

PROBABLE

Questions

& (Part-I)

Answers

Subject - ADC

Semester - 5th

Branch - ETC

Module - ALL

1. a) Suggest a suitable Mathematical Model for a Mobile radio channel. Justify the Model.

Sol: Mathematical Model for a Mobile radio channel can be suggested which results in time variant multipath propagation of the transmitted signal which may be characterized as time variant linear filter.

- Such linear filters are characterized by time variant channel impulse response $h(\tau; t)$ where $h(\tau; t)$ is the response of channel at time t due to an impulse applied at time $t - \tau$.

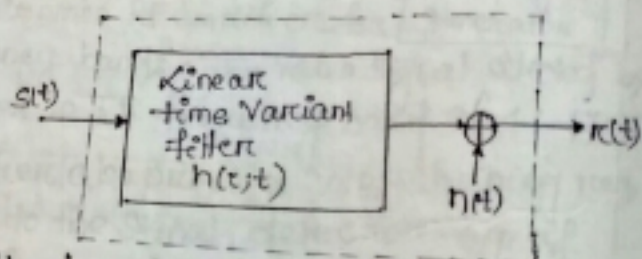
' τ ' → Age (Elapsed time) Variable.

→ For an input signal $s(t)$, the channel output signal

is:

$$r(t) = s(t) \otimes h(\tau; t) + n(t)$$

$$r(t) = \int_{-\infty}^{\infty} h(\tau; t) s(t - \tau) d\tau + n(t)$$



For a Mobile radio channels, time variant impulse response has the form.

$$h(\tau; t) = \sum_{k=1}^K a_k(t) \delta(\tau - \tau_k)$$

Where, $\{a_k(t)\}$ → possibly time-variant attenuation factor for the multipath propagation path.

→ The received signal has the form.

$$r(t) = \sum_{k=1}^K a_k(t) s(t - \tau_k) + n(t)$$

Hence, received signal consists of K multipath components where each component is attenuated by $\{a_k\}$ & delayed by $\{\tau_k\}$.

b) What are the two essential functions of a transmitter?

i) Transmitter converts the electrical signal into a form which is suitable for transmission through the physical channel or transmission medium.

ii) Transmitter must translate the information signal to be transmitted into the appropriate frequency range that matches the frequency allocation of assigned to the transmitter. These signals transmitted by multiple radio stations do not interfere with one another.

c) What are the impairments caused by a channel?

→ The transmitted signal is corrupted in a random manner in the channel by a variety of possible mechanisms. Most common one is additive noise which is generated at the front end of the receiver, where signal amplification is performed. This noise is called thermal noise.

→ Multipath propagation which manifests itself as the time variation in the signal amplitude is called fading. It occurs in long range, short wave radio transmission.

d) Give the AM broadcast range of frequencies. Justify the use of those frequencies.

AM broadcast range of frequencies is 0.3-3 MHz. In AM broadcast, the range of frequencies with ground wave propagation of even more powerful radio station is limited to about 100 miles. Again, powerful AM radio broadcast stations can propagate over large distances, via sky wave over the F-layer of the ionosphere, which ranges from 90-250 miles above the earth surface.

e) What kind of signal is a periodic sine wave? power or energy? Why?

A periodic sine wave is a power signal, as its average power is finite & non-zero. The power of a signal can be defined as:

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

So this indicates, almost all practical signals & periodic signals are power signal.

Q) What is the Fourier transform of a periodic impulse train with a period of 1 sec?

Ans: A periodic impulse train with a period of 1 sec can be represented as: $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$

$$\text{At } T=1 \rightarrow x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_{n=-\infty}^{\infty} \delta(t - n)$$

The Fourier Transform can be expressed as:

$$F[x(t)] = X(\omega) \\ = F\left[\sum_{n=-\infty}^{\infty} \delta(t - nT)\right] = \omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$$\text{Where, } \omega_0 = 2\pi/T = 2\pi$$

$$\text{So } X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n)$$

Q) Is AM a linear system? Justify

Ans: Amplitude Modulation is a linear system, as in AM the carrier signal is a linear function of the message signal. Considering the equation of the message signal & amplitude modulated signal, $s(t) = x(t) \cos \omega_c t + A \cos \omega_c t$

$$\rightarrow S(t) = [A + x(t)] \cos \omega_c t$$

So, as this equation is linear, hence, amplitude modulation is called a linear modulation system.

Q) Is sampling essential for TDM of 'N' number of signals? Justify

Ans: Sampling is essential for TDM of 'N' number of signals as in TDM, the multiplexer is a signal pole rotating switch or commutator. This commutator will rotate at f_s rotations per second. Since, each rotation corresponds to one frame, therefore one frame is completed in T_s seconds.

$$\text{Where, } T_s = 1/f_s$$

$T_s \rightarrow$ Sampling interval

$f_s \rightarrow$ Sampling frequency

→ Again the function of Commutator is:-

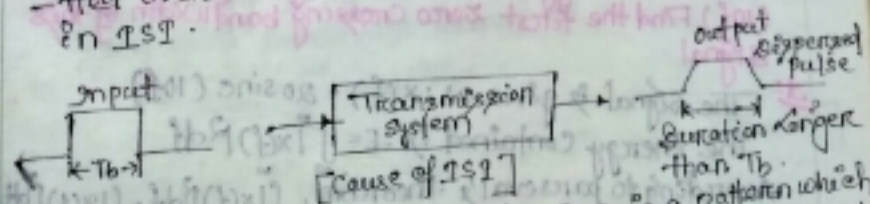
- i) To take narrow samples of each input message at a rate f_s which is higher than $2f_m$.
- ii) To sequentially Interleave the N Samples inside the interval T_s , $T_s = 1/f_s$

→ Another reason being is, TDM System supposed to have its signaling rate as high as possible & the signaling rate can be increased by increasing the sampling rate f_s &/or the number of input signals N .

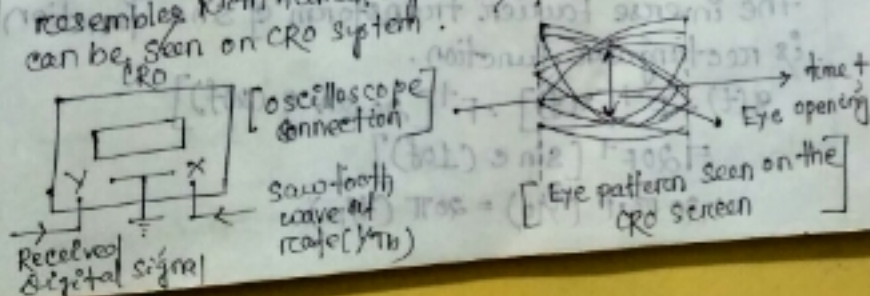
Q. What is intersymbol interference? Sketch a diagram to illustrate.

ISI arises due to the imperfections in the overall frequency response of the system. When a short pulse of duration T_b seconds is transmitted through a band-limited channel, then the frequency component contained in the input pulse are differentially attenuated & more importantly differentially delayed by the system.

→ Due to this the pulse appearing at the output of the system will be dispersed over an interval which is longer than T_b seconds. Due to this dispersion, duration T_b will interfere with each other which transmitted over the communication channel. This will result in ISI.

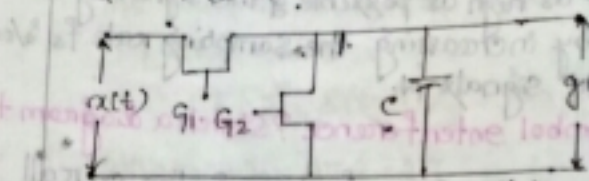


→ The practical way to study ISI is a pattern which resembles with human eye. This is called eye pattern & can be seen on CRO system.

