FINITE ELEMENT METHOD
(FEM)

SEMESTER : 5TH

BRANCH : MECHANICAL ENGG.

MODULE – III

Short-Type Questions & Answers

Long-Type Questions & Answers

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LONG -TYPE

Questions & Answers

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Q. What are the stages for finite element analysis using computer programming ?

In FEA or any type of analysis of engineering simulation there are few basic steps to be followed,

1. Preprocessing
2. Solver
3. post-processing
4. validation by Analytical or Experimental method.

***preprocessing**

Dividing a our domain into finite number of parts for imposing mathematical equations on it called as Discretization In industry it also called as a Meshing.

***Solver**

Solver is like a black box of commercial softwares. It involves applying boundary conditions and initial value generation in it. Number of codes are prepared to solve this mathematical equation by software campany directly.

***Post processing**

All solved problems are in numeric format. For visualization purpose it is mandatory to represent this results in Graphics. Stress contour,Pressure contour ,Strain contour are plotted.

***Validation**

It is very important step in engineering analysis point of view. Result obtain by FEA or any other method should be validated by any one of analytical or experimental method.

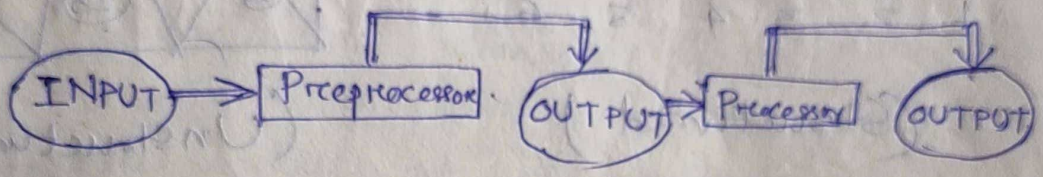
Preprocessing & Postpreocessing

A complete finite element (code) analysis is ~~logic~~ a logical analysis interaction of the following three stages:

(1) Preprocessore
(2) Processore or Main Programm
(3) Post preocessore.

1- Preprocessore:

A preprocessore is a program that pocsess an input data to produce output, which is then used as the input to the processore of the FEM code.



⇒ Preprocessore needs following inforemations:

- Geometrey
- Material
- Loading
- Boundary Conditions

⇒ Preprocessore perforems the following tasks

- Generate a finite element mesh.
- It takes material properties and Loading as an input.

(i) Mesh Generation:

In this task of preprocessore, a finite element mesh is generated. There are different techniques of mesh generations.

(a) Fully Automatic (Directly by the computer)

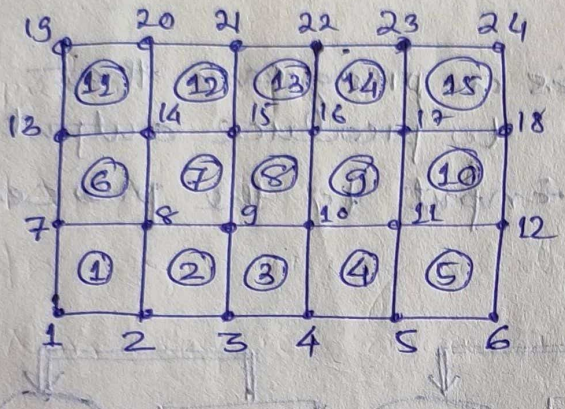
(b) Semi automatic (the model domain is subdivided ~~mainly~~ manually into simple subregions called multi-blocks)

Types of Mesh:

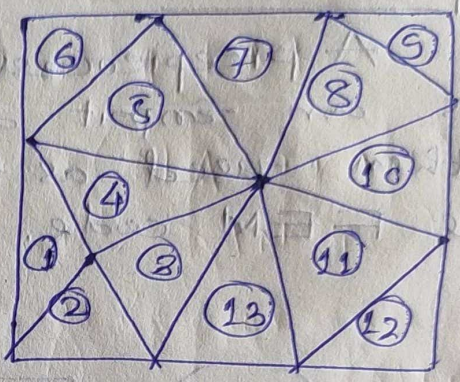
There are two types of mesh:

1- Structured

2- Unstructured

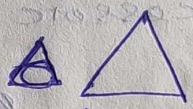


(Structured)

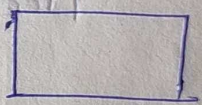


(Unstructured)

For two dimensional,



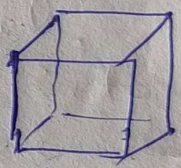
(C-S-T)



(quadrilateral)

and for three dimensional

(tetrahedral)



(Bricks)

are used for mesh generations.

