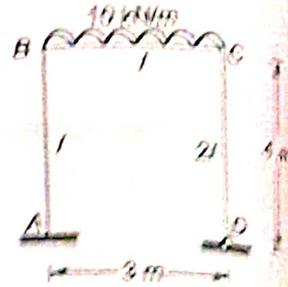


Q.1 Analyse the frame loaded as shown in figure below and draw the BMD. The frame is fixed at A and hinged at D. The relative second moment of the areas are also indicated in the figure.

[20 marks : 1995]



Solution:

Analysing the given frame using moment distribution method.

(i) Distribution Factors

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution Factors
B	BA	$\frac{I}{4} = \frac{3I}{12}$	$\frac{7I}{12}$	$\frac{3}{7}$
	BC	$\frac{I}{3} = \frac{4I}{12}$		$\frac{4}{7}$
C	CB	$\frac{I}{3} = \frac{8I}{24}$	$\frac{17I}{24}$	$\frac{8}{17}$
	CD	$\frac{3}{4} \times \frac{2I}{4} = \frac{3I}{8} = \frac{9I}{24}$		$\frac{9}{17}$

(ii) Non sway Analysis

Fixed end moments

$$\bar{M}_{AB} = 0; \bar{M}_{BA} = 0$$

$$\bar{M}_{BC} = -\frac{10 \times 3^2}{12} = -7.50 \text{ kN-m}$$

$$\bar{M}_{CB} = +\frac{10 \times 3^2}{12} = +7.50 \text{ kN-m}$$

$$\bar{M}_{CD} = 0; \bar{M}_{DC} = 0$$

		B		C		
		7.95	7.95	0		
	5.97	3.26	3.63	3.47		
1.825		1.785	2.485			
	5.785	1.895	1.895	1.198		
0.378		0.895	0.895			
	5.215	0.995	0.995	0.287		
0.188		0.415	0.415			
	5.025	0.585	0.585	0.077		
0.098		0.294	0.294			
	5.015	0.298	0.298	0.018		
2.117	4.248	4.248	5.468	5.468	0	

Taking moments about B, we get

$$M_{AB} + M_{BA} - H_A \times L_{AB} = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{L_{AB}}$$

$$= \frac{2.117 + 4.248}{4}$$

$$= 1.59 \text{ kN} (\rightarrow)$$

Taking moment about C, we get

$$M_{CD} + M_{DC} - H_D \times L_{CD} = 0$$

$$H_D = \frac{M_{CD} + M_{DC}}{L_{CD}}$$

$$= \frac{-5.468 + 0}{4}$$

$$H_D = -1.37 \text{ kN} (\rightarrow)$$

$$H_D = 1.37 \text{ kN} (\leftarrow)$$

Since the horizontal reaction (H_A) is greater than the horizontal reaction (H_D), there is an unbalanced force equal to $H_A - H_D$ which will act in the direction of H_A . Let it be S.

$$S = H_A - H_D = 1.59 - 1.37$$

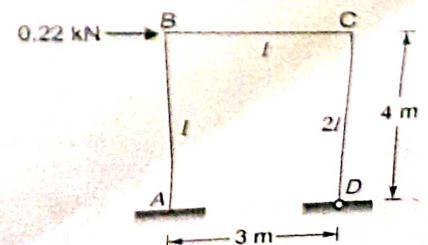
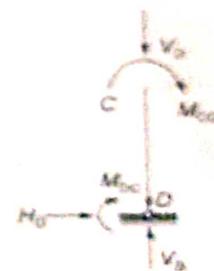
$$S = 0.22 \text{ kN} (\rightarrow)$$

(iii) Sway Analysis

Since sway of the frame is from left to right, the initial fixing moments due to deflection Δ will be in the anticlockwise direction

$$\bar{M}_{AB} = \bar{M}_{BA} = -\frac{6EI\Delta}{L^2} = -\frac{6EI\Delta}{4^2} = -\frac{6EI\Delta}{16}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = 0$$



$$M_{CB} = M_{DC} = \frac{6EI\delta}{l^2} = \frac{6 \times E \times 2I \times \delta}{4^2} = \frac{3EI\delta}{8}$$

	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
1	7	7	0	0	1	1
76	76	76	0	0	8	8
-2	-2	-2	0	0	-4	-4
-10	-10	-10	0	0	-20	-20

A	B		C		D	
	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$		
-10	-10	0	0	-20	-20	
-10	-10	0	0	+10	+20	
	+8.298	+5.714	+8.706	+5.294		
+0.143		+2.353	+2.857			
	-1.008	-1.345	-1.344	-1.513		
-0.504		-0.672	-0.673			
+0.144	+0.288	+0.384	+0.317	+0.356		
		+0.159	+0.192			
	-0.068	-0.091	-0.091	-0.101		
-0.034		-0.046	-0.046			
	+0.02	+0.026	+0.022	+0.024		
col. (a)	-8.251	-6.482	+6.482	+5.94	-5.94	0

$$H'_A = \frac{M_{AB} + M_{BA}}{L_{AB}} = \frac{-8.251 - 6.482}{4} = -3.68 \text{ kN } (\rightarrow)$$

$$H_A = 3.683 \text{ kN } (\leftarrow)$$

$$H'_D = \frac{M_{CD} + M_{DC}}{L_{CD}} = \frac{-5.94 + 0}{4}$$

$$H_D = -1.485 \text{ kN } (\rightarrow)$$

$$H_D = 1.485 \text{ kN } (\leftarrow)$$

Let the sway force responsible for these reactions be S' .

$$S' = H'_A + H'_D = 3.683 + 1.485 = 5.168 \text{ kN}$$

This sway force S' will act from left to right.

col. (a)						
Moments due to sway force $S' = 5.168 \text{ kN}$	-8.251	-6.482	+6.482	+5.94	-5.94	0
Moments due to actual sway force $S = \frac{0.22}{5.18} \times \text{col. (a)}$	-0.351	-0.276	+0.276	+0.253	-0.253	0

	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
Non sway moments	+2.177	+4.248	-4.248	+5.468	-5.468	0
Actual sway moments	-0.351	-0.276	+0.276	+0.253	-0.253	0
Total	+1.766	+3.972	-3.972	+5.721	-5.721	0

Actual horizontal reaction at A, $H_A = \frac{1.766 + 3.972}{4} = 1.44 \text{ kN } (\rightarrow)$

Actual horizontal reaction at D, $H_D = \frac{-5.721 + 0}{4} = -1.44 \text{ kN } (\rightarrow) = 1.44 \text{ kN } (\leftarrow)$

Vertical reaction at A, $V_A = \frac{3.972 - 5.721 + 10 \times 3 \times 1.5}{3} = 14.42 \text{ kN}$

Vertical reaction at D, $V_D = 10 \times 3 - 14.42 = 15.58 \text{ kN}$

Now taking outer face of the portal frame as reference face.