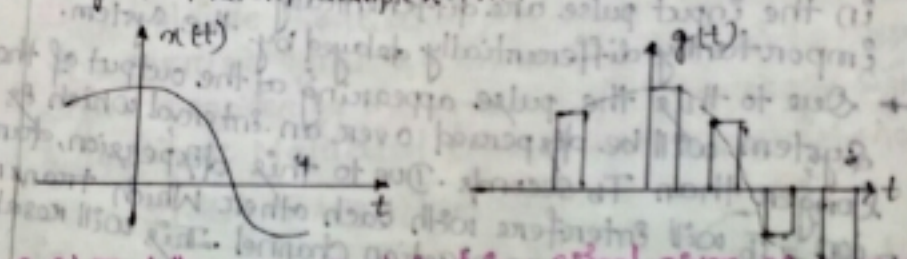


1) Draw a circuit to generate PAM signal
 PAM (Pulse Amplitude Modulation) is of two types:
 i) Flat top Sampled PAM
 ii) Natural Sampled PAM

Flat top Sampled PAM can be generated by sample & hold circuit. The circuit can be drawn as follows:



$G_1 \rightarrow$ Sampling switch
 $G_2 \rightarrow$ Discharge switch
 $x(t) =$ Base band signal
 $g(t) =$ Flat top Sampled PAM



2.a) Find the energy contained in a signal given as $20 \text{ sinc}(10f)$
 (10f) Find the first zero crossing bandwidth of this signal

The signal is given as: $x(f) = 20 \text{ sinc}(10f)$

The energy contained is: $E = \int |x(f)|^2 df$

According to Parseval's theorem, $\int |x(f)|^2 df = \int |x(t)|^2 dt$

$x(t)$ is the inverse Fourier transform of $x(f)$. So the inverse Fourier transform of sinc function is rectangular function.

$$\begin{aligned} x(t) &= F^{-1}[x(f)] = F^{-1}[20 \text{ sinc}(10f)] \\ &= 20 F^{-1}[\text{sinc}(10f)] \\ &= 20 \text{ rect}(t/10) = 20\pi(t/10) \end{aligned}$$

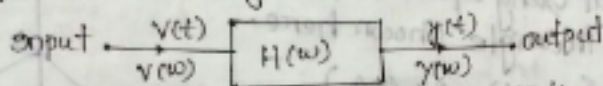
Energy can be found out by:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = 20 \int_{-\infty}^{\infty} \text{rect}(t/10) dt = 20 \cdot 10 = 200$$

→ The first zero crossing band width of the signal is $\omega = \frac{1}{2T} = \frac{1}{20} = 0.5 \text{ Hz}$

b) Find the Energy contained in the signal $v(t)$ when it is passed through an RC high pass filter of time constant τ_c . Derive the expression used.

Ans: Let us consider a signal $v(t)$ which will be passed through an ideal RC high pass filter which is shown hence:



The transfer function of the ideal high pass filter can be shown here:

The response or output of a system is expressed as:

$$y(w) = H(w) v(w)$$

$v(w)$ → Fourier transform of $v(t)$

$y(w)$ → Fourier transform of $y(t)$

→ The energy E_o of the output signal $y(t)$ may be expressed using Parseval's theorem as,

$$E_o = \frac{1}{2\pi} \int_{-\infty}^{\infty} |y(w)|^2 dw$$

$$\Rightarrow E_o = \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(w) v(w)|^2 dw \quad [\because y(w) = H(w) v(w)]$$

$$\text{Hence, } H(w) = \begin{cases} 1 & \omega_m \leq \omega \leq \infty \\ 0 & -\omega_m \leq \omega \leq \omega_m \end{cases}$$

$$\text{Therefore, } E_o = \frac{1}{2\pi} \int_{\omega_m}^{\infty} |1 \cdot v(w)|^2 dw = \frac{1}{2\pi} \int_{\omega_m}^{\infty} |v(w)|^2 dw$$

Assuming that Fourier Transform $v(w)$ is constant

With frequency for a narrow band $\omega_m \rightarrow \infty$. Therefore the energy of the signal over this will be:

$$E_0 = \frac{1}{2\pi} \left[\int_{\omega_m}^{\infty} d\omega \right] |V(\omega)|^2 \rightarrow [E_0 = \infty]$$

Hence, $v(t)$ is a power signal.

Q. Derive & sketch the spectrum of a triangular pulse of height 'A' & duration T from the corresponding rectangular pulses.

A: The spectrum of the rectangular pulse can be obtained by direct evaluation of the Fourier Transform integral using equation for the piecewise linear lines of the triangle shown here.

→ Taking, $w(t) = A \wedge (t/T)$

$$\frac{dw(t)}{dt} = \frac{1}{T} u(t+T) - \frac{2}{T} u(t) + \frac{1}{T} u(t-T)$$

$$\rightarrow \frac{d^2w(t)}{dt^2} = \frac{1}{T} \delta(t+T) - \frac{2}{T} \delta(t) + \frac{1}{T} \delta(t-T)$$

The Fourier transform pair of the second derivative

$$\frac{d^2w(t)}{dt^2} \leftrightarrow \frac{1}{T} e^{j\omega T} - \frac{2}{T} + \frac{1}{T} e^{-j\omega T}$$

This can be written as

$$\frac{d^2w(t)}{dt^2} \leftrightarrow \frac{1}{T} (e^{j\omega T/2} - e^{-j\omega T/2})^2 = \frac{4}{T} (\sin \pi f T)^2$$

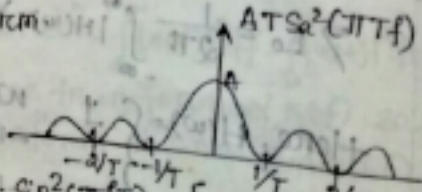
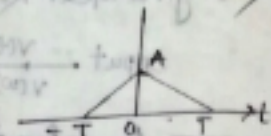
Applying integral theorem twice, the Fourier Transform for the original wave form

$$W(f) \leftrightarrow \frac{-4}{T} \frac{(\sin \pi f T)^2}{(j2\pi f)^2}$$

This,

$$W(f) = A \wedge (t/T) \leftrightarrow AT \cdot \frac{\sin^2(\pi f T)}{(2\pi f)^2}$$

$$\rightarrow W(f) = A \wedge (t/T) \leftrightarrow AT \text{Sa}^2(\pi f T)$$



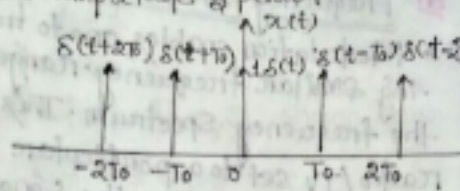
[spectrum of triangular pulse]

b) Give the Fourier Series of an impulse train of Unit Amplitude & periodic T_0 sec. Sketch the same.

Impulse train of unit amplitude & period T_0 sec. can be represented as

The signal can be defined as:-

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$



The Fourier Series can be represented by,

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

As the above signal represents an even signal, hence

$$b_n = 0$$

The co-efficients can be expressed as

$$a_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) dt = \frac{1}{T_0}$$

$$a_0 = \frac{1}{T_0}$$

Similarly, the even co-efficient

$$a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \cos n\omega_0 t dt$$

$$= \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) \cos n\omega_0 t dt = \frac{2}{T_0} [\cos n\omega_0 t]_{t=0} = \frac{2}{T_0} \cdot 1$$

$$= \frac{2}{T_0} \cdot \text{so, } a_n = \frac{2}{T_0}$$

So the geometrical Fourier Series representation is

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

$$\Rightarrow x(t) = \frac{1}{T_0} + \sum_{n=1}^{\infty} \frac{2}{T_0} \cos n\omega_0 t$$

4. a) Explain the meaning of frequency translation. Why is it used?

