

STUDY MATERIAL

SUBJECT : NETWORK THEORY (NT)

MODULE -

SEMESTER : - 3RD

BRANCH : EE / EEE

**DEPARTMENT OF ELECTRICAL ENGINEERING
SRINIX COLLEGE OF ENGINEERING, BALASORE
(www.srinix.org)**

MODULE - I

NETWORK THEORY

51

①

3RD SEMESTER

SHORT TYPE

BRANCH \rightarrow EEE + EEE

ER. P. K. TRIPATHY

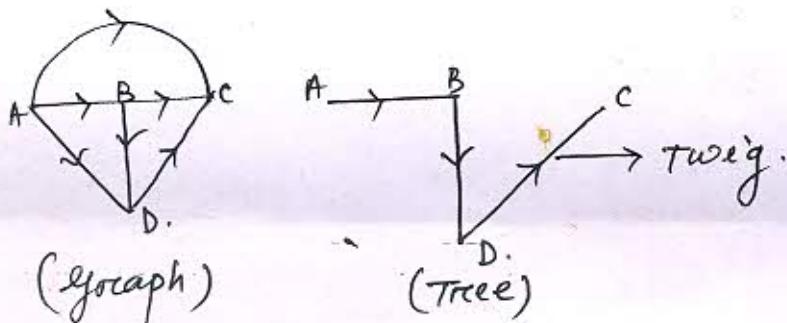
(1) Write properties of incidence matrix. (2011, 2012-13)

Soln \rightarrow The algebraic sum of a column of A_i is zero.
 \rightarrow The determinant of A_i of a closed loop is zero.

where $A_i = \text{incidence matrix}$.

(2) What do you understand by 'twigs'? Briefly explain by a diagram (2012)

Soln \rightarrow The branches of the tree are known as twigs.



(3) Relate link, node and branch of a graph.

Soln $n \rightarrow$ no. of nodes in the graph.

$$\text{Twigs} = n-1.$$

$l =$ Total number of links.

$b =$ Total number of branches.

$$l = b - (n-1) = b - n + 1$$

(4) State and explain Maximum power Transfer Theorem? (2012-13)

Soln In a linear bilateral network containing an independent voltage source in series with resistance (R_s) delivers maximum power to the load resistance (R_L) when R_L is equal to R_s ($R_s = R_L$)

\rightarrow Similarly the independent current source parallel with source resistance (R_s) delivers maximum power to the load resistance when ($R_L = R_s$)

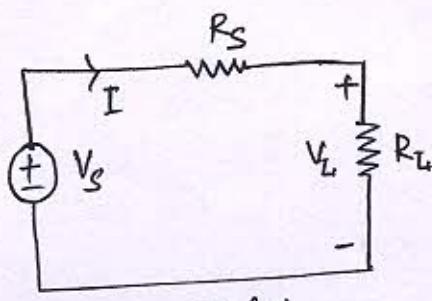


Fig (a)

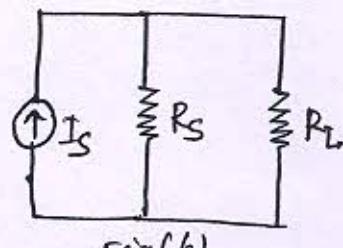


Fig (b)

(5) What do you mean by coefficient of coupling? Explain. (2012, 13, 10)

Solu It is defined as the fraction of total flux that links the coils i.e K, the coefficient of coupling = $\frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$.

where ϕ_1 and $\phi_2 \rightarrow$ Total flux in the corresponding coils.

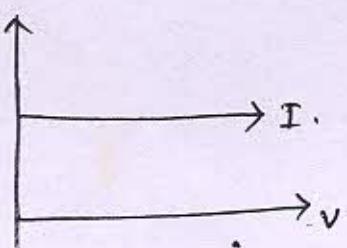
ϕ_{12} and $\phi_{21} \rightarrow$ Fluxes linked with the coils.

$$\phi_{12} < \phi_1 \text{ and } \phi_{21} < \phi_2$$

Hence maximum value of 'K' is unity

(6) Give the phasor diagram of series resonance? (2011)

Solu"



$$I = \frac{V}{Z}$$

$Z = \text{minimum}$

$I = \text{maximum under resonance condition}$

(7) State Tellegen's Theorem? (2011)

Solu For any given time, the sum of power delivered to each branch of any electric network is zero.

Thus for K^{th} branch, this theorem states that

$$\sum_{k=1}^n V_k i_k = 0, n \text{ being the number of branches.}$$

$V_k \rightarrow$ is the voltage drop in the branch.

$i_k \rightarrow$ is the current through the branch.

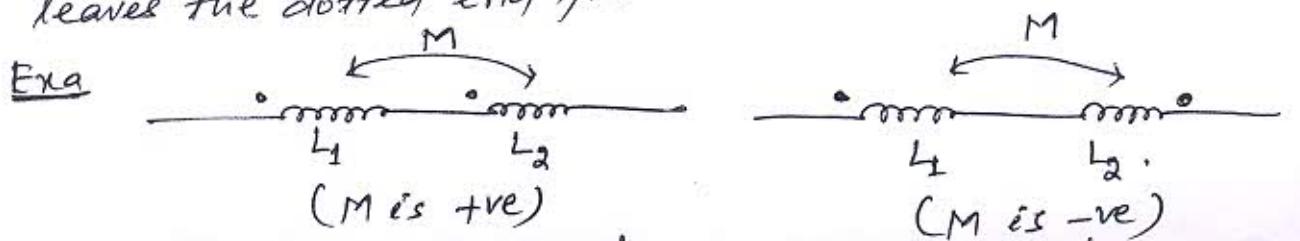
(3)

(8) Explain dot convention in coupled coils. (2011, 2007)

Soluⁿ To determine the relative polarity of the mutually induced voltage a convention is used. This is known as dot convention.

→ According to this convention,

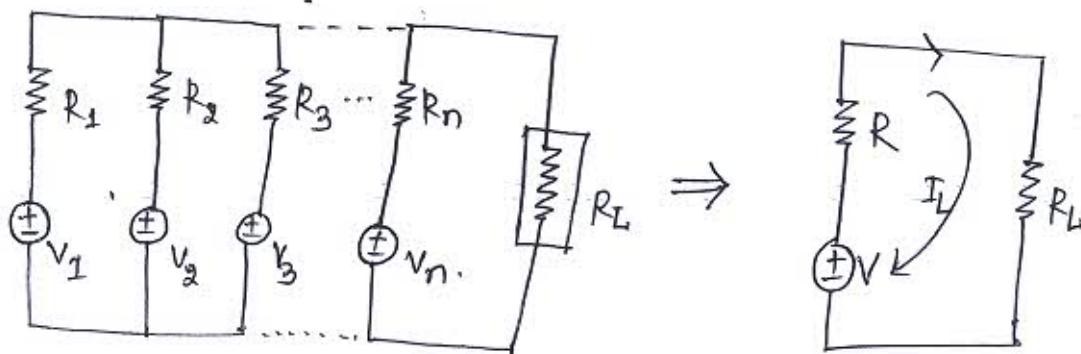
polarity of mutual inductance of a couple coil may be treated as +ve, if the loop currents enter into the respective coils at the dotted ends of respectively coils or simultaneously coming out from the dotted ends of the respective coils and -ve if current enters the dotted end for one coil and leaves the dotted end for other coil.