Taking simply supported span BC
Maximum moment

$$= \frac{10 \times 3^2}{8} = 11.25 \text{ kN-m at centre (Sagging)}$$

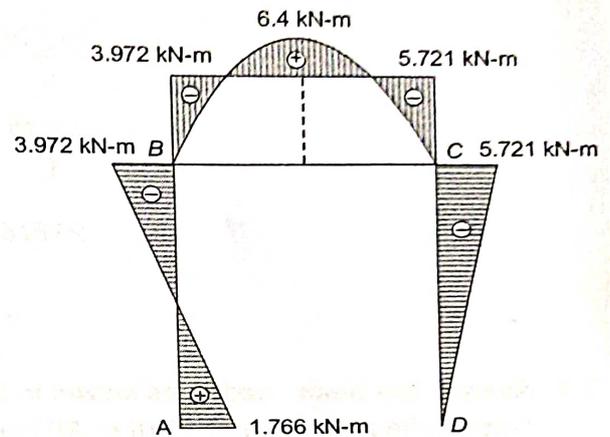
Fixed moment at the centre

$$= 3.972 + \frac{(5.721 - 3.972)}{3} \times 1.5$$

$$= 4.85 \text{ kN-m (Hogging)}$$

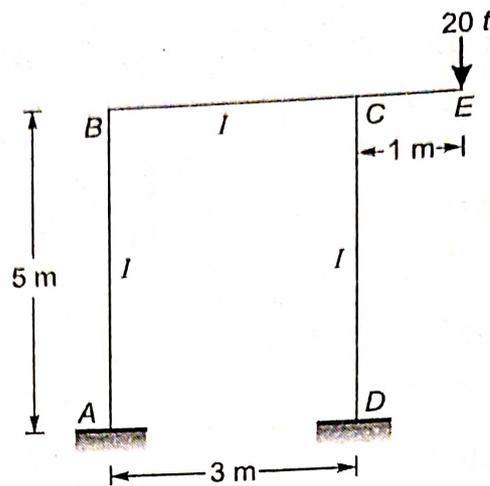
\therefore Resultant moment at the centre of span BC

$$= 11.25 - 4.85 = 6.4 \text{ kN-m (Sagging)}$$



Bending Moment Diagram

Q.4 Analyse the rigid jointed frame in figure below and draw SFD and BMD. Assume constant EI for all members.



[20 marks : 1

Solution:

Analysing the given portal frame by moment distribution method.

(i) Distribution Factors

Joint	Members	Relative Stiffness	Total Relative Stiffness	Distribution Factors
B	BA	$\frac{I}{5} = \frac{3I}{15}$	$\frac{8I}{15}$	$\frac{3}{8}$
	BC	$\frac{I}{3} = \frac{5I}{15}$		$\frac{5}{8}$
C	CB	$\frac{I}{3} = \frac{5I}{15}$	$\frac{8I}{15}$	$\frac{5}{8}$
	CD	$\frac{I}{5} = \frac{3I}{15}$		$\frac{3}{8}$

(ii) Non-sway Analysis
Fixed end moments

$$\bar{M}_{AB} = 0; \quad \bar{M}_{BA} = 0$$

$$\bar{M}_{BC} = 0; \quad \bar{M}_{CB} = 0, \quad \bar{M}_{CE} = -20 \text{ t-m} = M_{CE}$$

$$\bar{M}_{CD} = 0; \quad \bar{M}_{DC} = 0$$

∴ Equilibrium condition at C,

$$M_{CB} + M_{CD} + M_{CE} = 0$$

$$M_{CB} + M_{CD} - 20 = 0$$

$$M_{CB} + M_{CD} = 20$$

$$M_{BA} + M_{BC} = 0$$

⇒

⇒

Also,

A	B		C		D	
	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{8}$		
0	0	0	0	0	0	
		+6.25	+12.5	+7.5	+3.75	
0	0	+6.25	+12.5	+7.5	+3.75	
	-2.34	-3.91	0	0		
-1.17		0	-1.96		0	
	0	0	+1.23	+0.73		
0		+0.62	0		+0.37	
	-0.23	-0.39	0	0		
-0.12		0	-0.2		0	
		0	+0.13	+0.07		
0		+0.07	0		+0.04	
	-0.03	-0.04	0	0		
-0.015		0	-0.02		0	
			+0.013	+0.007		
Non sway moments	-1.31	-2.6	+2.6	+11.69	+8.31	+4.16

$$\text{Horizontal reaction at A, } H_A = \frac{-1.31 - 2.6}{5} = -0.782 \text{ t } (\rightarrow) = 0.782 \text{ t } (\leftarrow)$$

$$\text{Horizontal reaction at D, } H_D = \frac{8.31 + 4.16}{5} = 2.494 \text{ t } (\rightarrow)$$

Let the sway force be S.

$$\therefore S = 2.494 - 0.782 = 1.712 \text{ t } (\rightarrow)$$

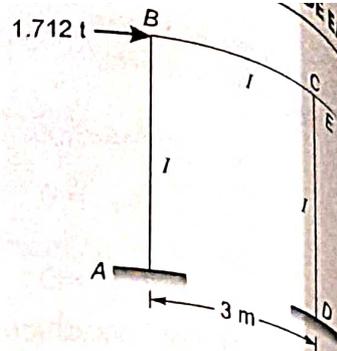
(iii) Sway Analysis

The initial fixing moments will be anticlockwise because the sway is from left to right.