31 Frequency Translation:

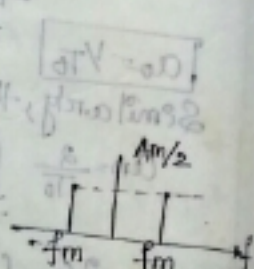
→ Modulation enables one to translate the signals occupying similar frequency range to different regions in the frequency spectrum. This allows a user to tune his radio/TV set to a particular broadcasting station. Without Modulation, the signals from various stations would have resulted in interfering the signals.

→ So, frequency translation is the process where the low frequency message signal will be translated to a higher frequency spectrum with the high frequency carrier signal. The method of frequency translation can be shown as follows:

Method of Frequency Translation:

A signal may be translated to a new spectral range by multiplying the signal with an auxiliary sinusoidal signal. Considering the signal as sinusoidal in nature which is given by,

$$\begin{aligned} V_m(t) &= A_m \cos \omega_m t = A_m \cos 2\pi f_m t \\ &= \frac{A_m}{2} [e^{j\omega_m t} + e^{-j\omega_m t}] \\ &= \frac{A_m}{2} [e^{j2\pi f_m t} + e^{-j2\pi f_m t}] \end{aligned}$$



The spectrum can be drawn here:

→ Considering the auxiliary sinusoidal signal

$$\begin{aligned} V_c(t) &= A_c \cos \omega_c t = A_c \cos 2\pi f_c t \\ &= \frac{A_c}{2} [e^{j\omega_c t} + e^{-j\omega_c t}] \\ &= \frac{A_c}{2} [e^{j2\pi f_c t} + e^{-j2\pi f_c t}] \end{aligned}$$

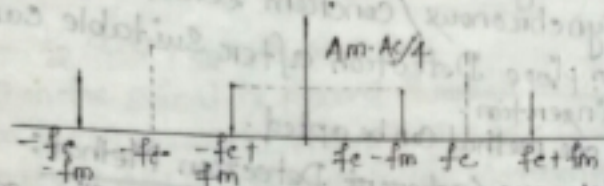
Considering the result of Multiplication,

$$V_m(t) \cdot V_c(t) = A_m A_c \cos \omega_m t \cdot \cos \omega_c t$$

$$= \frac{A_m \cdot A_c}{2} [\cos(\omega_m + \omega_c)t + \cos(\omega_c - \omega_m)t]$$

$$= \frac{A_m \cdot A_c}{4} [e^{j(\omega_c + \omega_m)t} + e^{-j(\omega_c + \omega_m)t} + e^{j(\omega_c - \omega_m)t} + e^{-j(\omega_c - \omega_m)t}]$$

The new Spectral Amplitude pattern is shown here.



→ There are now 4 spectral components resulting in two sinusoidal wave forms. one of frequency $f_c + f_m$ & other frequency $f_c - f_m$.

Need of Frequency Translation:-

By frequency translation, a no. of useful purpose can be solved.

i) Narrow Banding:-

→ Assuming an audio signal extends from 50 - 10⁴ Hz & transmitted directly from the antenna, the ratio of highest to lowest component is 200. Now to reduce the length of the antenna, audio spectrum is translated, so that the range would be (10⁶ + 50) to (10⁶ + 10⁴) Hz.

→ Now the ratio of highest to lowest frequency would be only 1.01 so the frequency translation may be used to change a 'wide band' signal into a 'narrow band' signal which can be more conventionally processed.

ii) Common Processing:-

→ Sometimes, a number of signal of same character have to be processed but occupying different spectral range. So, it is necessary to adjust the frequency range of processing apparatus to correspond to the frequency range of the signal to be processed.

iii) Frequency Multiplexing.

iv) Feasibility of Antenna.

b) Suggest a suitable receiver to receive a signal generated by a balanced Modulator. Draw the appropriate Spectra.

Ans. The signal generated by a balanced Modulator is a DSB-SC signal. Now DSB-SC signal can be received by two demodulators/detectors which are:

- i) Synchronous/coherent Detection Method
- ii) Envelope Detection after suitable carrier regeneration.

→ AM one method can be opted.

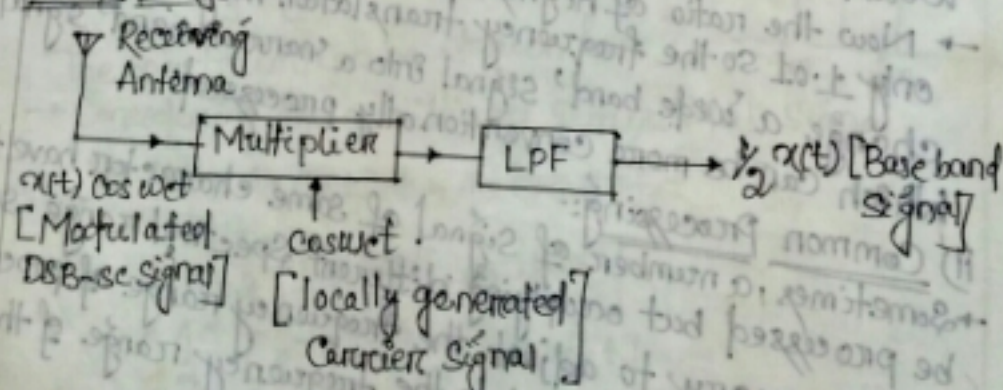
i) Synchronous/coherent Detection Method:-

→ DSB-SC system is used at the transmitter end to shift the Modulating signal (frequency ω_m) to a higher carrier frequency ω_c . Now, this Modulated signal is transmitted from the transmitter & it reaches the receiver through a transmission medium.

→ At the receiver end, the original Modulating signal $\alpha(t)$ is recovered from modulated signal which can be achieved by simply retranslating the base band or modulating signal from a higher spectrum centered at ω_c to the original spectrum.

→ The process of retranslation is called demodulation/detection.

Block Diagram:-



Working Principle:-

→ Here the received signal is first multiplied with a locally generated carrier signal $\cos \omega_c t$ & then passed through a LPF. At the output of LPF, the original Modulating signal is recovered.

→ Mathematically, $S(t) = \{x(t) \cos \omega_c t\} \cos \omega_c t$

$$\Rightarrow S(t) = x(t) \cos^2 \omega_c t = x(t) \left(\frac{1 + \cos 2\omega_c t}{2} \right)$$

$$= \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos 2\omega_c t$$

→ When the signal is passed through LPF, then the term $\frac{1}{2} x(t) \cos 2\omega_c t$, centered at $\pm 2\omega_c$ is suppressed & the output of LPF will be original Modulating signal $\frac{1}{2} x(t)$.

→ The frequency & phase of locally generated carrier signal & the carrier signal at the transmitter must be identical; which means the local oscillator signal must be exactly coherent or synchronized with carrier at the transmitter both in phase & frequency, otherwise the detected signal would get distorted. Hence the method of recovery is called synchronous/coherent detection.

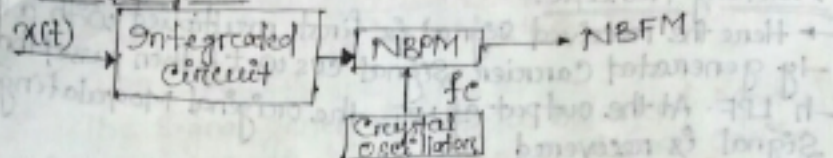
5.0) Explain how Armstrong's Method generates an NBFM signal. Draw the phasor of such an NBFM signal.