(In both cases coupled coils are connected in series.)

(9) State Millman's Theorem and explain with a suitable examples?

Soluⁿ When a number of voltage source V_1, V_2, \dots, V_n are in parallel having resistances R_1, R_2, \dots, R_n respectively. The arrangement can be replaced by a single equivalent voltage source 'V' with series a equivalent resistance R' .



$$V = \frac{V_1 G_1 \pm V_2 G_2 \pm V_3 G_3 \pm \dots \pm V_n G_n}{G_1 + G_2 + G_3 + \dots + G_n}.$$

$G_i = \frac{1}{R_i}$ = conductance

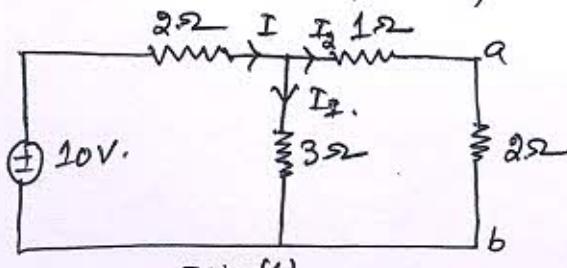
$$R' = \frac{1}{G'} = \frac{1}{G_1 + G_2 + G_3 + \dots + G_n}.$$

(4)

(10) State and explain Reciprocity Theorem with a suitable example. (2010, 2011)

Solu" statement : \rightarrow If a source of emf located at one point in a network composed of linear bilateral circuit elements, produces a current I' at a selected point in the network, the same source of emf acting at the second point will produce the same current at the first point.

Problem



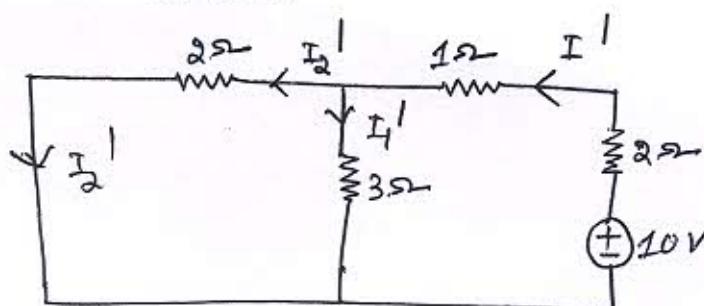
Show Reciprocity Theorem.

Fog-(1)

$$R_{eq} = \left\{ (2+1) // 3 \right\} + 2 = \frac{9}{7} + 2 = \frac{21}{7} = 3.5$$

$$I = \frac{V}{R_{eq}} = \frac{10}{3.5} = \frac{20}{7} \text{ Ampere.}$$

$$I_2 = I \times \frac{3}{3+1+2} = \frac{20}{7} \times \frac{3}{6} = \frac{10}{7} \text{ Amp.}$$



Fog-(2)

$$R_{eq} = (2//3) + 1 + 2 = \frac{6}{5} + 3 = \frac{21}{5} \text{ ohm}$$

$$I' = \frac{V}{R_{eq}} = \frac{10}{21/5} = \frac{50}{21} \text{ Amp.}$$

$$I_2' = \frac{50}{21} \times \frac{3}{3+2} = \frac{10}{7} \text{ Amp.}$$

$$\boxed{I_2 = I_2'}$$

So reciprocity Theorem is verified.

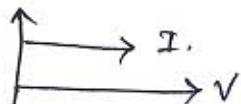
(5)

(11) Give at least four properties of parallel resonance circuit?

Solu" → power factor of circuit is unity.

→ current at resonance is minimum and it is in the phase of applied voltage.

→ Net impedance at resonance is maximum (L/CR)



→ The admittance is minimum.

(12) Define Q-factor, Band width and selectivity and how they are related. (2010, 2011, 2013)

Solu" Quality factor (Q-factor).

It is defined as the ratio of voltage across the inductor or capacitor to the applied voltage. It is denoted by Q_0 .

$$Q_0 = \frac{V_L}{V}, Q_0 = \frac{V_C}{V}$$

V_L and V_C → voltage across inductor and capacitor.

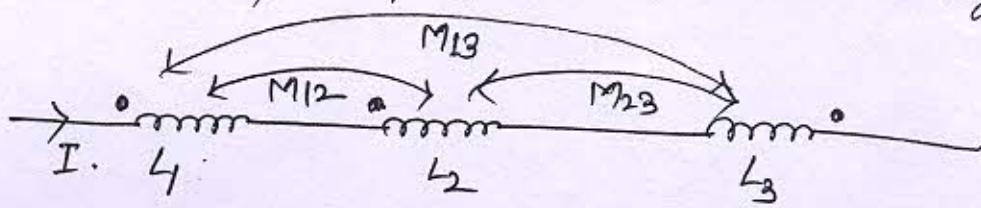
$$Q_0 = \frac{\omega_0 L}{R}, Q_0 = \frac{1}{\omega_0 C R}$$

$$\text{Band width} = f_2 - f_1$$

f_1 and f_2 → lower and upper half frequency.

Selectivity It is defined as the ratio of resonance frequency to quality factor = $\frac{f_0}{Q_0}$.

(13) What is the expression for the total inductances of the three series connected coupled coils as shown in figure (2007)



Soluⁿ

$$V_{\text{Total}} = V_{L_1} + V_{L_2} + V_{L_3}.$$

$$V_{L_1} = L_1 \frac{dI}{dt} + M_{12} \frac{dI}{dt} - M_{13} \frac{dI}{dt} = (L_1 + M_{12} - M_{13}) \frac{dI}{dt}.$$

$$V_{L_2} = (L_2 + M_{12} - M_{23}) \frac{dI}{dt}.$$

$$V_{L_3} = (L_3 - M_{23} - M_{13}) \frac{dI}{dt}.$$

$$\text{Req } \frac{df}{dt} = (L_1 + L_2 + L_3 + 2M_{12} - 2M_{13} - 2M_{23}) \frac{dI}{dt}.$$

$$\Rightarrow \text{Req} = L_1 + L_2 + L_3 + 2M_{12} - 2M_{13} - 2M_{23}$$

(14) Write down the limitation of reciprocity theorem and also mention its application ? (2010).

Soluⁿ Limitations of Reciprocity Theorem :-

- This theorem is not applicable when network consisting of any time varying element.
- This theorem is not applicable of non-linear elements like diode, transistor etc.
- This theorem is not applicable more than one energy source.

Application of Reciprocity Theorem :-

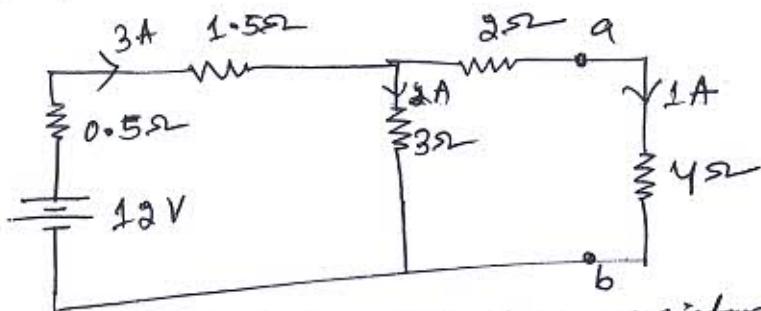
- This theorem is applicable to linear, time invariant n/w consisting of passive n/w elements.
- This theorem is applicable in dc as well as ac circuit.
- This theorem allows interchange the position of excitation and response.
- It provides bilateral property of the network.
- It provides great convenience in design and measurement problems.