$$\bar{M}_{AB} = -\frac{6EI\Delta}{L^2} = -\frac{6EI\Delta}{25} = \bar{M}_{BA}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = 0$$

$$\bar{M}_{CD} = -\frac{6EI\Delta}{L^2} = -\frac{6EI\Delta}{25} = \bar{M}_{DC}$$



$\therefore M_{AB} : M_{BA} : M_{BC} : M_{CB} : M_{CD} : M_{DC}$
-1 : -1 : 0 : 0 : -1 : -1
-10 : -10 : 0 : 0 : -10 : -10

	B		C		
	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	
A	-10	-10	0	0	-10
		+3.75	+6.25	+6.25	+3.75
	+1.88		+3.13	+3.13	
		-1.17	-1.96	-1.96	-1.17
	-0.59		-0.98	-0.98	
		+0.37	+0.61	+0.61	+0.37
	+0.19		+0.31	+0.31	
		-0.12	-0.19	-0.19	-0.12
	-0.06		-0.1	-0.1	
		+0.04	+0.06	+0.06	+0.04
	+0.02		+0.03	+0.03	
		-0.01	-0.02	-0.02	-0.01
Col. (a)	-8.56	-7.14	+7.14	+7.14	-7.14
					-8.56
D					

Horizontal reaction at A, $H'_A = \frac{-8.56 - 7.14}{5} = -3.14 \text{ t} (\rightarrow) = 3.14 \text{ t} (\leftarrow)$

Horizontal reaction at D, $H'_D = \frac{-8.56 - 7.14}{5} = -3.14 \text{ t} (\rightarrow) = 3.14 \text{ t} (\leftarrow)$

Let the sway force for these reactions be S' .

$$S' = 3.14 + 3.14 = 6.28 \text{ t} (\rightarrow)$$

	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
col.(a)	-8.56	-7.14	+7.14	+7.14	-7.14	-8.56
Actual sway moment $\left(\text{col. (a)} \times \frac{S}{S'} \right)$	-2.33	-1.95	+1.95	+1.95	-1.95	-2.33
Non-sway moments	-1.31	-2.6	+2.6	+11.69	+8.31	+4.16
Total	-3.64	-4.55	+4.55	+13.64	+6.36	+1.83

Actual horizontal reaction at A, $H_A = \frac{-3.64 - 4.55}{5} = -1.638 \text{ t} (\rightarrow) = 1.638 \text{ t} (\leftarrow)$

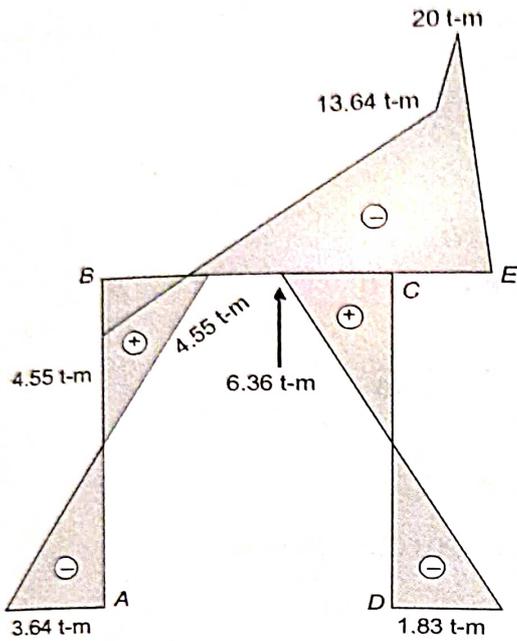
Actual horizontal reaction at D, $H_D = \frac{6.36 + 1.83}{5} = 1.638 \text{ t} (\rightarrow)$

Vertical reaction at A, $V_A = -\left(\frac{4.55 + 13.64}{3}\right) = -6.063 \text{ t} (\uparrow) = 6.063 \text{ t} (\downarrow)$

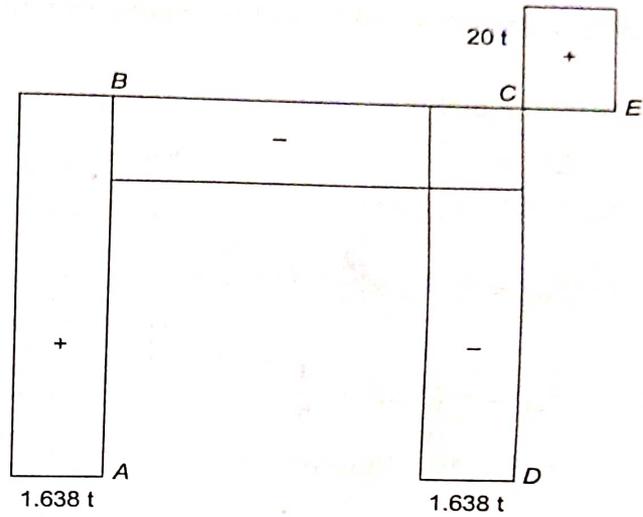
Vertical reaction at D, $V_D = 20 + 6.063 = 26.063 \text{ t} (\uparrow)$

BMD and SFD

Assuming outer face of the portal frame as reference face.

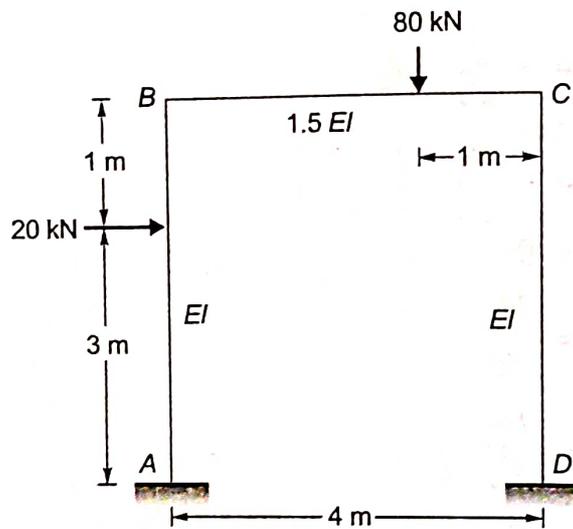


Bending Moment Diagram



Shear Force Diagram

Q.31 Analyse the rigid jointed frame shown in the figure using slope deflection method. Find all the reaction components at the supports *A* and *D*. Draw BMD and SFD for all the members.