Armstrong Method:-

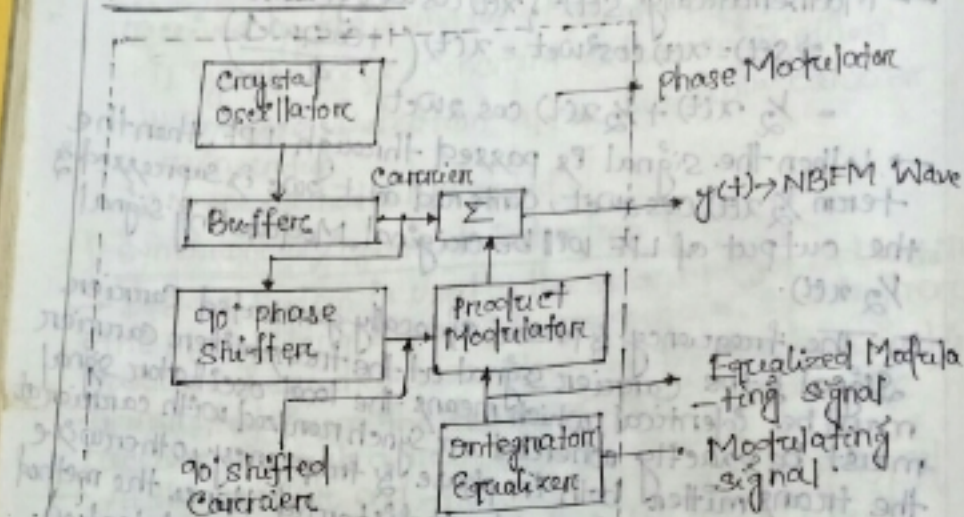
→ It is used to generate an angle Modulated signal. First it will generate a narrow band angle Modulated signal & then change it to a wide band signal. This method is usually known as the indirect Method for generation of FM & PM.

→ Hence, frequency stability will be high as crystal oscillator is used. So the working principle is to generate NBFM indirectly by utilizing the phase Modulation technique then changing NBFM to WBFM.

Block Diagrams-



Practical Circuit:-



→ The modulating signal passed through an integrated circuit before applying to phase Modulator. Let NBFM wave produced at the output of phase Modulator.

$$y(t) = V_c \cos[2\pi f_c t + \phi(t)]$$

$V_c \rightarrow$ Amplitude
 $f_c \rightarrow$ Frequency of carrier produced by crystal oscillator.

→ The phase angle $\phi_1(t)$ of $y(t)$ is related to $x(t)$ as:

$$\phi_1(t) = 2\pi K_f \int_0^t x(t) dt$$

$K_f \rightarrow$ Frequency sensitivity of the modulator.

If $\phi_1(t)$ will be small then.

$$\cos[\phi_1(t)] \approx 1, \sin[\phi_1(t)] \approx \phi_1(t)$$

So the expression for $x(t)$ can be obtained as,

$$y(t) = V_c \cos[2\pi f_c t + \phi_1(t)]$$

$$= V_c [\cos(2\pi f_c t) \cos \phi_2(t) - \sin(2\pi f_c t) \sin \phi_2(t)]$$

$$= V_c \cos(2\pi f_c t - \phi_2(t))$$

Substituting, $\phi_2(t) = 2\pi k_f \int x(t) dt$

$$\text{So, } y(t) = V_c \cos(2\pi f_c t - 2\pi k_f \int x(t) dt)$$

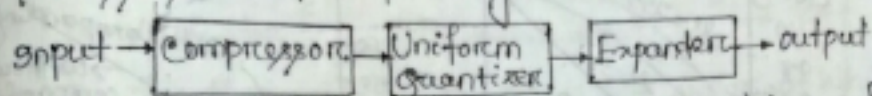
This represents NBFM which means at the out put of the phase Modulator, NBFM will be obtained.

Implementation of Phase Modulator:-

- The crystal oscillator produces a stable unmodulated carrier which is applied to the 90° phase shift for as well as the combining net through a buffer.
- The 90° phase shifted carrier is applied to the balance Modulator along with Modulating signal. At the output of product modulator, DSB-SC signal (AM without carrier) will be obtained.
- The signal consists of only two side bands with their resultant in phase with 90° shifted carrier. The two side bands & original carrier without any phase shift are applied to a combining network (Σ). The resultant of vector addition of carrier & two side bands will be obtained at the output.

b) Suggest a method to implement compression of a signal having a large dynamic range.

Compression is the process where the weak signals are amplified & strong signals are attenuated before applying them to uniform quantizer. At the receiver exactly opposite is followed which is called expansion & circuit is called expander. So, the combined process is called Companding.



→ Signals having large amplitude are having large dynamic range. These signals can be compressed which are having logarithmic c/s compression. This compression can be achieved by two methods:-

- i) μ -law
- ii) A-law

→ Any one of the above method can be explained.

u-law Companding

- The companding μ is continuous here, it is approximately linear for smaller values of μ levels & logarithmic for high μ levels.

Mathematically

$$Z(x) = (\text{Sgn } x) \frac{\ln(1 + \mu|x|/x_{\max})}{\ln(1 + \mu)} \quad \text{Where, } 0 \leq |x|/x_{\max} \leq 1$$

Where, $Z(x) \rightarrow$ output
 $x \rightarrow$ input

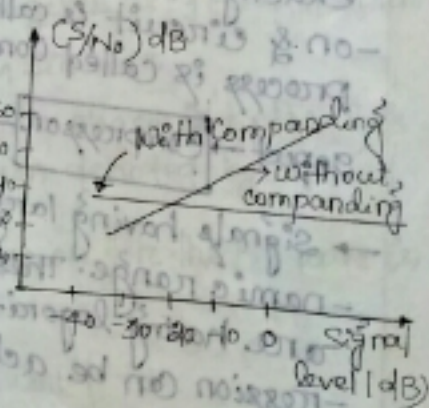
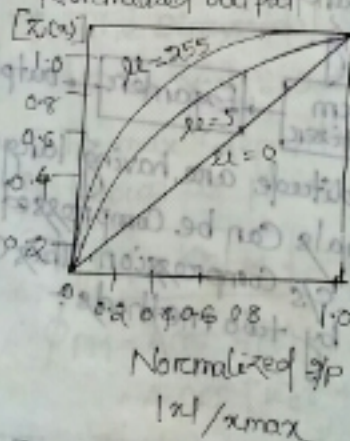
$|x|/x_{\max} \rightarrow$ Normalized value of input with respect to max^m value of $x_{\max}$

- $(\text{Sgn } x)$ represents ± 1 , i.e. +ve & -ve values of μ & μ practical value of μ is 255. The μ -law Companding μ values of μ have been shown.

Further, $\mu=0$, corresponds to the Uniform quantization. μ -law is used for speech & music signal.

- Variation of SNR w.r.t signal level with & without companding is shown here. SNR is almost constant at all signal levels when companding is used.

Normalized output



Comparison of μ -law Companding

PCM performance with μ -law Companding

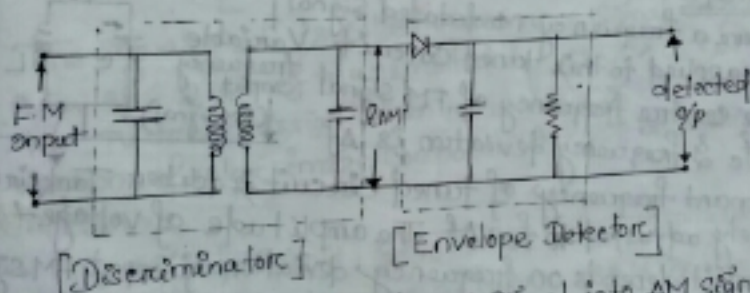
Q.1) Draw and Explain an FM Receiver. What is the role of the discriminator?

Ans: FM receiver/detectors depend upon the slope of the frequency response of a frequency selective network. Two types of detectors are there.