(11) Taking material property & loading as an input:

Input regarding material properties and loadings are called from user and stored in a particular locations. These inputs are then used by preocessors and postpreocessores

2- Processore or Main Programm : The preprocessor ~~runs the input fole when data is fed~~ without any bugs and obtains the soln. Main programm ~~is~~ designed for evaluating.

Main program consists of following programms :

- evaluation of co-efficient matrix
- right hand side (i.e. force vector) of global elemental equation
- assembly
- application of boundary conditions
- Solution of linear system of equations

3- Post Processore :

Post processore consists of following routines

- Output of the processer is in the form of discreete value at the nodes.
- Post processore may convert them into continuous form by multiplying with force vector.
- Routine for obtaining the secondary quantity (i.e. stress, strain etc.)

⇒ After the solution is obtained the post-processing stage, depending on the type of data and method of presentation chosen, the result will be displayed.

Popular Softwares used in FEM

(i) NASA structural analysis (NASTRAN)
(ii) An engineering analysis system (ANSYS)
(iii) Non-Linear incremental structural analysis (NISA)
(iv) IDEAS etc.

Mesh Refinement

Mesh refinement are implemented to improve the accuracy of a model. the geometry can be divided into smaller elements so that the mass density is increased, Usually mesh refinement techniques are used, in case of higher order elements. Increasing the order of the element leads to a significant increase in the computer time needed to obtain the solution.

When the mesh refinement is used all the previous meshes should be included in the finer meshes for achieving convergence of model

the two adaptive techniques of mesh refinements are
(i) Frontal solver technique
(ii) Skyline solver technique.

Example 1.30 ***Consider a 1 mm diameter, 50 mm long aluminium pin-fin as shown in Fig.(i) used to enhance the heat transfer from a surface wall maintained at 300° C. Calculate the temperature distribution in a pinfin by using Rayleigh-Ritz method. Take, k = 200 w/m°C for aluminium h = 20 w/m²°C, T_∞ = 30°C.***

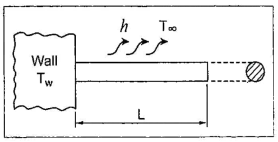


Fig. (i).

$$k\frac{d^2T}{dx^2} = \frac{P\,h}{A}(T - T_\infty)$$

$$T(0) = T_w = 300°C$$

$$q_L = kA\frac{dT}{dx}(L) = 0 \text{ (Insulated tip)}$$

[Anna University, M.E. (CAD/CAM) Apr/May 2006]

Given: The governing differential equation,

$$k\frac{d^2T}{dx^2} = \frac{P\,h}{A}(T - T_\infty)$$

Diameter, $d = 1 \text{ mm} = 1 \times 10^{-3}$ m

Length, $l = 50 \text{ mm} = 50 \times 10^{-3}$ m

Thermal conductivity, $k = 200$ W/m °C

Heat transfer coefficient, $h = 20$ W/m² °C

Fluid temperature, $T_\infty = 30°C$

Boundary conditions, $T(0) = T_w = 300°C$

$$q_L = kA\frac{dT}{dx}(L) = 0$$

To find: Ritz parameters.

☺ ***Solution:*** The equivalent functional representation is given by

π = Strain energy – Work done

π = U – W

$$\pi = \left[\int_0^L \frac{1}{2}k\left(\frac{dT}{dx}\right)^2 dx + \int_0^L \frac{1}{2}\frac{P\,h}{A}(T - T_\infty)^2\,dx\right] - q_L\,T_L \quad \ldots(1)$$

$$\pi = \int_0^L \frac{1}{2}k\left(\frac{dT}{dx}\right)^2 dx + \int_0^L \frac{1}{2}\frac{P\,h}{A}(T - T_\infty)^2\,dx \quad \ldots(2)$$

$[\because q_L = 0]$

Assume a trial function, let $T(x) = a_0 + a_1 x + a_2 x^2$ ...(3)

Apply boundary condition,

at $x = 0$, $T(x) = 300$

$$300 = a_0 + a_1(0) + a_2(0)^2$$

$$\boxed{a_0 = 300}$$

Substituting a_0 value in equation (3),

$$T(x) = 300 + a_1 x + a_2 x^2 \quad \ldots(4)$$

$$\Rightarrow \frac{dT}{dx} = a_1 + 2\,a_2\,x \quad \ldots(5)$$

Substitute the equation (4), (5) in (2)

$$\pi = \int_0^l \frac{1}{2}k(a_1 + 2\,a_2\,x)^2\,dx + \int_0^l \frac{1}{2}\frac{P\,h}{A}(300 + a_1 x + a_2 x^2 - 30)^2\,dx$$

$$\pi = \int_0^l \frac{1}{2} k (a_1 + 2 a_2 x)^2 dx + \int_0^l \frac{1}{2} \frac{P h}{A} (270 + a_1 x + a_2 x^2)^2 dx$$