(7)

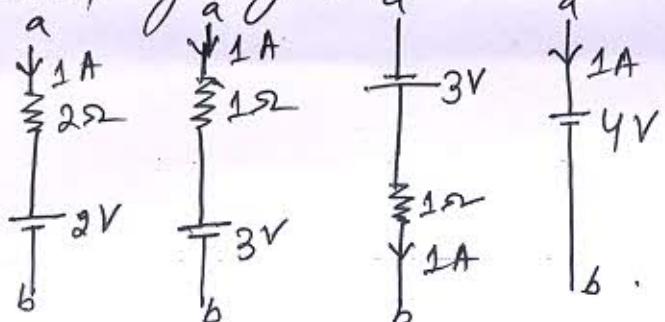
(5) State and explain Substitution Theorem with a suitable examples?

Soluⁿ Statement : \rightarrow In a network any branch may be substituted by another branch without disturbing the current and voltages in other branches of the network, provided the new branch has the same internal voltage when carrying the same current.

Explanation : \rightarrow Consider a circuit as shown in figure.



The branch containing the 4Ω resistance has a current of 1A. According to Substitution theorem this branch may be replaced by any of the branches as shown in figure.



XOX

(8)

LONG TYPE

(1) For the given matrix draw the graph and also calculate the total number of possible trees.

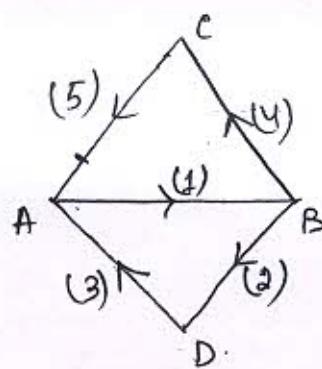
$$A = \begin{bmatrix} +1 & 0 & -1 & 0 & -1 \\ -1 & +1 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 & +1 \end{bmatrix}$$

Solution : →

incidence matrix.

$$A_i = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ A & +1 & 0 & -1 & 0 & -1 \\ B & -1 & +1 & 0 & +1 & 0 \\ C & 0 & 0 & 0 & +1 & 0 \\ D & 0 & -1 & +1 & 0 & +1 \end{bmatrix}$$

graph



The no. of possible trees = $\det \{ [A] [A]^T \}$

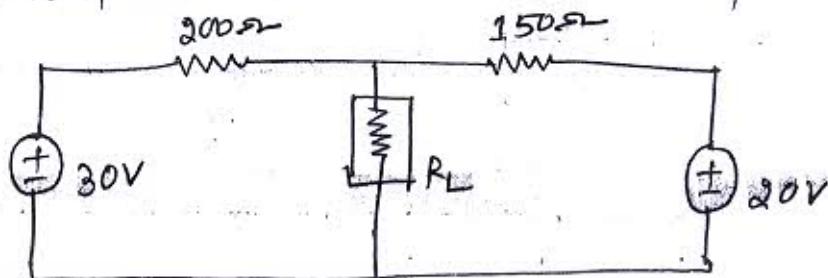
$$\begin{aligned} [A] [A]^T &= \begin{bmatrix} +1 & 0 & -1 & 0 & -1 \\ -1 & +1 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 & +1 \end{bmatrix} \times \begin{bmatrix} +1 & -1 & 0 \\ 0 & +1 & 0 \\ -1 & 0 & 0 \\ 0 & +1 & -1 \\ -1 & 0 & +1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \det \{ [A] [A]^T \} &= \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{vmatrix} = 3(6-1)+1(-2-1) \\ &\quad -1(1+3) \\ &= 15 - 3 - 4 = 8 \end{aligned}$$

The no. of possible trees = 8

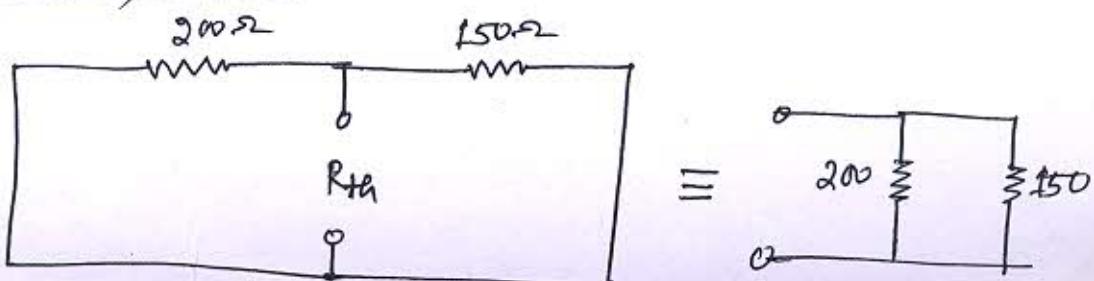
(9)

(2) For the given network calculate the value of load for maximum power flow and also calculate the value of maximum power?

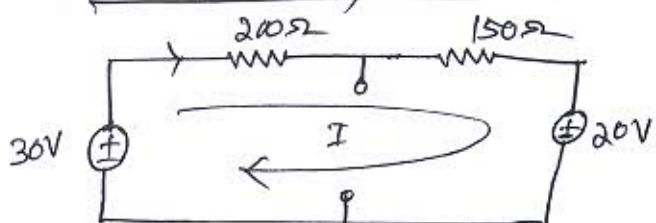
Soluⁿ

We know

$$P_{\max} = \frac{V_{th}^2}{4R_{th}}$$

calculation of R_{th} :

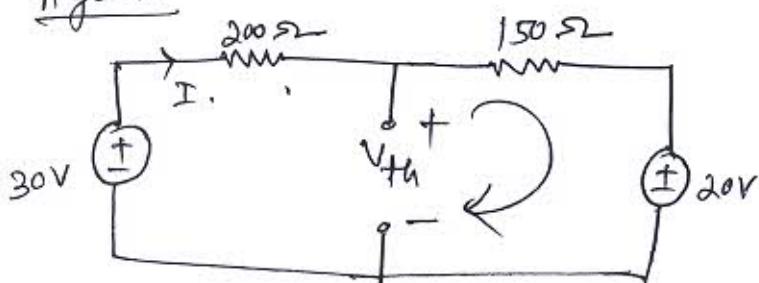
$$R_{th} = 200 // 150 = \frac{200 \times 150}{200 + 150} = 85.714 \Omega$$

Calculation of V_{th} :

apply KVL to the Loop.

$$30 - 200I - 150I - 20 = 0$$

$$\begin{aligned} 10 &= 350I \\ \Rightarrow I &= \frac{10}{350} = 0.02857 \text{ Amp.} \end{aligned}$$

Again:

$$V_{th} - 150I - 20 = 0$$

$$V_{th} = 20 + 150I$$

$$= 20 + 150 \times 0.02857$$

$$= 24.2855 \text{ Volt.}$$

Hence

$$P_{\max} = \frac{(24.2855)^2}{4 \times 85.714} = 1.7202 \text{ watt.}$$

(3) Find out the value of current I_L which passing through the load resistance by using Millman's theorem for the following circuit?