- i) Single tuned detector / simple slope detector.
- ii) St tuned / Balanced slope detector.

i) Single Tuned Detector / simple slope detector

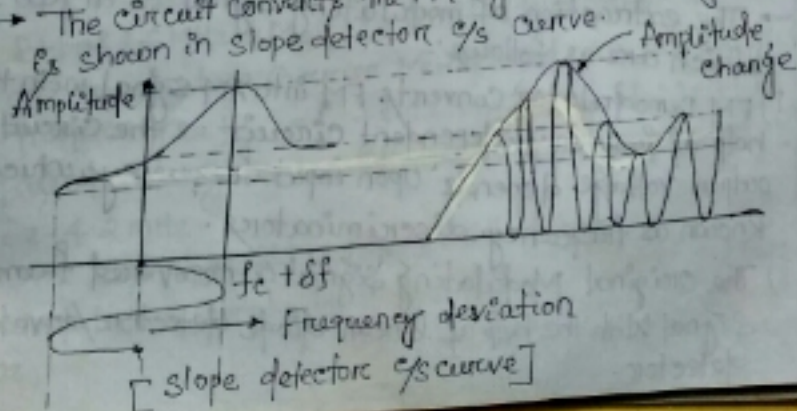
→ The circuit consists of a tuned circuit which is slightly detuned from carrier frequency ω_c . The circuit uses two tuned circuits having different frequencies. First one is tuned to incoming FM carrier ω_c where as the second one is tuned to a frequency slightly different from carrier frequency ω_c . Hence, this portion of circuit which contains two tuned circuits, tuned to different frequencies is called discriminator.



[Discriminator]

[Envelope Detector]

→ The circuit converts the FM signal into AM signal which is shown in slope detector's curve.



→ Another portion of the circuit is envelope detector. The AM signal from the output of discriminator is applied at the input of the envelope detector where original modulating signal is obtained.

Advantages:-

i) Simple circuit & inexpensive.

Disadvantages:-

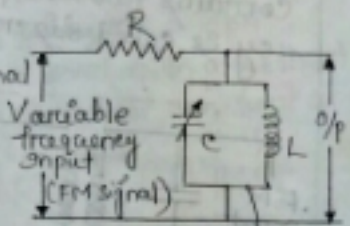
- The circuit is non-linear c/s produces a harmonic distortion as the slope is not same at each point of c/s.
- The circuit does not eliminate the amplitude variation & the output is sensitive to any amplitude variation in the input F.M. signal which is not a desirable feature.

Discriminator:-

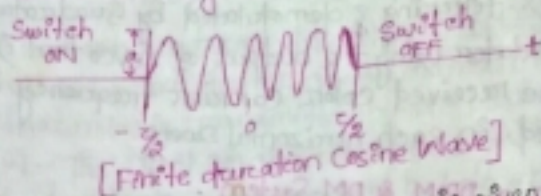
→ Discriminator operates on the principle of slope detection which is explained below.

Principle:-

- Here a frequency modulated signal is applied to this tuned circuit. The centre frequency of FM signal is f_c & frequency deviation is Δf . Resonant frequency of tuned circuit is deliberately adjusted to $f_c + \Delta f$. The amplitude of voltage tank circuit depends on frequency deviation of input FM signal.
- The extraction of modulating signal is in two steps which are as follows:-
 - i) FM Demodulator converts FM into AM signal with the help of frequency dependent circuit i.e. the circuit whose output voltage depends upon input frequency, which are known as frequency discriminators.
 - ii) The original modulating signal is recovered from AM signal with the help of linear diode detector/envelope detector.



1. a) Find the spectrum of the finite duration cosine wave shown in the figure below?



A finite duration cosine wave is given as:

The waveform can be represented by

$$f(t) = \begin{cases} a \cos \omega_0 t, & -T/2 \leq t \leq T/2 \\ 0, & \text{otherwise} \end{cases}$$

Its Fourier Spectrum can be found by,

$$F[f(t)] = F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$\Rightarrow F(\omega) = a \int_{-T/2}^{T/2} \cos \omega_0 t e^{-j\omega t} dt$$

$$\Rightarrow F(\omega) = a \int_{-T/2}^{T/2} e^{-j(\omega - \omega_0)t} dt$$

So, this integration will give

$$F(\omega) = \frac{2a}{(\omega - \omega_0)} \sin(\omega - \omega_0) \frac{T}{2}$$

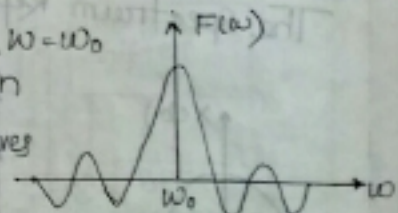
Hence the centre will be peaked at $\omega = \omega_0$

The value of $f(\omega)$ at its peak $\omega = \omega_0$

is found by expanding the sin

before setting $\omega = \omega_0$, which gives

$$f(\omega_0) = aT$$



[Spectrum of Finite cosine wave]

b) Calculate the band width required for transmission of digital data at a rate of 1 Mega pulses per second with a pulse duration of 1 microsecond.

Digital data is transmitted at a rate of 1 mega pulses per second So, $r = 10^6$ pulses/second.

pulse duration (T_b) = $1 \mu s = 1 \times 10^{-6}$ second.

Band width can be calculated by,

$$R_b = \left(\frac{2}{1+\pi} \right) BW \Rightarrow \frac{1}{T_b} = \left(\frac{2}{1+\pi} \right) BW$$

$$\Rightarrow BW = \frac{1+\pi}{2T_b} = \frac{1+10^6}{2 \times 1 \times 10^{-6}} = 500 \times 10^9 \text{ Hz}$$

So, the BW required will be $500 \times 10^9 \text{ Hz}$.

2) How many minimum number of samples are required to exactly describe the signal $x(t) = 10 \cos(6\pi t) + 4 \sin(8\pi t)$

3) The signal is given as $x(t) = 10 \cos(6\pi t) + 4 \sin(8\pi t)$. Comparing the above equation with the below equation,

$$x(t) = 10 \cos \omega_1 t + 4 \sin \omega_2 t$$

$$\text{So, } \omega_1 t = 6\pi t \Rightarrow \omega_1 = 6\pi$$

$$\Rightarrow 2\pi f_1 = 6\pi \Rightarrow f_1 = 3 \text{ Hz}$$

Similarly, for the second signal,

$$\omega_2 t = 8\pi t \Rightarrow \omega_2 = 8\pi$$

$$\Rightarrow 2\pi f_2 = 8\pi \Rightarrow f_2 = 4 \text{ Hz}$$

So, $f_m \rightarrow$ Maximum frequency present in the signal.

$$\text{Here, } f_m = f_2 = 4 \text{ Hz}$$

$$\text{So, Nyquist rate} = f_s = 2f_m = 2f_2 = 8 \text{ Hz}$$

So, 8 samples/sec are required to exactly describe the above signal.

4) A 400W carrier is modulated by a 1kHz tone to a depth of 75%. Find the total side band power and the Modulation efficiency.

$$\text{Carrier power } (P_c) = 400 \text{ W}$$

$$\text{Depth is 75\%} \Rightarrow m_a = 0.75$$

So, the total power can be calculated by, $P_t = P_c \left(1 + \frac{m_a^2}{2} \right)$

$$\Rightarrow P_t = 400 \left\{ 1 + \frac{(0.75)^2}{2} \right\} = 512.5 \text{ W}$$

$$\text{So, Total power, } P_t = P_{SB} + P_c$$

$$\Rightarrow 512.5 = P_{SB} + 400 \Rightarrow P_{SB} = 112.5 \text{ W}$$

$$\text{Modulation Efficiency } (\eta) = \left(\frac{P_{SB}}{P_t} \right) \times 100$$

$$\Rightarrow \eta = \left(\frac{112.5}{512.5} \right) \times 100 = 21.95\%$$

So, total side band power is 112.5W & modulation Efficiency is 21.95% (Ans)

Q) A superhetrodyne receiver is designed to receive signals from 540-1600 KHz with $f_{IF} = 455$ KHz. It is set to receive 540 KHz signal. Assume that the local oscillator (L_o) has a significant third harmonic output. Find the possible carrier frequency received if $f_{LO} = f_c - f_{IF}$ is used.