[30 marks : 2005]

Solution:

Fixed end moments:

$$\bar{M}_{AB} = -\frac{20 \times 3 \times 1^2}{4^2} = -3.75 \text{ kN-m}$$

$$\bar{M}_{BA} = + \frac{20 \times 1 \times 3^2}{4^2} = + 11.25 \text{ kN-m}$$

$$\bar{M}_{BC} = - \frac{80 \times 3 \times 1}{4^2} = - 15 \text{ kN-m}$$

$$\bar{M}_{CB} = + \frac{80 \times 1 \times 3^2}{4^2} = 45 \text{ kN-m}$$

$$\bar{M}_{CD} = \bar{M}_{DC} = 0$$

Slope deflection equations

Member AB

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L} \left(2\theta_A + \theta_B - \frac{3\delta}{L} \right)$$

\Rightarrow

$$M_{AB} = -3.75 + \frac{2EI}{4} \left(\theta_B - \frac{3\delta}{4} \right)$$

\Rightarrow

$$M_{AB} = -3.75 + \frac{EI}{2} \theta_B - \frac{3EI\delta}{8}$$

and

$$M_{BA} = \bar{M}_{BA} + \frac{2EI}{L} \left(2\theta_B + \theta_A - \frac{3\delta}{L} \right)$$

\Rightarrow

$$M_{BA} = + 11.25 + \frac{2EI}{4} \left(2\theta_B - \frac{3\delta}{4} \right)$$

\Rightarrow

$$M_{BA} = 11.25 + EI\theta_B - \frac{3EI\delta}{8}$$

Member BC

$$M_{BC} = \bar{M}_{BC} + \frac{2EI}{L} (2\theta_B + \theta_C)$$

\Rightarrow

$$M_{BC} = -15 + \frac{2 \times 1.5 EI}{4} (2\theta_B + \theta_C)$$

\Rightarrow

$$M_{BC} = -15 + 1.5 EI\theta_B + 0.75 EI\theta_C$$

$$M_{CB} = \bar{M}_{CB} + \frac{2EI}{L} (2\theta_C + \theta_B)$$

\Rightarrow

$$M_{CB} = 45 + \frac{2 \times 1.5 EI}{4} (2\theta_C + \theta_B)$$

\Rightarrow

$$M_{CB} = 45 + 0.75 EI\theta_B + 1.5 EI\theta_C$$

Member CD

$$M_{CD} = \bar{M}_{CD} + \frac{2EI}{L} \left(2\theta_C + \theta_D - \frac{3\delta}{L} \right)$$

\Rightarrow

$$M_{CD} = 0 + \frac{2EI}{4} \left(2\theta_C - \frac{3\delta}{4} \right)$$

\Rightarrow

$$M_{CD} = EI\theta_C - \frac{3EI\delta}{8}$$

$$M_{DC} = \bar{M}_{DC} + \frac{2EI}{4} \left(2\theta_D + \theta_C - \frac{3\delta}{L} \right)$$

$$\Rightarrow M_{DC} = 0 + \frac{2EI}{4} \left(\theta_C - \frac{3\delta}{4} \right)$$

$$\Rightarrow M_{DC} = 0.5 EI \theta_C - \frac{3EI\delta}{8}$$

Equilibrium condition at B,

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow 11.25 + EI \theta_B - \frac{3EI\delta}{8} + 1.5 EI \theta_B + 0.75 EI \theta_C - 15 = 0$$

$$\Rightarrow 0.75 EI \theta_C + 2.5 EI \theta_B - \frac{3EI\delta}{8} = 3.75$$

Equilibrium condition at C,

$$M_{CB} = M_{CD} = 0$$

$$\Rightarrow (45 + 0.75 EI \theta_B + 1.5 EI \theta_C) + \left(EI \theta_C - \frac{3EI\delta}{8} \right) = 0$$

$$\Rightarrow 0.75 EI \theta_B + 2.5 EI \theta_C - \frac{3EI\delta}{8} = -45$$

For horizontal equilibrium, we have

$$H_A + H_D + 20 = 0$$

$$\Rightarrow \frac{M_{AB} + M_{BA} - 20 \times 1}{4} + \frac{M_{CD} + M_{DC}}{4} + 20 = 0$$

$$\Rightarrow M_{AB} + M_{BA} - 20 + M_{CD} + M_{DC} + 80 = 0$$

$$\Rightarrow -3.75 + \frac{EI}{2} \theta_B - \frac{3EI\delta}{8} + 11.25 + EI \theta_B - \frac{3EI\delta}{8} + EI \theta_C - \frac{3EI\delta}{8} + 0.5 EI \theta_C - \frac{3EI\delta}{8} + 60 = 0$$

$$\Rightarrow 1.5 EI \theta_B + 1.5 EI \theta_C - \frac{12EI\delta}{8} = -67.5$$

$$\Rightarrow EI \theta_B + EI \theta_C - EI \delta = -45$$

Solving (i), (ii) and (iii), we get

$$EI \theta_B = 12.43; EI \theta_C = -15.43; EI \delta = 42$$

Final moments

$$M_{AB} = -3.75 + 0.5 EI \theta_B - 0.375 EI \delta$$

$$= -3.75 + 0.5 \times 12.43 - 0.375 \times 42$$

$$\Rightarrow M_{AB} = -13.285 \text{ kN-m}$$

$$M_{BA} = 11.25 + 12.43 - 0.375 \times 42 = +7.93 \text{ kN-m}$$

$$M_{BC} = -15 + 1.5 \times 12.43 - 0.75 \times 15.43 = -7.93 \text{ kN-m}$$

$$M_{CB} = 45 + 0.75 \times 12.43 - 1.5 \times 15.43 = +31.18 \text{ kN-m}$$

$$M_{CD} = -15.43 - 0.375 \times 42 = -31.18 \text{ kN-m}$$

$$M_{DC} = -0.5 \times 15.43 - 0.375 \times 42 = -23.465 \text{ kN-m}$$

$$\text{Horizontal reaction at A} = \frac{M_{AB} + M_{BA} - 20 \times 1}{4} = \frac{-13.285 + 7.93 - 20}{4}$$

$$= -6.34 \text{ kN } (\rightarrow) = 6.34 \text{ kN } (\leftarrow)$$

$$\text{Horizontal reaction at D} = \frac{M_{CD} + M_{DC}}{4} = \frac{-31.18 - 23.465}{4}$$

$$= -13.66 \text{ kN } (\rightarrow) = 13.66 \text{ kN } (\leftarrow)$$

$$\text{Vertical reaction at A} = -\left(\frac{M_{BC} + M_{CB} - 80 \times 1}{4}\right) = -\left(\frac{-7.93 + 31.18 - 80}{4}\right) = 14.1875 \text{ kN } (\uparrow)$$

$$\text{Vertical reaction at D} = \frac{M_{BC} + M_{CB} + 80 \times 3}{4} = \frac{-7.93 + 31.18 + 240}{4} = 65.8125 \text{ kN } (\uparrow)$$