Solution

We Know $V = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3 + V_4 G_4}{G_1 + G_2 + G_3 + G_4}$

$$G = \frac{1}{R}$$

Solution

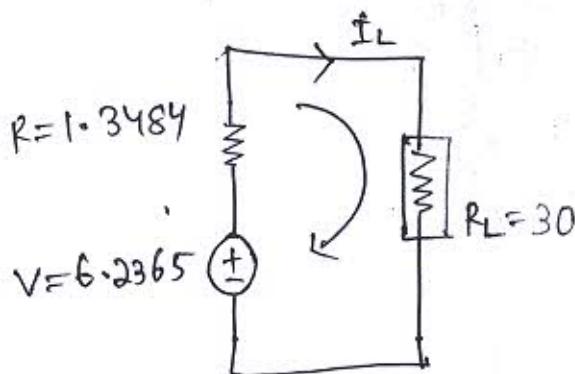
We Know $I_L = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3 + V_4 G_4}{G_1 + G_2 + G_3 + G_4}$

$$G = \frac{1}{R}$$

$$V = \frac{10 \times \frac{1}{4} - 15 \times \frac{1}{6} + 5 \times \frac{1}{8} + 20 \times \frac{1}{5}}{\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{5}} = \frac{2.5 - 2.5 + 0.625 + 4}{0.7416} = 6.2365 \text{ Volts}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + G_3 + G_4}$$

$$= \frac{1}{\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{5}} = \frac{1}{0.7416} = 1.3484$$

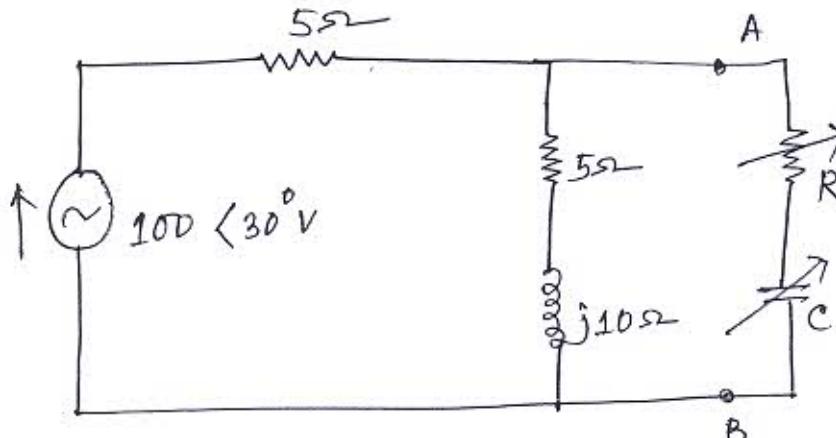


$$I_L = \frac{V}{R_L + R} = \frac{6.2365}{30 + 1.3484} = \frac{6.2365}{31.3484} = 0.1989 \approx 0.2$$

(7) (b) For the given network, determine

(a) The values of R and x that will result in maximum power using transferred across terminals AB and

(b) The value of maximum power.

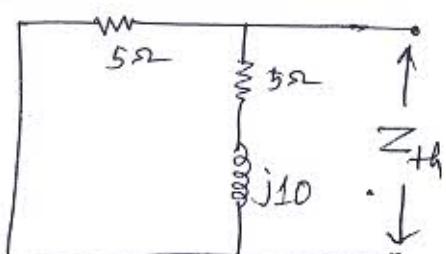


$$Z = R + jx$$

Solution

$$V_{AB} = V_{OC} = 100 \angle 30^\circ \times \frac{5 + j10}{10 + j10}$$

$$= 79.07 \angle 48.43^\circ$$



$$Z_{TH} = 5 \parallel (5 + j10)$$

$$= \frac{5(5 + j10)}{10 + j10}$$

$$= 3.95 \angle 18.43^\circ$$

$$= 3.75 + j1.25$$

$$Z_L = Z_{TH} = 3.75 - j1.25$$

$$R = 3.75\Omega, X = 1.25\Omega$$

$$P_{max} = \frac{(79.07)^2}{4 \times 3.95} = 395.07 \text{ watt.}$$

(7) Show for RLC series circuit $\frac{\omega_0}{R} = \frac{\omega_0 L}{R} = \frac{f_0}{\text{Bandwidth}}$.
where ω_0 is quality factor, f_0 = resonance frequency.

Soluⁿ on resonating circuit. At half power frequencies, the net reactance of either X_L or X_C depending on the ckt is R and given by $X = \pm (X_L - X_C) = R$

$$R = \pm \left(\omega L - \frac{1}{\omega C} \right) = \pm X$$

Let f_1 be the frequency when the net circuit reactance be -ve and f_2 be frequency when the net circuit reactance is +ve.

$$\left(\omega_2 L - \frac{1}{\omega_2 C} \right) = R \quad \text{(1)} \quad \text{and} \quad \left(\omega_1 L - \frac{1}{\omega_1 C} \right) = -R \quad \text{(2)}$$

Adding equation (1) and (2)

$$(\omega_2 + \omega_1)L - \frac{1}{C} \left(\frac{1}{\omega_2} + \frac{1}{\omega_1} \right) = 0$$

$$\text{or } (\omega_2 + \omega_1)L - \frac{1}{C} \left(\frac{\omega_2 + \omega_1}{\omega_2 \omega_1} \right) = 0$$

Thus either $L = \frac{1}{C} \cdot \frac{1}{\omega_2 \omega_1}$ or $\omega_2 \omega_1 = \frac{1}{LC}$ — (3)

Again subtracting eqn (2) from eqn (1)

$$(\omega_2 - \omega_1)L + \frac{1}{C} \left(\frac{1}{\omega_1} - \frac{1}{\omega_2} \right) = 2R$$

$$\Rightarrow (\omega_2 - \omega_1)L + \frac{1}{C} \left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \right) = 2R \quad \text{--- (4)}$$

Dividing eqn (4) by '4'

$$(\omega_2 - \omega_1) + \frac{1}{LC} \left(\frac{\omega_2 - \omega_1}{\omega_2 \omega_1} \right) = \frac{2R}{4}$$

$$\Rightarrow (\omega_2 - \omega_1) + \frac{1}{LC} \left(\frac{\omega_2 - \omega_1}{\frac{1}{LC}} \right) = \frac{2R}{4} \quad \left[\because \omega_2 \omega_1 = \frac{1}{LC} \right]$$

$$\Rightarrow 2(\omega_2 - \omega_1) = \frac{2R}{L} \quad \text{or} \quad (\omega_2 - \omega_1) = \frac{R}{L} \quad \text{--- (5)}$$

Again $\frac{1}{R} = \frac{\omega_0}{L} \Rightarrow \frac{R}{L} = \frac{\omega_0}{\omega_0}$. comparing with eqn (5)

$$\frac{\omega_0}{\omega_0} = (\omega_2 - \omega_1) \Rightarrow \omega_0 = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}$$

$\therefore \omega_0 = \frac{f_0}{f_2 - f_1}$ - resonant frequency

(9) A series RLC circuit has $R = 5\Omega$, $L = 10mH$ and $C = 15\mu F$. Calculate (i) Q factor of the circuit, (ii) the bandwidth, (iii) the resonant frequency (iv) the half power frequency f_1 and f_2 .