* Superhetrodyne receiver is designed to receive 540-1600 KHz with $f_{IF} = 455$ KHz

→ It is tuned to receive 540 KHz

Carrier frequency received will be $f_{LO} = f_c - f_{IF}$

$$\text{So, } f_{LO} = 540 - 455 = 85 \text{ KHz}$$

→ As the mixer is a non-linear device, the following frequencies will be generated at the output:

→ Assuming 10 KHz bandwidth of IF amplifier, the 3rd harmonic output will be:

$$f_{LO} = 3f_c - f_{IF} = (3 \times 540) - 455$$

$$= 1165 \text{ KHz}$$

So, 1165 KHz will be received

f) A carrier of frequency 5 KHz and amplitude 5V is frequency modulated by a sinusoidal modulating waveform of the frequency 1 KHz & amplitude of 10V with $k_f = 10 \text{ KHz/V}$. Write down the expression for the frequency modulated (FM) wave. Find the approximate band of frequency occupied by the FM wave form.

Carrier Frequency $\rightarrow (f_c) = 5 \text{ kHz}$ & Amplitude $A_c = 5 \text{ V}$
 Modulating frequency $\rightarrow (f_m) = 1 \text{ kHz}$ & Amplitude $A_m = 10 \text{ V}$
 $K_f = 10 \text{ Hz/V}$

The expression of frequency modulated signal will be

$$\begin{aligned} s(t) &= A \cos \left[\omega_c t + K_f \int x(t) dt \right] \\ &= A \cos \left[(2\pi f_c t) + K_f \int A_m \cos \omega_m t dt \right] \\ &= A \cos \left[(2\pi f_c t) + (K_f \cdot A_m) \int \cos \omega_m t dt \right] \\ &= 5 \cos \left[(2\pi \times 5 \times 10^3) t + \frac{(10 \times 10^0)}{2\pi \times 1 \times 10^3} \sin(2\pi f_m t) \right] \\ &= 5 \cos \left[(2\pi \times 5 \times 10^3) t + \frac{10 \times 10^0}{2\pi \times 1 \times 10^3} \sin(2\pi \times 1 \times 10^3 t) \right] \\ s(t) &= 5 \cos \left[(10\pi \times 10^3) t + 0.159 \sin(2\pi \times 1 \times 10^3 t) \right] \end{aligned}$$

The above equation represents frequency modulated wave.

→ Approximation band of frequencies available is:
 $f_c \pm \Delta f$ so,

$(5+1)$ to $(5-1) = 6 \text{ kHz}$ & 4 kHz are available.

8) of a TV signal of 4.5 MHz bandwidth & transmitted using 8 bit binary PCM. determine the max^m bit rate.

Band width of TV signal $\Rightarrow BW = 4.5 \text{ MHz}$.

PCM of 8 bit so, $n=8$

$$BW = V \cdot f_m \Rightarrow f_m = \frac{BW}{V} \Rightarrow f_m = \frac{4.5 \times 10^6}{8}$$

$$\Rightarrow f_m = 562.5 \text{ kHz}$$

$$\text{Now, bit rate } (R) = V \cdot f_s = 2 \cdot V \cdot f_m [f_s = 2f_m]$$

$$\Rightarrow R = (2 \times 8 \times 562.5 \times 10^3) \Rightarrow R = 9 \text{ Mbps}$$

Write down the advantages & disadvantages of Delta Modulation.

Advantages:

Delta Modulation has certain advantages which are
 i) Since, DM transmits only one bit for one sample,

therefore the signaling rate & transmission channel bandwidth is quite small.

ii) The transmitter & receiver implementation is very much simple for DM. There is no analog to digital converter required here.

Disadvantages:-

DM has two major drawbacks which are:

- i) Slope overload distortion.
- ii) Granular Noise / slope Noise.

i) Give a time domain expression for narrow band additive White Gaussian noise.

AWGN (Additive White Gaussian Noise) channel:

→ Channel is assumed to have following c/s:

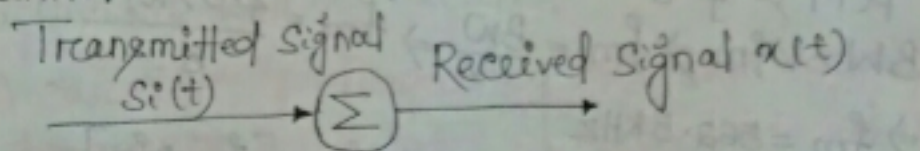
i) The channel is linear with a BW that is wide enough to accommodate the transmission of signal $s_i(t)$ with negligible or no distortion.

ii) The channel noise, $w(t)$ is the sample function of a zero mean white Gaussian noise process.

→ The Received signal $x(t)$ is expressed as:-

$$x(t) = s_i(t) + w(t) \quad \begin{cases} 0 \leq t \leq T \\ i = 1, 2, \dots, M \end{cases}$$

So, an additive white Gaussian noise (AWGN) channel can be modelled as:



White Gaussian Noise

2020-11-27 16:02

4) a) An angle modulated signal has the form $v(t) = 100 \cos [2\pi f_c t + 4 \sin(2000\pi t)]$, where $f_c = 10 \text{ MHz}$.

- i) Determine the average transmitted power
- ii) Determine the peak phase deviation
- iii) Determine the peak frequency deviation
- iv) Is this signal a FM or PM signal? Explain

* An angle modulated signal given by
 $v(t) = 100 \cos [2\pi f_c t + 4 \sin(2000\pi t)]$ where $f_c = 10 \text{ MHz}$

$$\rightarrow v(t) = 100 \cos [2\pi \times 10 \times 10^6 t + 4 \sin(2000\pi t)]$$

→ Comparing this signal with the conventional angle modulated signal given by,

$$v(t) = A_c [\cos(2\pi f_c t) + m_f \sin(2\pi f_m t)]$$

- i) The average transmitted power can be calculated as $P = \frac{A_c^2}{2} = \frac{(100)^2}{2} = 5000 \text{ W} = 5 \text{ kW}$

- ii) The peak phase deviation ϕ .

$$\Delta \phi_{\max} = \max |m_f \sin(\omega_m t)|$$

$$= \max |4 \sin(2000\pi t)| = 4$$

- iii) The peak frequency deviation Δf ,

$$\Delta f_{\max} = \max |f_i - f_c|$$

Where, f_i = instantaneous frequency

f_c = carrier frequency

So, the instantaneous frequency can be calculated as

$$f_i = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

$$\rightarrow f_i = f_c + \frac{1}{2\pi} \cdot \frac{d}{dt} [4 \sin(2000\pi t)]$$

$$= f_c + \frac{4}{2\pi} \cos(2000\pi t) \cdot 2000\pi$$

$$= f_c + 4000 \cos(2000\pi t)$$

$$\text{So, } \Delta f_{\max} = \max |f_i - f_c|$$

$$= \max |4000 \cos(2000\pi t)|$$

$$= 4000 \text{ Hz} = 4 \text{ kHz}$$

i) The angle modulated signal can be interpreted both as a PM and an FM signal. It is a PM signal with phase deviation constant $K_p = 4$ and message signal, $m(t) = \cos(2000\pi t)$.

It is also taken as FM signal with frequency deviation constant $K_f = 4000$ and message signal $m(t) = \cos(2000\pi t)$.

b) Compute the bandwidth requirement for the transmission of an FM signal having a frequency deviation of 75 kHz and an audio bandwidth of 10 kHz.

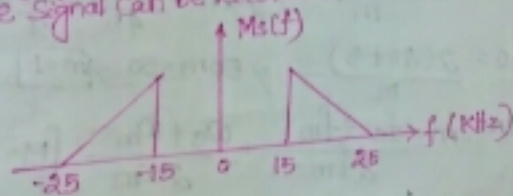
For an FM signal, frequency deviation, $\Delta f = 75 \text{ kHz}$
Audio Bandwidth, $f_m = 10 \text{ kHz}$

According to Carson's Rule, the required Bandwidth can be calculated by, $BW = 2(\Delta f + f_m)$

$$\Rightarrow BW = 2(75 + 10) = 170 \text{ kHz}$$

So, Required Bandwidth is 170 kHz.