Alternatively

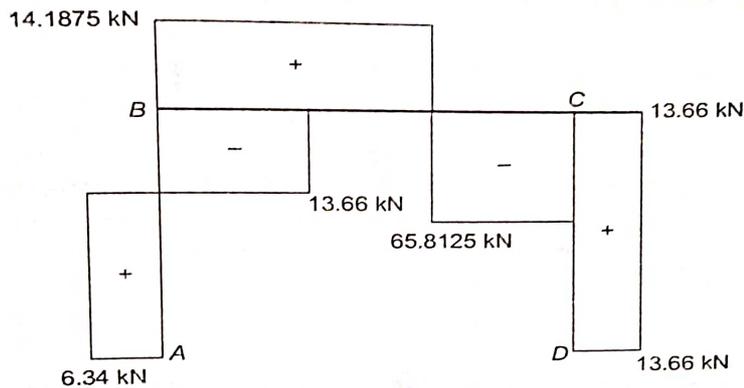
$$V_D = 80 - V_A = 80 - 14.1875 = 65.8125 \text{ kN}$$

Shear force and bending moment diagrams:

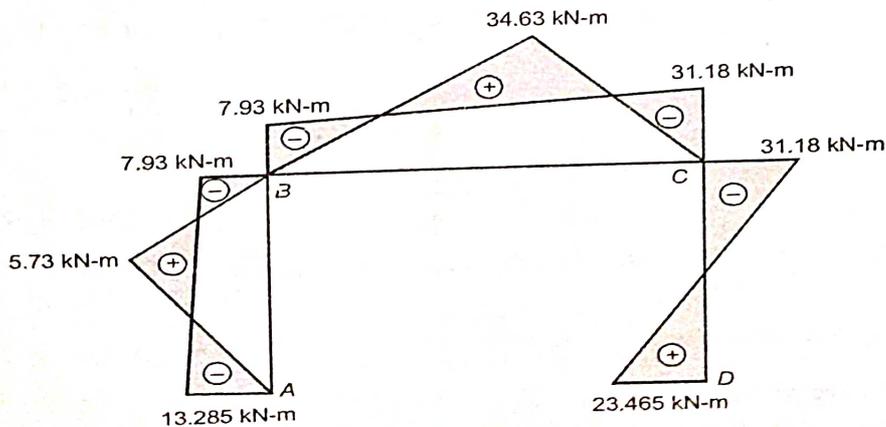
Taking outer surface of the portal frame as reference face.

$$\text{Maximum simply supported moment in AB} = \frac{20 \times 3 \times 1}{4} = 15 \text{ kN-m}$$

$$\text{Maximum simply supported moment in BC} = \frac{80 \times 3 \times 1}{4} = 60 \text{ kN-m}$$

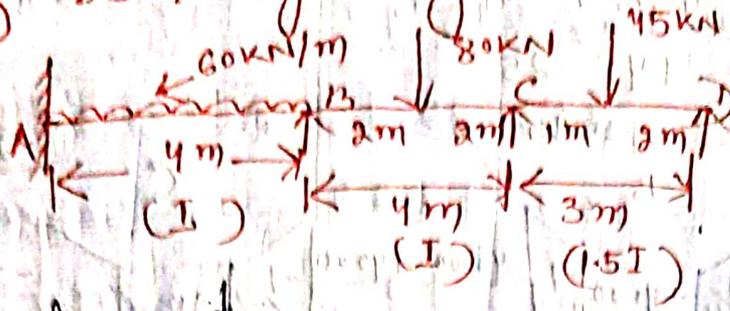


Shear Force Diagram



Bending Moment Diagram

Q. Find the support moment at A, B, C, D for the continuous beam by using Kar's method.



Sol. Fixed end moment

$$M_{FAB} = -wl^2/12 = \frac{-60 \times 4^2}{12} = -80 \text{ kNm}$$

$$M_{FBA} = +wl^2/12 = \frac{+60 \times 4^2}{12} = +80 \text{ kNm}$$

$$M_{FBC} = -wl/8 = \frac{-80 \times 4}{8} = -40 \text{ kNm}$$

$$M_{FCB} = +wl/8 = \frac{+80 \times 4}{8} = +40 \text{ kNm}$$

$$M_{FCD} = \frac{-wab^2}{l^2} = \frac{-45 \times 1 \times 2^2}{2^2} = -20 \text{ kNm}$$

$$M_{FDC} = \frac{+wa^2b}{l^2} = \frac{+45 \times 1^2 \times 2}{2^2} = +10 \text{ kNm}$$

Rotation factor

Joint	Member	K	ΣK	$\frac{K}{\Sigma K}$	RF = $(U) = \frac{1}{2} \times \frac{K}{\Sigma K}$
B	BA	$\frac{I}{4}$	$\frac{2I}{4}$	$\frac{I/4 \times 4}{2I} = \frac{1}{2}$	$-\frac{1}{2} \times \frac{1}{2} = -\frac{1}{4}$
	BC	$\frac{I}{4}$		$\frac{I/4 \times 4}{2I} = \frac{1}{2}$	$-\frac{1}{2} \times \frac{1}{2} = -\frac{1}{4}$
C	CB	$\frac{I}{4}$	$\frac{5I}{8}$	$\frac{I/4 \times 8}{5I} = \frac{2}{5}$	$-\frac{1}{2} \times \frac{2}{5} = -\frac{1}{5}$
	CD	$\frac{3I}{4}$		$\frac{3I/4 \times 8}{5I} = \frac{3}{5}$	$-\frac{1}{2} \times \frac{3}{5} = -\frac{3}{10}$