Solution $R = 5\Omega$, $L = 10mH$, $C = 15\mu F$

$$(i) Q \text{ factor} = \frac{1}{R} \sqrt{L/C} = \frac{1}{5} \sqrt{\frac{10 \times 10^{-3}}{15 \times 10^{-6}}} = 5.16$$

$$(ii) \text{ Band width} = \frac{f_0}{Q} = \frac{411}{5.16} = 79.66 \text{ Hz}$$

$$(iii) f_0 = \text{Resonant frequency} = \frac{1}{2\pi\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{10 \times 10^{-3} \times 15 \times 10^{-6}}} \\ = \frac{1}{2\pi \times 3.872 \times 10^{-4}} = 411 \text{ Hz}$$

(iv) Half power frequency

$$f_1 = f_0 - \frac{B\omega}{2} = 411 - \frac{79.66}{2} = 371.17 \text{ Hz}$$

$$f_2 = f_0 + \frac{B\omega}{2} = 411 + \frac{79.66}{2} = 450.83 \text{ Hz}$$

— XOX —

MODULE
mmmmmm

SHORT TYPE

(1) Determine the Laplace Transform of the functions. (2010)

$$f(t) = e^{at} \text{ and } f(t) = \bar{e}^{at}.$$

Soluⁿ given $f(t) = e^{at}$.

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{at}\}$$

$$\Rightarrow F(s) = \frac{1}{s-a}.$$

$$f(t) = \bar{e}^{at}$$

Taking Laplace on both side

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\bar{e}^{at}\}$$

$$\Rightarrow F(s) = \frac{1}{s+a}.$$

(2) Find the Laplace transform of the functions.

$$f(t) = \sin \omega t \text{ and } f(t) = \cos \omega t.$$

Soluⁿ given $f(t) = \sin \omega t$.

$$\Rightarrow F(s) = \frac{\omega}{s^2 + \omega^2}$$

$$f(t) = \cos \omega t$$

$$\Rightarrow F(s) = \frac{s}{s^2 + \omega^2}$$

(3) Draw the pole plot of sine and cosine functions. (2007)

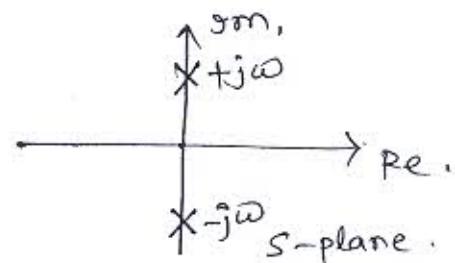
Soluⁿ $f(t) = \sin \omega t$.

$$F(s) = \frac{\omega}{s^2 + \omega^2}$$

No zero's.

$$s^2 = -\omega^2$$

$$s = \sqrt{-\omega^2} = \pm j\omega$$

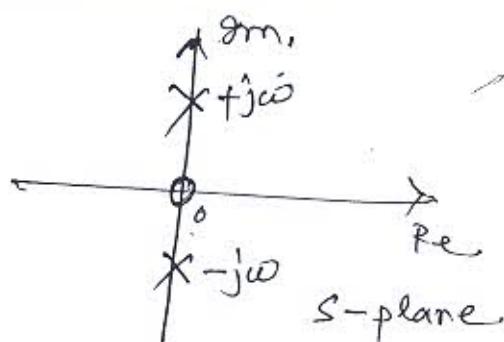


$$f(t) = \cos \omega t$$

$$F(s) = \frac{s}{s^2 + \omega^2}$$

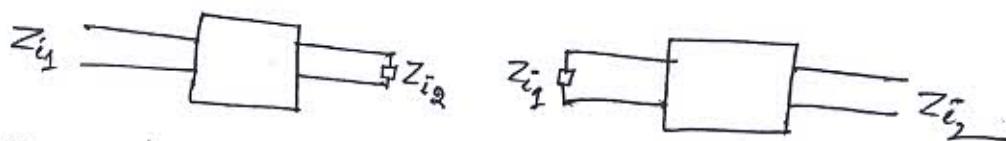
$$\text{Zero's } [s = 0]$$

$$\text{pole } s = \sqrt{-\omega^2} = \pm j\omega$$



(4) What is image and stereotrophic impedances.

Soluⁿ Image impedance :→



In a two port Network, when secondary port is closed Z_{i_2} and the impedance at the input port is Z_{i_1} . Similarly if the input port is closed by the impedance Z_{i_1} and the impedance at the output port is Z_{i_2} . So Z_{i_1} and Z_{i_2} are known as image impedances.

stereotrophic impedance.

$$\text{We Know } Z_{i_1} = \sqrt{\frac{AB}{CD}}, Z_{i_2} = \sqrt{\frac{BD}{AC}}$$

→ The image impedance is said to be stereotrophic impedance when the 2-port network is symmetrical, i.e $A = D$.

$$\text{So that } Z_{i_1} = Z_{i_2} = \sqrt{B/C} = Z_0$$

$$\text{So } Z_{i_1} = Z_{i_2} = \sqrt{B/C} = Z_0$$

Z_0 is known as stereotrophic impedance.

(5) A Two port Network has the following equations given below

$$V_1 = 3V_2 - 4I_2 \quad (2008, 2012-13)$$

$$I_1 = 2V_2 - I_2, \text{ find the required parameters.}$$

Soluⁿ We know $V_1 = AV_2 - BI_2$

$$I_1 = CI_2 - DI_2$$

By comparing $A = 3, B = 4, C = 2, D = 1$,
ABC_D parameters.

(21)

(6) If the Z -parameters of a two port Network are given as
 $Z_{11} = 5\Omega$, $Z_{22} = 7\Omega$, $Z_{12} = Z_{21} = 3\Omega$, then find the A, B, C, D parameters.
[2010]

Solution $A = \frac{Z_{11}}{Z_{21}} = \frac{5}{3} \Omega$

$$B = \frac{AZ}{Z_{21}} = \frac{35 - 9}{3} = 8.66 \Omega$$

$$C = \frac{1}{Z_{21}} = \frac{1}{3} \Omega$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{7}{3} \Omega$$

(7) For a two port network, give the expression for Z -parameters in terms of transmission ($ABCD$) parameters.

Solution $Z_{11} = \frac{A}{c}$, $Z_{12} = \frac{AD - BC}{c} = \frac{AT}{c}$

$$Z_{21} = \frac{1}{c}, Z_{22} = \frac{D}{c}.$$

(8) Why Z -parameters are known as open circuit parameters.

Soluⁿ The Z -parameters are also known as open circuit parameters because all the parameters are obtained by opening input port ($I_1 = 0$) or opening the output port ($I_2 = 0$).

(9) Define (a) Transfer function of a circuit.

(b) pole's and zero's of a Network.

Soluⁿ (a) The Transfer function is defined as, a quantity at one port to the quantity at another port, thus the transfer function is defined as the ratio of an o/p quantity to an input quantity.