5-a) The Nyquist Sampling theorem states that the minimum sampling rate must be equal to or greater than twice the highest frequency component of a low pass signal. When band pass signals are to be sampled, a lower sampling rate can sometimes be used. Consider the band pass signal $m(t)$ whose spectrum is shown in the figure below. Sketch the spectrum of the ideally sampled signal $m_s(t)$ when the sampling frequency f_s is 1) 25 kHz, 2) 45 kHz, 3) 50 kHz. Indicate if the signal can be recovered in each case.



The band pass signal is given as here. This signal can be compared with the original band pass spectrum as follows:

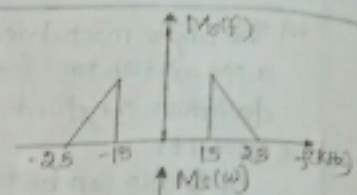
So by comparing we can get

$$\omega_c = 20 \text{ KHz}$$

$$\omega_H = 25 \text{ KHz}$$

$$\omega_L = 15 \text{ KHz}$$

$$\omega_m = 5 \text{ KHz}$$



→ So, the modulating signal frequency is 5 KHz. In order to satisfy the Nyquist Criteria, minimum sampling rate must be

$$\omega_s = 2(\omega_H - \omega_L) = 2 \times 2\omega_m = 4\omega_m$$

$$\Rightarrow \omega_s = 20 \text{ KHz}$$

→ So, the min^m sampling frequency must be 20 KHz in order to satisfy the sampling theorem. Here, as the sampling frequencies are either i) 25 KHz (5ω_m)

ii) 45 KHz (9ω_m) iii) 50 KHz (10ω_m)

→ Here, ω_L = 15 KHz or ω_H = 25 KHz is not harmonic of sampling frequencies i.e. 25 KHz, 45 KHz or 50 KHz. The minimum sampling rate is given by

$$\omega_s = \frac{2(\omega_c + \omega_m)}{m}$$

$$\Rightarrow 25 = \frac{2(20+5)}{m} \Rightarrow m = 2$$

$$\Rightarrow 45 = \frac{2(20+5)}{m} \Rightarrow m = \frac{50}{45} = \frac{10}{9} \approx 1.12$$

$$\Rightarrow 50 = \frac{2(20+5)}{m} \Rightarrow 50m = 50 \Rightarrow m = 1$$

$$\text{Where, } m = \frac{f_c + f_m}{2f_m} = \frac{\omega_c + \omega_m}{2\omega_m} \quad [m = \text{largest Integer}]$$

$$m = \frac{20+5}{2 \times 5} = \frac{25}{10} \Rightarrow m = 2.5$$

→ Here, minimum sampling rate is twice the band width i.e. $2 \times 2\omega_m = 2 \times 2 \times 5 = 20 \text{ KHz}$

→ Here as the sampling band pass signal can be recovered from its sampled version by passing the sampled signal through a band pass filter with arbitrarily sharp cut-off & having pass band from ω_L to ω_H . i.e. (15KHz-25KHz)

Q) Explain why a non-uniform quantizer is used in a practical PCM system rather than a uniform quantizer.

Ans) In Uniform quantizer, linear c/s & there betⁿ output & input as the step size remains same through out the range of quantizer. Therefore, over the complete range of inputs, the max^m quantization error also remains same.

→ As the quantization error is given as: $E_{\max} = \frac{\Delta}{2}$

Where, $\Delta \rightarrow$ Step size. $\Delta = \frac{x_{\max} - (-x_{\max})}{q} = \frac{2x_{\max}}{q}$

If $x(t)$ is normalized, its max^m value i.e. $x_{\max} = 1$.

Therefore $\Delta = \frac{2}{q}$

→ Considering an example of practical PCM system where $n = 4$ bits. So, the number of levels 'q' will be $q = 2^n = 16$ levels.

So, step size, $\Delta = \frac{2}{q} = \frac{2}{16} = \frac{1}{8}$

Hence quantization error is given by,

$$E_{\max} = \left| \frac{\Delta}{2} \right| = \left| \frac{1/8}{2} \right| = \frac{1}{16}$$

→ So, here the quantization error is $1/16$ part of the full voltage range which is the VP signal. For simplicity assume that full range voltage is 16V. Then max^m quantization error will be 1V.

However, for the low signal amplitudes like 2V, 3V etc the max^m quantization error of 1V is quite high i.e. about 30 to 50%. Which means for signal amplitude which are close to 15V, 16V etc.

the max^m quantization error of $\Delta/2$ can be considered to be small. This problem arises because of uniform quantization. Therefore, non-uniform quantization should be used.

→ In other words, it can be said that, SNR_q remains essentially constant over a wide range of input power levels. A quantizer that satisfies all these requirements is known as a Robust quantizer. In other words, such a robust performance can be obtained by using a non-uniform quantizer which indicates the necessity of it in practical PCM circuit.

(b) List and discuss in brief the two distinct and independent sources of noise in a PCM system.

(i) Noise In PCM System:

→ There are two distinct & independent sources of noise occur in PCM system which are:

- i) Transmission noise introduced outside the transmitter
- ii) Quantization noise introduced in the transmitter

→ The transmission noise has no effect so long as the peak noise amplitude is less than half the pulse size a bit error occurs i.e. a symbol '1' is received as symbol '0' & vice versa giving rise to probability of error (P_e).

(ii) Quantization Noise:

→ Quantization Noise will be expressed for linear/uniform quantization, because of quantization inherent errors are introduced in the signal. This error is called quantization error which is given as:

$$e = x_q(nT_s) - x(nT_s)$$

→ Total amplitude range of the input signal is divided into q levels. So, the step size can be calculated as:

$$\Delta = \frac{x_{\max} - (-x_{\max})}{q} = \frac{2x_{\max}}{q}$$

→ If the signal $x(t)$ is normalized to minimum & maximum values equal to 1, then $x_{\max} = 1 \rightarrow -x_{\max} = -1$.
So, step size, $\Delta = \frac{2}{N}$ For Normalized signal

→ Now if, step size ' Δ ' is considered as sufficiently small, then it may be assumed that the quantization error ' e ' will be an uniformly distributed random variable. Max^m quantization error is given as:

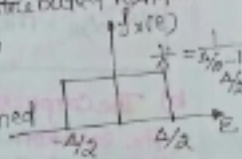
$$E_{\max} = |\Delta/2| \quad -\Delta/2 \leq E_{\max} \leq \Delta/2$$

Hence, over the interval $(-\Delta/2, \Delta/2)$ quantization error may be assumed as an uniformly distributed random variable. Which can be shown here,

→ PDF (Probability density function)

for quantization error may be defined as:

$$f_e(e) = \begin{cases} 0 & \text{for } e < -\Delta/2 \\ 1/\Delta & \text{for } -\Delta/2 \leq e \leq \Delta/2 \\ 0 & \text{for } e > \Delta/2 \end{cases}$$



→ From the PDF, it may be observed that the quantization error ' e ' has zero average value. So, mean value will be zero. Signal to quantization noise ratio of the quantizer is defined as: $\frac{S}{N} = \frac{\text{Signal Power (Normalized)}}{\text{Noise Power (Normalized)}}$

→ If type of signal at input i.e. $x(t)$ is known, it is possible to calculate the signal power. The noise power is expressed as Noise Power = $\overline{v_{\text{noise}}^2}$

Here, v_{noise} is taken as the mean square value of noise voltage. Since, noise is defined by random variable ' e ', PDF $f_e(e)$ there fore, its mean square value is given as: $E[e^2] = \overline{e^2} = v_{\text{noise}}$

→ Mean square value of a random variable ' x ' can be expressed as: $\overline{x^2} = E[x^2] = \int_{-\infty}^{\infty} x^2 f_x(x) dx$

Hence, $E[e^2] = \int_{-\infty}^{\infty} e^2 f(e) de$

$$E[e^2] = \int_{-A/2}^{A/2} e^2 \frac{1}{A} de = \frac{1}{A} \left[\frac{e^3}{3} \right]_{-A/2}^{A/2}$$

$$\Rightarrow E[e^2] = \frac{1}{A} \left[\frac{(A/2)^3}{3} + \frac{(A/2)^3}{3} \right] = \frac{1}{3A} \left[\frac{A^3}{8} + \frac{A^3}{8} \right]$$

$$= \frac{2A^3}{24A} = \frac{A^2}{12}$$

So, the mean square value of noise voltage would be,

$$V_{\text{noise}}^2 = \text{mean square value} = \frac{A^2}{12}$$

b) The Compression c/s for a μ -law companding follows the equation:

$$\frac{y(s)}{y_{\text{max}}} = \pm \ln \left(1 + \mu \left| \frac{s}{s_{\text{max}}} \right| \right) \quad \text{find the expansion (inverse)}$$

function $y(s)$.