Iteration table

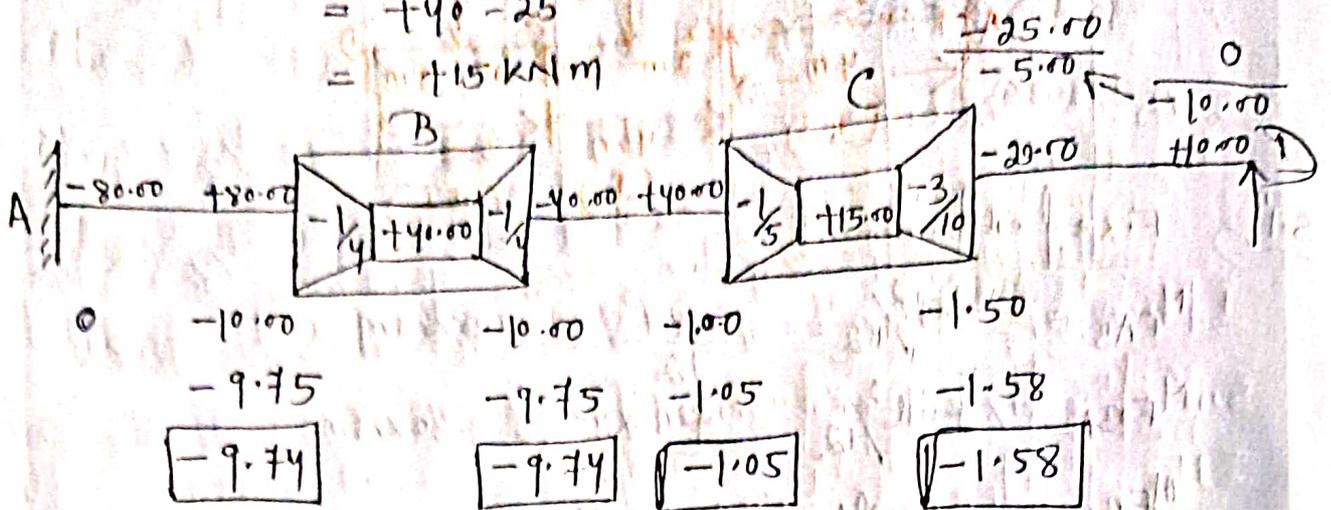
sum of fixed end moment at B

$$\Sigma M_{FB} = M_{FBA} + M_{FBC}$$

$$= +80 - 40 = 40 \text{ kNm}$$

Sum of fixed end moment at C

$$\begin{aligned} \Sigma M_{FC} &= M_{FCB} + M_{FCD} \\ &= +40 - 25 \\ &= +15 \text{ kNm} \end{aligned}$$

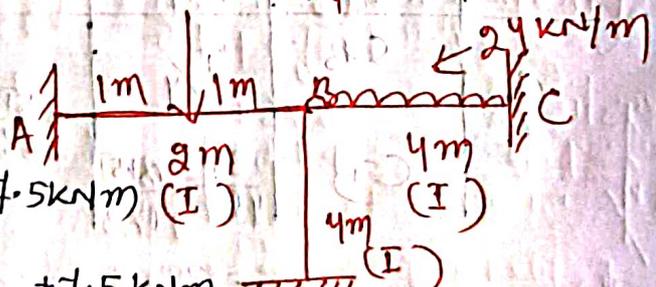


Final moment

FEM	-80.00	+80.00	-40.00	+40.00	-25.00	0
NEC	2x0	-2x9.74	-2x9.74	-2x1.05	-2x1.58	2x0
FEC	-9.74	0	+1.05	-9.74	0	
Total moments	-89.74	+60.53	-60.53	+28.16	-28.16	0

Q.) Find the support moments for the frame shown in fig.

soln. Fixed end moments



$$M_{FAB} = -wl^2/8 = -\frac{30 \times 2}{8} = -7.5 \text{ kNm (I)}$$

$$M_{FBA} = +wl^2/8 = \frac{30 \times 2}{8} = +7.5 \text{ kNm (I)}$$

$$M_{FBC} = -wl^2/12 = -\frac{24 \times 4^2}{12} = -32 \text{ kNm}$$

$$M_{FCB} = +wl^2/12 = \frac{24 \times 4^2}{12} = +32 \text{ kNm}$$

$$M_{FBD} = 0$$

$$M_{FDB} = 0$$

Rotation factor

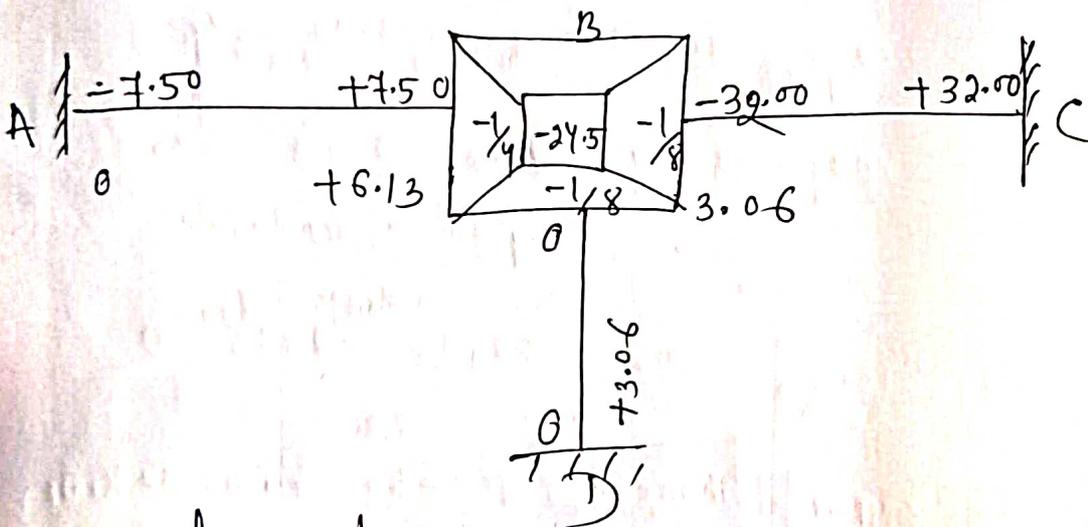
Joint	Member	K	ΣK	$\frac{K}{\Sigma K}$	$V = -\frac{1}{2} \times D.F.$
B	BA	$I/2$	I	$1/2$	$-1/4$
	BC	$I/4$		$1/4$	$-1/8$
	BD	$I/4$		$1/4$	$-1/8$

