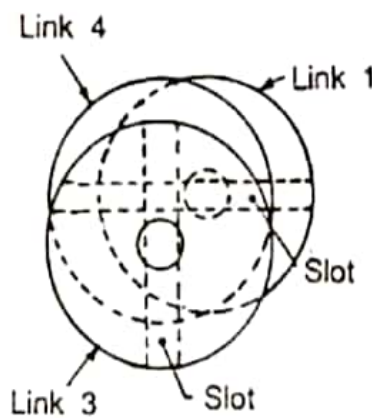
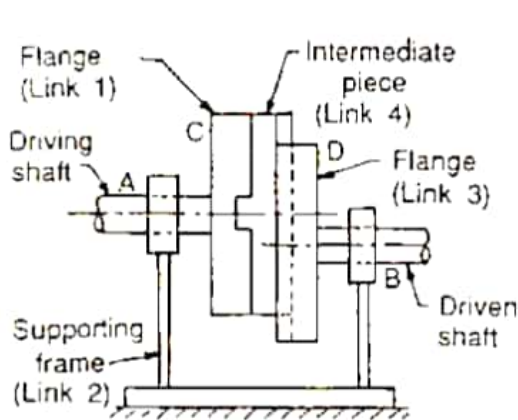
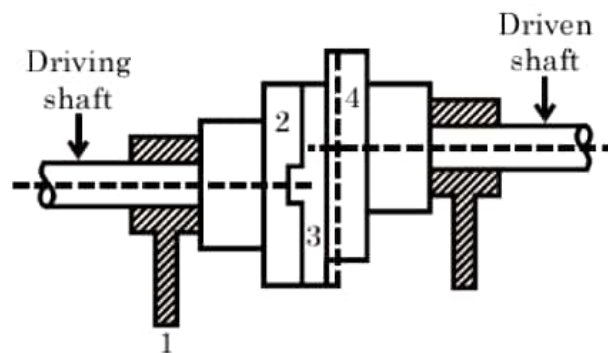
1. revolute pair (between 1 & 2)
2. revolute pair (between 2 & 3)
3. prismatic pair (between 3 & 4)
4. prismatic pair (between 4 & 1)

This mechanism is used for converting rotary motion into a reciprocating motion.



## Oldham's coupling

- It is used for transmitting angular velocity between two parallel but eccentric shafts.
- An Oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart.
- Oldham's coupling is the inversion of double slider crank mechanism.
- The shafts are coupled in such a way that if one shaft rotates the other shaft also rotates at the same speed.



- The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces.
- The intermediate piece (link 4) which is a circular disc, have two tongues (i.e. diametrical projections)  $T_1$  and  $T_2$  on each face at right angles to each other.
- The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3).
- The link 4 can slide or reciprocate in the slots in the flanges.

Let the distance between the axes of the shafts is constant, the centre of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shafts. Then the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path.

$\therefore$  **Maximum** sliding' speed of each tongue (in m/s),

$$v = \omega \cdot r$$

Where  $\omega$  = Angular velocity of each shaft in rad/s, and  
 $r$  = Distance between the axes of the shafts in metres.

$$\text{Now } F_Q = \frac{F_P}{\cos \phi} = 43257.8 \text{ N}$$

$$\begin{aligned}\text{Now } F_T &= F_Q \sin(\theta + \phi) \\ &= 39524.1 \text{ N}\end{aligned}$$

$$\begin{aligned}\therefore T &= F_T \times r. \\ &= 7904.8 \text{ Nm}\end{aligned}$$

$$\therefore \boxed{T = 7.904 \text{ kNm}}$$

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