* The Compression c/s for μ -law companding follows the equation:

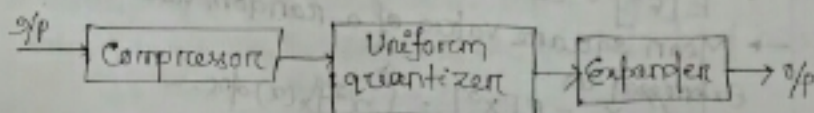
$$\frac{y(s)}{y_{\text{max}}} = \pm \frac{\ln \left(1 + \mu \left| \frac{s}{s_{\text{max}}} \right| \right)}{\ln(1 + \mu)}$$

Here, $y(s) \rightarrow$ output & $s \rightarrow$ input.

$s/s_{\text{max}} \rightarrow$ Normalized value of input w.r.t max^m value

' $\pm$ ' $\rightarrow$ Represents +ve & -ve values of input & output

$\rightarrow$ Practical value for $\mu = 255$. Companding is the combination of compressing & expanding. This equation represents compressing process. Here, the weak signals are amplified & strong signals are attenuated before applying them to uniform quantizer.



→ So, at the receiver exactly opposite is followed which is called expansion & the circuit is called expander. The equation that represents expander circuit, i.e. inverse of the above equation can be written as:

$$J^{-1}(s) = \pm \frac{1}{2\pi} ((1-u)^{|s|-1})$$

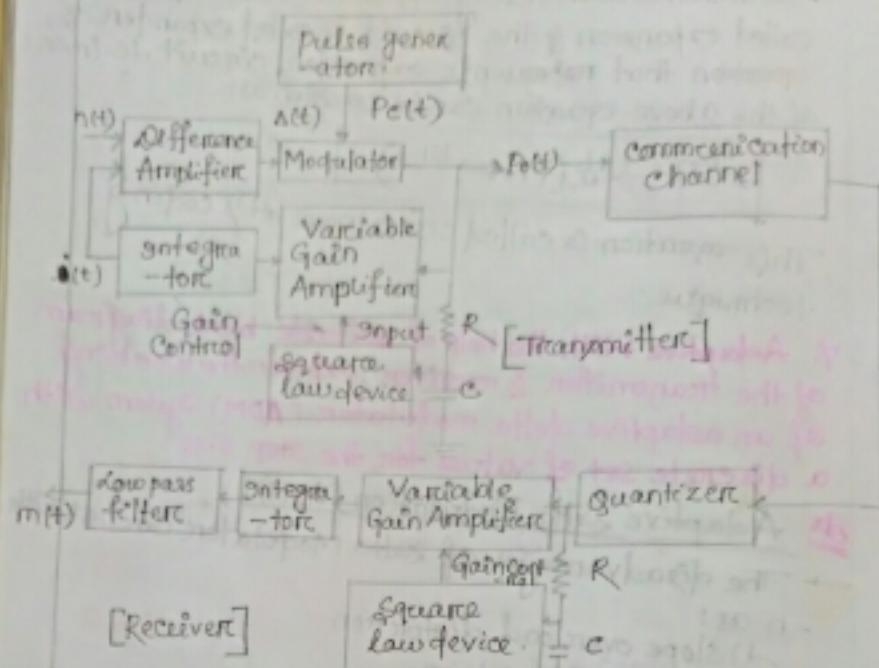
This equation is called codes wave let's coding technique.

17. Adaptive With the help of suitable block diagram of the transmitter & receiver, explain the working of an adaptive delta modulator (ADM) system with a discrete set of values for the step size.

Ans Adaptive Delta Modulation (ADM):

→ The disadvantages of Delta Modulator can be written as:

- i) Slope overload distortion.
 - ii) Granular Noise.
- Occurs due to small step size. The stair case approximate signal cannot approximate the message signal.
- Occurs due to large step size. Stair case approximate signal is very large in approximating the message signal.
- So the limitations of PM can be overcome by suitably changing the step size. Slope overload distortion, granular noise will be reduced by making $\Delta(t)$ or step size adaptive to variation in the message signal.
- Hence, called adaptive delta modulation as it can take continuous & discrete change in step size.



[Adaptive Delta Modulation]

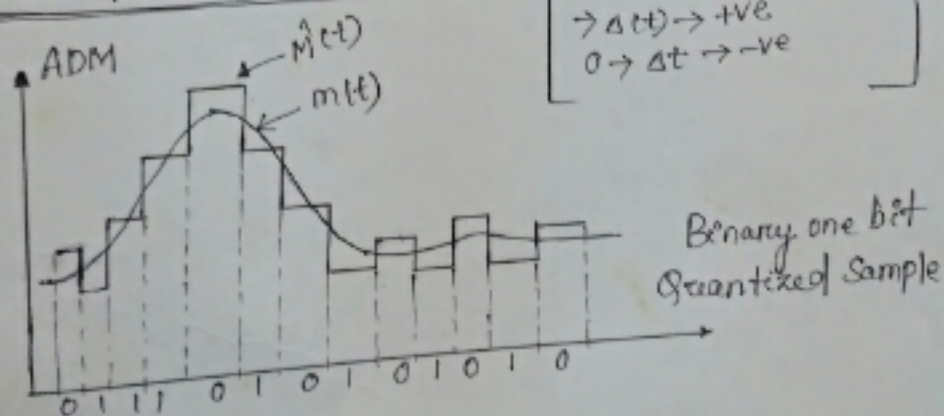
- On the transmitter side, a variable gain amplifier is used before the integrator with $P(t)$ as its input. The gain of this amplifier depends on gain control input which is obtained by integrating $P(t)$ in an RC network, & then passing the integrator output through a square law device.
- Under slope overload condition, $P(t)$ is a long sequence of either +ve or -ve pulses. The RC integrator integrates these pulses. Thus the output of this integrator is either of a large +ve or large negative value.
- The square law device output is of a large +ve value irrespective of whether the input is positive or -ve thus the gain control of the variable gain amplifier is large & its gain increases. Hence, the step

size increases which can then take care of slope overload.

→ When the signal variations are within step size V_{step} is a sequence of alternate +ve & -ve pulse. Re-integrator output is zero & gain control input of variable gain amplifier is also zero.

→ The gain of variable gain amplifier decreases, resulting in a reduced step size, which takes care of the situation on the receiver side, the o/p of quantizer is fed to a variable gain amplifier whose gain control ip is derived from an RC integrator & a square law device. Thus an adaptive adjustment of step size is obtained at receiver, resulting in an undistorted reception of transmitted signal.

Waveform for ADM:-



$$\left[\begin{array}{l} \Delta t = m(t) - m^{\wedge}(t) + 1 \\ \rightarrow \Delta t \rightarrow +ve \\ 0 \rightarrow \Delta t \rightarrow -ve \end{array} \right]$$