Iteration table

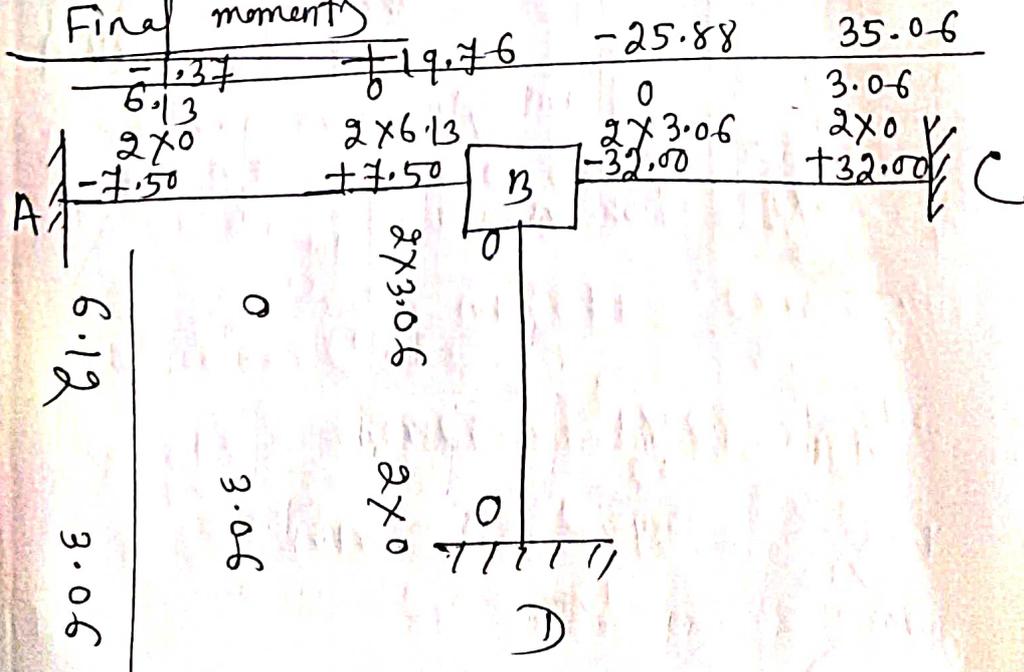
sum of fixed end moments at 'B'.

$$\Sigma M_B = M_{FBA} + M_{FBC} + M_{FBD}$$

$$= 7.5 - 32 + 0 = -24.5 \text{ kNm}$$



Final moments



$$M_{AB} = -7.50 + 2 \times 0 + 6.13 = -1.37 \text{ kNm}$$

$$M_{BA} = 7.50 + 2 \times 6.13 + 0 = +19.76 \text{ kNm}$$

$$M_{BC} = -32 + 2 \times 3.06 + 0 = -25.88 \text{ kNm}$$

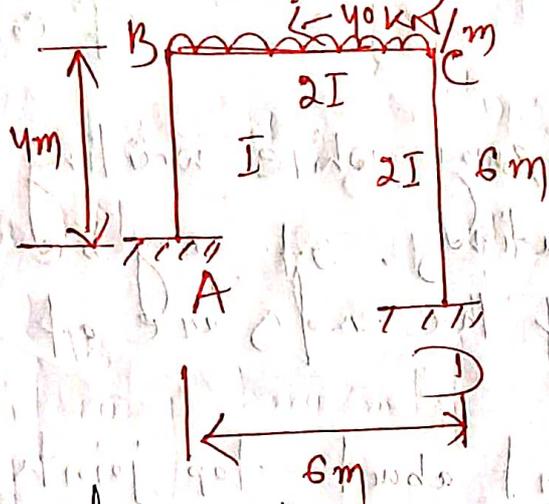
$$M_{CB} = 32 + 2 \times 0 + 3 \times 0.6 = 35.06 \text{ kNm}$$

$$M_{BD} = 0 + 2 \times 3.06 + 0 = 6.12 \text{ kNm}$$

$$M_{DB} = 0 + 2 \times 0 + 3 \times 0.6 = 3.06 \text{ kNm}$$



Q.7) Analyse the frame shown in fig. and draw B.M.D



Solⁿ

Fixed end moment

$$M_{AB} = 0$$

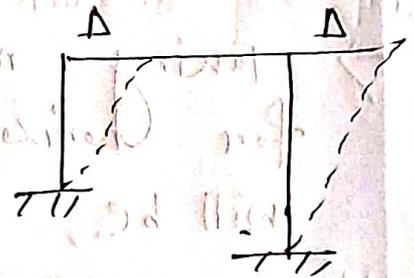
$$M_{CD} = 0$$

$$M_{BA} = 0$$

$$M_{DC} = 0$$

$$M_{BC} = \frac{-wl^2}{12} = \frac{-40 \times 6^2}{12} = -120 \text{ kNm}$$

$$M_{CB} = \frac{+wl^2}{12} = \frac{+40 \times 6^2}{12} = +120 \text{ kNm}$$



slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l} \left(2\theta_A + \theta_B - \frac{3\delta}{l} \right)$$

$$0 = 0 + \frac{2EI}{4} \left(\theta_B - \frac{3\delta}{4} \right) \quad (\theta_A = 0 \text{ at 'A' fixed})$$

$$\theta_B = \frac{EI\theta_B}{2} - \frac{3}{8} EI\delta \quad \text{--- (1)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l} \left(2\theta_B + \theta_A - \frac{3\delta}{l} \right)$$