$[\because (a+b)^2 = a^2 + b^2 + 2ab;\ (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$$\pi = \frac{k}{2} \int_0^l [a_1^2 + 4 a_2^2 x^2 + 4 a_1 a_2 x] +$$

$$\frac{P h}{2A} \int_0^l [(270)^2 + a_1^2 x_1^2 + a_2^2 x^4 + 540 a_1 x + 2 a_1 a_2 x^3 + 540 a_2 x^2)] dx$$

$$\pi = \frac{k}{2} \left[a_1^2 x + \frac{4 a_2^2 x^3}{3} + \frac{4 a_1 a_2 x^2}{2} \right]_0^{50 \times 10^{-3}}$$

$$+ \frac{P h}{2A} \left[72900 x + \frac{a_1^2 x^3}{3} + \frac{a_2^2 x^5}{5} + \frac{540 a_1 x^2}{2} + \frac{2 a_1 a_2 x^4}{4} + \frac{540 a_2 x^3}{3} \right]_0^{50 \times 10^{-3}}$$

$[\because l = 50 \times 10^{-3}\ \text{m}]$

$$\pi = \frac{k}{2} \left[(50 \times 10^{-3}) a_1^2 + \frac{4 a_2^2 (50 \times 10^{-3})^3}{3} + \frac{4 a_1 a_2 (50 \times 10^{-3})^2}{2} \right]$$

$$+ \frac{P h}{2A} \left[72900 (50 \times 10^{-3}) + \frac{a_1^2 (50 \times 10^{-3})^3}{3} + \frac{a_2^2 (50 \times 10^{-3})^5}{5} \right.$$

$$\left. + \frac{540 a_1 (50 \times 10^{-3})^2}{2} + \frac{2 a_1 a_2 (50 \times 10^{-3})^4}{4} + \frac{540 a_2 (50 \times 10^{-3})^3}{3} \right]$$

$$\pi = \frac{200}{2} [50 \times 10^{-3} a_1^2 + 1.666 \times 10^{-4} a_2^2 + 5 \times 10^{-3} a_1 a_2] + \frac{\pi \times 10^{-3} \times 20}{2 \times \frac{\pi}{4} \times (10^{-3})^2}$$

$$[3645 + 4.166 \times 10^{-5} a_1^2 + 6.25 \times 10^{-8} a_2^2 + 0.675 a_1 + 3.125 \times 10^{-6} a_1 a_2 + 0.0225 a_2]$$

$$\pi = [5 a_1^2 + 0.0166 a_2^2 + 0.5 a_1 a_2] + [14.58 \times 10^7 + 1.66 a_1^2 + 2.5 \times 10^{-3} a_2^2$$

$$+ 2700 a_1 + 0.125 a_1 a_2 + 900 a_2]$$

$$\pi = [6.66 a_1^2 + 0.0191 a_2^2 + 0.625 a_1 a_2 + 27000 a_1 + 900 a_2 + 14.58 \times 10^7]$$

Apply, $\frac{\partial \pi}{\partial a_1} = 0$

$\Rightarrow \quad 13.32 a_1 + 0.625 a_2 + 27000 = 0$

$\Rightarrow \quad 13.32 a_1 + 0.625 a_2 = -27000 \quad \ldots(6)$

Apply, $\frac{\partial \pi}{\partial a_2} = 0$

$\Rightarrow \quad 0.625 a_1 + 0.382 a_2 + 900 = 0$

$\Rightarrow \quad 0.625 a_1 + 0.0382 a_2 = -900 \quad \ldots(7)$

Solve the equations (6) and (7)

$$13.32 a_1 + 0.625 a_2 = -27000 \quad \ldots(6)$$

$$0.625 a_1 + 0.0382 a_2 = -900 \quad \ldots(7)$$

(6) × 0.625

$\Rightarrow \quad 8.325 a_1 + 0.3906 a_2 = -16875 \quad \ldots(8)$

(7) × − 13.32

$\Rightarrow \quad -8.325 a_1 - 0.5088 a_2 = +11988 \quad \ldots(9)$

$$-0.1182 a_2 = -4887$$

$$0.1182 a_2 = 4887$$

$$\boxed{a_2 = 41345}$$

Substitute a_2 value in equation (6),

$$13.32 a_1 + 0.625 (41345) = -27000$$

$$\Rightarrow a_1 = -3967.01$$

Substitute a_0, a_1 and a_2 values in equation (3)

$$T = 300 - 3967.01 x + 41345 x^2$$

Result: Temperature distribution in a pin-fin

$$T = 300 - 3967.01 x + 41345 x^2$$