(22)

(b) When a variable s' has value equal to any of the roots, the network functions becomes zero. Hence these complex frequencies are called the zero's.

→ When the variable s' has any of the values, the N/ω functions becomes infinite, hence these complex frequency are called poles of the N/ω function.

(10) Give the conditions for reciprocity and symmetry in terms of various parameters of two port network. (2011)

Solution.Z-parameter

$$\text{Reciprocity} \quad Z_{21} = Z_{12}$$

$$\text{Symmetrical} \quad Z_{11} = Z_{22}$$

H-parameter

$$\text{Reciprocity} \quad h_{21} = -h_{12}$$

$$\text{Symmetrical} \quad \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$

Y-parameter

$$\text{Reciprocity} \quad Y_{21} = Y_{12}$$

$$\text{Symmetrical} \quad Y_{11} = Y_{22}$$

ABCD parameters

$$\text{Reciprocity: } AD - BC = 1 \Rightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix} = 1$$

$$\text{Symmetrical: } A = D.$$

(11) Mention the properties of RC driving point impedance.

Solution: →

→ All the poles and zero's are simple and lies on the negative real axis.

→ The poles and zero's should be alternate.

→ The lowest critical frequency is a pole nearer to the origin or at the origin.

→ The highest critical frequency is a zero which may be at infinite.

→ The residue of all the poles are real and +ve.

→ $Z(\infty) \leq Z(0)$.

(23)

$$Y_a = s, Y_b = \frac{1}{s}, Y_c = \frac{1}{s+2} = \frac{1}{s+2}$$

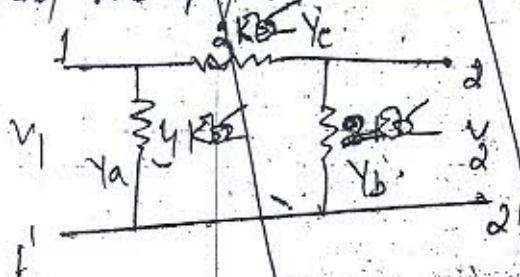
As per the formula

$$Y_{11} = Y_a + Y_c = s + \frac{1}{s+2} = \frac{s^2+2s+1}{s+2}$$

$$Y_{21} = Y_{12} = -Y_c = -\frac{1}{s+2}$$

$$Y_{22} = Y_b + Y_c = \frac{1}{s} + \frac{1}{s+2} = \frac{s+7}{5(s+2)}$$

P-5 Find the Y-parameters shown in fig?



Solution As it is a T network the admittance parameters can be found out by taking the formula for $Y_a = 4K\Omega, Y_b = 3K\Omega, Y_c = 2K\Omega$

$$Y_{11} = Y_a + Y_c = 4 + 3 = 7 K\Omega$$

$$Y_{12} = Y_{21} = -Y_c = -3 K\Omega$$

$$Y_{22} = Y_b + Y_c = 3 + 2 = 5 K\Omega$$

LONG TYPE

① Condition for symmetrical and Reciprocal \Rightarrow

(1) For Z-parameters \Rightarrow

$$\text{Reciprocal: } V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

case-I: when $V_2 = 0$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$0 = Z_{21}I_1 + Z_{22}I_2 \rightarrow$$

$$\begin{cases} Z_{21}I_1 = -Z_{22}I_2 \\ \Rightarrow \frac{I_1}{I_2} = \frac{-Z_{22}}{Z_{21}} \end{cases}$$

$$\Rightarrow \frac{V_1}{I_1} = Z_{11} \frac{I_1}{I_2} + Z_{12}$$

$$\Rightarrow \frac{V_1}{I_2} = Z_{11} \left(\frac{Z_{22}}{Z_{21}} \right) + Z_{12}$$

$$= -Z_{11}Z_{22} + Z_{12}Z_{21}$$

case - 2 When $V_2 = 0$

(24)

$$0 = z_{11}I_1 + z_{12}I_2 \rightarrow z_{11}I_1 = -z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2 \rightarrow \frac{I_2}{I_1} = -\frac{z_{11}}{z_{12}}$$

$$\Rightarrow \frac{V_2}{I_1} = z_{21} + z_{22} \frac{I_2}{I_1}$$

$$= z_{21} + z_{22} \left(-\frac{z_{11}}{z_{12}} \right)$$

$$= \frac{z_{21}z_{12} - z_{11}z_{22}}{z_{12}}$$
(2)

The network will be $\overset{z_{12}}{\text{reciprocal}}$ when.

$$\boxed{\frac{V_1}{I_1} = \frac{V_2}{I_2}}$$

$$\Rightarrow -\frac{z_{11}z_{22} + z_{12}z_{21}}{z_{21}} = \frac{z_{21}z_{12} - z_{11}z_{22}}{z_{12}}$$

$$\boxed{z_{21} = z_{12}} \quad \checkmark$$

Symmetrical.

Case - I. When $I_2 = 0$

$$V_2 = z_{21}I_1 + z_{22}I_2, V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 \Rightarrow V_1 = z_{11}I_1$$

$$\text{so } \frac{V_1}{I_1} = z_{11}$$

Case - II When $I_1 = 0$

$$V_1 = z_{12}I_2, V_2 = z_{22}I_2$$

$$\text{so } \frac{V_2}{I_2} = z_{22}$$

The network will be symmetrical when

$$\boxed{\frac{V_1}{I_1} = \frac{V_2}{I_2}}$$

$$\boxed{z_{11} = z_{22}} \quad \checkmark$$

25

(2) For h-parameters \rightarrow

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Reciprocal (when $V_1 \neq 0$)

$$0 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\boxed{\frac{I_2}{V_2} = -\frac{h_{12}}{h_{11}}} \quad \text{--- (1)}$$

(When $V_2 = 0$)

$$V_1 = h_{11}I_1$$

$$I_2 = h_{21}I_1$$

$$\Rightarrow \boxed{\frac{I_2}{V_1} = \frac{h_{21}}{h_{11}}} \quad \text{--- (2)}$$

The network will be reciprocal if

$$\boxed{\frac{I_1}{V_2} = \frac{I_2}{V_1}}$$

$$\Rightarrow -\frac{h_{12}}{h_{11}} = \frac{h_{21}}{h_{11}}$$

$$\Rightarrow \boxed{h_{21} = -h_{12}}$$

Symmetrical(Make $I_1 = 0$)

$$V_1 = h_{12}V_2$$

$$I_2 = h_{22}V_2$$

$$\boxed{\frac{I_2}{V_2} = h_{22}} \quad \text{--- (1)}$$

(Make $I_2 = 0$)

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$0 = h_{21}I_1 + h_{22}V_2 \Rightarrow \boxed{\frac{V_2}{I_1} = -\frac{h_{21}}{h_{22}}}$$

Divide by I_1, V_2

$$\begin{matrix} V_1 & V_2 \\ I_1 & I_2 \end{matrix}$$
Reciprocal ABCD

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$$

Symmetrical $A = D$

$$\boxed{\frac{I_1}{V_2} = \frac{I_2}{V_1}}$$

$$\boxed{\frac{I_1}{V_1} = \frac{I_2}{V_2}}$$

$$\begin{aligned} \Rightarrow \frac{V_1}{I_1} &= h_{11} + h_{12} \left(-\frac{h_{21}}{h_{22}} \right) \\ &= \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} \end{aligned}$$

$$\Rightarrow \boxed{\frac{I_1}{V_1} = \frac{h_{22}}{h_{11}h_{22} - h_{12}h_{21}}} \quad \text{--- (2)}$$

The network will be symmetrical if,

$$\boxed{\frac{I_1}{V_1} = \frac{I_2}{V_2}}$$

$$\Rightarrow \frac{h_{22}}{h_{11}h_{22} - h_{12}h_{21}} = h_{22}$$

$$\Rightarrow \frac{1}{h_{11}h_{22} - h_{12}h_{21}} = 1$$

$$\Rightarrow h_{11}h_{22} - h_{12}h_{21} = 1$$

$$\Rightarrow \boxed{\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1}$$

(2)

(26)

Problem calculate the z-parameters, if the values of other parameters are given below

(i) $A = 2, B = -1, C = 3$ and $D = -2$.

$$\text{Ans: } Z_{11} = \frac{A}{C} = \frac{2}{3} \Omega$$

$$Z_{12} = \frac{AD - BC}{C} = \frac{-4 + 3}{3} = -\frac{1}{3} \Omega$$

$$Z_{21} = \frac{B}{C} = \frac{1}{3} \Omega$$

$$Z_{22} = \frac{D}{C} = -\frac{2}{3} \Omega$$

(ii) $h_{11} = 1, h_{12} = -2, h_{21} = -3, h_{22} = 2$

$$\text{Ans: } Z_{11} = \frac{\Delta h}{h_{22}} = \frac{h_{11}h_{22} - h_{21}h_{12}}{h_{22}} = \frac{2 - 6}{2} = -2 \Omega$$

$$Z_{12} = \frac{h_{12}}{h_{22}} = -1 \Omega$$

$$Z_{21} = \frac{-h_{21}}{h_{22}} = \frac{3}{2} \Omega$$

$$Z_{22} = \frac{1}{h_{22}} = \frac{1}{2} \Omega$$

(iii) $y_{11} = 1/3, y_{12} = 2/3, y_{21} = -1/3, y_{22} = 1/6$

$$Z_{11} = \frac{y_{22}}{\Delta Y} = \frac{1}{6} \times \frac{18}{5} = \frac{3}{5} \Omega$$

$$\Delta Y = y_{11}y_{22} - y_{12}y_{21} = \frac{1}{18} + \frac{2}{9} = \frac{5}{18}$$

$$Z_{12} = \frac{-y_{12}}{\Delta Y} = -\frac{2}{3} \times \frac{18}{5} = -\frac{12}{5} \Omega$$

$$Z_{21} = \frac{-y_{21}}{\Delta Y} = \frac{1}{3} \times \frac{18}{5} = \frac{6}{5} \Omega$$

$$Z_{22} = \frac{y_{11}}{\Delta Y} = \frac{1}{3} \times \frac{18}{5} = \frac{6}{5} \Omega$$

(3)

Restriction on Location of poles and zero's in a deering

Q7

Pole impedance function : →

$$H(s) = \frac{A(s)}{B(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_n}{b_0 s^m + b_1 s^{m-1} + \dots + b_m}$$

- (1) All the coefficient of $A(s)$ and $B(s)$ should be +ve.
- (2) If the poles and zero's should have complex and imaginary parts, they must occur in conjugate pairs.
- (3) The real part of the poles and zero's must be zero or -ve.
- (4) The polynomial $A(s)$ and $B(s)$ should not have any missing terms from highest order to lowest order of missing all the even order terms are odd order terms should be missing.
- (5) The degree of $A(s)$ and $B(s)$ should differ at most unity.
- (6) The lowest degree of $A(s)$ and $B(s)$ should differ at most one.

restriction / conditions Necessary for the location of pole and zero. in a Transfer function : →

- (1) The coefficient of $A(s)$ and $B(s)$ should real.
- (2) The coefficient of $B(s)$ should be +ve though few coefficient of $A(s)$ may be -ve.
- (3) The poles and zero's, if complex or imaginary must be in conjugate pair.

The real part of the poles must be -ve or zero
real part is zero, the pole must be simple i.e. $-j, +j, j, -2j$, Repeated poles $-2j, -2j, +2j, +2j$

There may be some missing term for the $A(s)$ from the higher order to lower order. But $B(s)$ should not have any missing term. If missing all the even orders or all the odd orders should be ...

(28)

Question (4) $P(s) = s^4 + s^3 + ks^2 + 2s + 3$. Determine the value of k for which $P(s)$ is stable?

Solution

$$P(s) = s^4 + s^3 + ks^2 + 2s + 3$$

$$\begin{array}{c|ccc} s^4 & 1 & k & 3 \\ s^3 & 1 & 2 & 0 \\ s^2 & \frac{k-2}{1} & 3 \\ s^1 & \frac{2k-4-3}{3} \\ s^0 & 3 & k-2 \end{array}$$

For a stable system

$$\frac{2k-7}{k-2} > 0$$

$$\Rightarrow 2k-7 > 0$$

$$\Rightarrow 2k > 7$$

$$\Rightarrow k > \frac{7}{2}$$

$$\Rightarrow k > 3.5$$

Range of k is

$$0 < k < 3.5$$

Prove that for a series R-L-C circuit during resonance, the quality factor is same as the ratio of resonant frequency and bandwidth.

Ans. In a series RLC ckt.

$$\text{Band width} = f_2 - f_1$$

$$\omega_1 L - \frac{1}{\omega_1 C} = R \quad \dots (1)$$

$$\omega_2 L - \frac{1}{\omega_2 C} = -R \quad \dots (2)$$

Adding we get

$$\omega_1, \omega_2 = 1/LC$$

Substracting we get

$$\omega_2 - \omega_1 = R/L$$

$$\text{Quality factor } Q = \frac{\omega_0}{R}$$

$$= \frac{\omega_0}{R/L} = \frac{\omega_0}{\omega_2 - \omega_1}$$

$$\frac{f_0}{f_2 - f_1} = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$$

(Proof)

A series R-L-C circuit with values $R = 10$ ohm, $L = 0.2$ H, and $C = 100$ microfarad is excited from a DC source of 50 V by sudden switching ON of a key at time $t = 0$ s. Find the expression for the resulting current.

Ans. $R = 10 \Omega \quad L = 0.2 \text{ H} \quad C = 100 \mu\text{F}$.

$v = 50$ volt

$$V = Ri + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

Taking Laplace transform.

$$\frac{V}{S} = R I(s) + SL [I(s) - I(0)] + \frac{1}{C} \left[\frac{I(s)}{s} + \frac{q(0)}{s} \right]$$

There is no initial current in the inductor and initial charge at capacitor.

$$\frac{V}{S} = RI(s) + SLI(s) + \frac{1}{C} \left[\frac{I(s)}{s} \right]$$

$$(\because I(0) = 0, q(0) = 0)$$

$$\frac{V}{S} = \left[R + SL + \frac{1}{CS} \right] I(s)$$

$$VC = [RCS + S^2 LC + 1] I(s)$$

$$VC = LC \left[S^2 + \frac{R}{L}s + \frac{1}{LC} \right] I(s)$$

$$\frac{V}{s^2 + R/L s + 1} = I(s)$$

$$I(s) = \frac{50/0.2}{s^2 + \frac{10}{0.2}s + \frac{1}{0.2 \times 100 \times 10^{-6}}}$$

$$\Rightarrow \frac{250}{s^2 + 50s + 5 \times 10^4}$$

Taking inverse Laplace transform

$$I(s) = \frac{250}{(s^2 + 2 \cdot S \cdot 25 + (25)^2 + 5 \times 10^4 - (25)^2)}$$

$$I(s) = \frac{250}{(s + 25)^2 + (222 \cdot 2)^2}$$

$$I(s) = \frac{250}{(s + 25)^2 + (222 \cdot 2)^2} \times \frac{222 \cdot 2}{222 \cdot 2}$$

$$i(t) = 1.25 e^{-25t} \sin 222 \cdot 2 t$$

5.(a) What is the role of poles and zeros of a network function? What are the possible restrictions that may be imposed on their location in an s-plane?

Ans. Pole gives the time domain behaviour of the network function and zero gives the amplitude of network function. It gives the stability of the network function.

Restriction of poles-zeros in Driving point functions :

→ The coefficient of polynomials N.r. and D.M. of the network function must be real and +ve.

→ Poles and zeros, if complex, must occur in conjugate pairs.

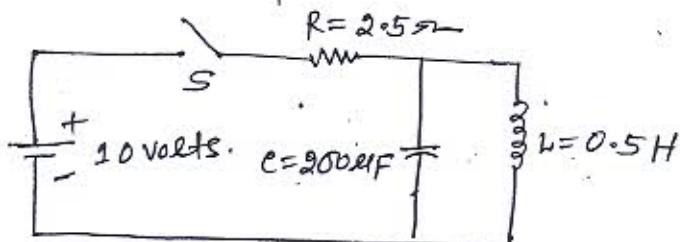
→ The real parts of all poles and zeros must be zero or negative.

(30)

(b) On the network shown in figure, the switch S is closed and steady state attained. At $t=0$, the switch is opened.

(a) Determine the current through the inductor.

(b) Voltage across the capacitor at $t=0.5$ second.



$$\text{Solu}^n \quad I(0) = \frac{10}{2.5} = 4A.$$

In this case at steady state conductors act as a short circuit and capacitor act as an open circuit. At $t=0$, switch is opened.

$$L \frac{dI}{dt} + \frac{1}{C} \int I dt = 0$$

Taking Laplace Transformation

$$SLI(s) - LI(0) + \frac{1}{Cs} I(s) = 0$$

$$\Rightarrow I(s) \left[SL + \frac{1}{Cs} \right] = LI(0)$$

$$I(s) \left[5 \times 0.5 + \frac{1}{200 \times 10^{-6}} \right] = 0.5 \times 4 = 2V$$

$$\Rightarrow I(s) = \frac{4s}{s^2 + 10^4}$$

Taking inverse Laplace Transformation.

$$I(t) = 4 \cos 100t$$

Voltage across the capacitor.

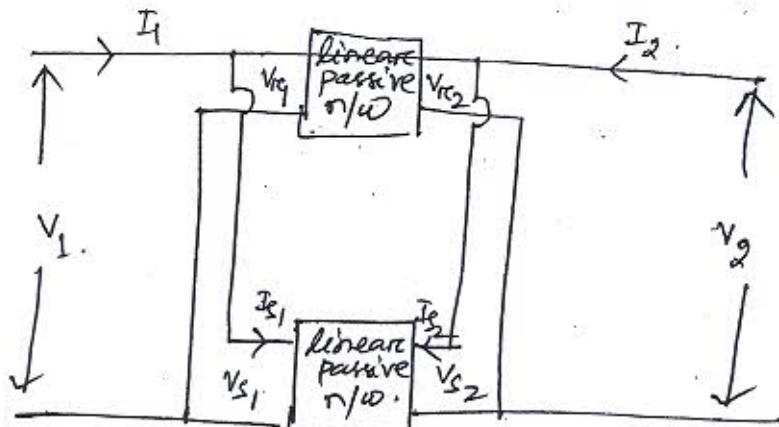
$$V_C = \frac{1}{C} \int_0^{0.5} 4 \cos 100t \, dt$$

$$= \frac{4}{200 \times 10^{-6} \times 100} \sin(100 \times 0.5) = 153.2 \text{ volt.}$$

(31)

~~(E)~~ Write short notes on parallel combination for interconnected of two port network.

Soluⁿ A parallel connection is defined when the voltage in the parallel elements are equal and the currents add up to give the resultant current.



If we consider 2 n/w 'rc' and 's' connected in parallel.

$$\text{Port-1} \quad I_{T1} + I_{S1} = I_1, \quad V_{T1} = V_{S1} = V_1$$

$$\text{Port-2} \quad I_{T2} + I_{S2} = I_2, \quad V_{T2} = V_{S2} = V_2$$

The two network 'rc' and 's' can be connected in following manner to be in parallel with each other. Under these conditions.

$$I_1 = (I_{T1} + I_{S1}) = (Y_{11rc} + Y_{11s}) V_1 + (Y_{12rc} + Y_{12s}) V_2$$

$$I_2 = (I_{T2} + I_{S2}) = (Y_{21rc} + Y_{21s}) V_1 + (Y_{22rc} + Y_{22s}) V_2$$

SHORT TYPE

(1) Define filter and classify it on the basis of operation?

Soluⁿ Definition A filter is a highly frequency selective device which passes the certain range of frequency (pass band) and suppress all other frequency (stop band).

Classification on the basis of operation :-

(a) Low pass filter (LPF)

(b) High pass filter (HPF)

(c) Band pass filter (BPF)

(d) Band stop filter or Band reject / elimination filter (BSF)

(2) Examine whether the network function is given by

$$F(s) = \frac{s+4}{s+6} \text{ is positive real?}$$

Soluⁿ $F(j\omega) = \frac{j\omega+4}{j\omega+6} = \frac{24+\omega^2}{36+\omega^2} + j \frac{3\omega}{36+\omega^2}$

$$\operatorname{Re}[F(j\omega)] = \frac{24+\omega^2}{36+\omega^2} > 0$$

(For all values of ω)

Hence the given function is a positive real function.

(3) What is Dierichlet condition's? Explain briefly?

Soluⁿ The conditions, under which a periodic functions $f(t)$ can be expanded in a convergent Fourier series, are known as Dierichlet's conditions.

These are as follows

(i) $f(t)$ is a single valued function.

(ii) $f(t)$ has a finite number of discontinuities in each period T .

(iii) $f(t)$ has a finite number of maxima and minima in each period.

(iv) The integral $\int_0^T |f(t)| dt$ exists and is finite or in other way $\int_0^\infty [f(t)]^2 dt < \infty$.

(34)

(4) List three properties of Fourier Transforms? [2010]

Soluⁿ → Linearity

→ scaling.

→ Time shifting

→ Frequency shifting

(5) give conditions for a polynomial $p(s)$ to be Hurwitz

Soluⁿ → $p(s)$ is real when s' is real.

→ The roots of $p(s)$ have real parts which are to be zero or negative.

X o X