$$0 = 0 + \frac{2EI}{4} \left(2\theta_B - \frac{3\delta}{4} \right)$$

$$\theta_B = \frac{EI\theta_B}{2} - \frac{3}{8} EI\delta \quad \text{--- (2)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l} \left(2\theta_B + \theta_C - \frac{3\delta}{l} \right)$$

$$= -120 + \frac{2E(2I)}{6} (2\theta_B + \theta_C)$$

$$= -120 + \frac{4EI}{3} \theta_B + \frac{2EI}{3} \theta_C \quad \text{--- (3)}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l} \left(2\theta_C + \theta_B - \frac{3\delta}{l} \right)$$

$$= 120 + \frac{2E(2I)}{6} (\theta_B + 2\theta_C)$$

$$= 120 + \frac{4EI}{3} \theta_C + \frac{2EI}{3} \theta_B \quad \text{--- (4)}$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l} \left(2\theta_C + \theta_D - \frac{3\delta}{l} \right)$$

$$= 0 + \frac{2E(2I)}{6} \left(2\theta_C - \frac{3\delta}{6} \right)$$

$$= \frac{4EI}{3} \theta_C - \frac{EI\delta}{3} \quad \text{--- (5)}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l} \left(2\theta_D + \theta_C - \frac{3\delta}{l} \right)$$

$$= 0 + \frac{2E(2I)}{6} \left(\theta_C - \frac{3\delta}{6} \right)$$

$$= \frac{2EI}{3} \theta_C - \frac{EI\delta}{3} \quad \text{--- (6)}$$

Joint equilibrium equation :-

$$\sum M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow EI Q_B - \frac{3}{8} EI \delta + \left(-120 + \frac{4EI Q_B}{3} + \frac{2}{3} EI Q_C \right) = 0$$

$$\Rightarrow \boxed{2.333 EI Q_B + 0.667 EI Q_C - 0.375 \delta = 120} \quad (7)$$

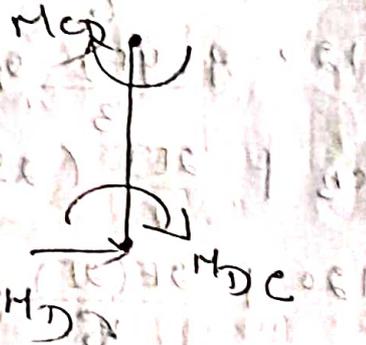
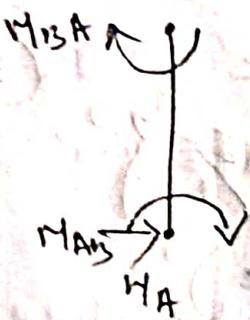
$$\sum M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

$$120 + \frac{4}{3} EI Q_C + \frac{2}{3} EI Q_B + \frac{4}{3} EI Q_C - \frac{EI \delta}{3} = 0$$

$$\Rightarrow \boxed{0.667 EI Q_B + 2.667 EI Q_C - 0.333 \delta = -120} \quad (8)$$

Considering free body diagram of column and taking moment about top joints, we get



$$H_A = \frac{M_{AB} + M_{BA}}{l_{AB}}$$

$$H_D = \frac{M_{CD} + M_{DC}}{l_{CD}}$$

Now considering horizontal equilibrium of a frame we get,

$$H_A + H_D = 0$$

$$\frac{M_{AB} + M_{BA}}{4} + \frac{M_{CD} + M_{DC}}{6} = 0$$

MULTIPLE CHOICE OBJECTIVE TYPE QUESTIONS

1. The moment area theorems in the structural analysis fall in the category of
(a) Force method (b) Displacement method
(c) Stiffness method (d) Iterative method
2. The analysis of statically indeterminate structures by the unit load method is based on
(a) Force method concept (b) Stiffness method
(c) Both of the above (d) None of the above
3. The analysis of statically indeterminate structures by the unit load method is based on
(a) Consistent deformation
(b) Stiffness method
(c) Consistent force (d) None of the above
4. The force method in structural analysis always ensures
(a) Equilibrium
(b) Kinematically admissible forces
(c) Equilibrium of forces (d) None of the above
5. Unequal settlements in the supports of a statically indeterminate structure develop
(a) Member forces (b) Reactions from supports
(c) No reactions
(d) Strains in some members only
6. The method of virtual work in the analysis of structures results in
(a) Computable deformations
(b) Equilibrium of forces
(c) Stress strain relations
(d) None of the above
7. Maxwell's reciprocal theorem in structural analysis can be applied in
(a) All elastic structures (b) Plastic structures
(c) Symmetrical structures only
(d) Prismatic element structures only
8. Castigliano's first theorem is applicable
(a) for elastic structures
(b) for all statically determinate structures
(c) only when principle of superposition is valid
(d) None of the above.
9. The Muller-Breslau principle in structural analysis is used for
(a) Drawing influence line diagram for any force function
(b) Superimposition of load effects
(c) Writing virtual work equation
(d) None of the above (IES 2012)
10. When a load is applied to a structure with rigid joints
(a) there is no rotation or displacement of joint
(b) there is no rotation of joint
(c) there is no displacement of joint
(d) there can be rotation and displacement of joint but the angle between the members connected to the joint remains same even after application of the load (IES 2008)
11. A determinate structure
(a) cannot be analyzed without the correct knowledge of modulus of elasticity
(b) must necessarily have roller support at one of its ends
(c) requires only statical equilibrium equations for its analysis
(d) will have zero deflection at its ends.

12. By which one of the following methods is an approximate quick solution possible for frames subjected to transverse loads?

- (a) By cantilever or portal method
- (b) By strain energy method
- (c) By moment distribution method
- (d) By matrix method

13. A statically indeterminate structure is the one which

- (a) cannot be analyzed at all
- (b) can be analyzed using equations of statics only
- (c) can be analyzed using equations of statics and compatibility equations
- (d) can be analyzed using equations of compatibility only

14. The three moment equation in structural analysis is basically a

- (a) Stiffness method
- (b) Displacement method
- (c) Energy method
- (d) Flexibility method

15. In moment distribution method the sum of distribution factors of all the members meeting at any joint is always

- (a) zero
- (b) < 1
- (c) > 1
- (d) $= 1$

16. For prismatic members, stiffness factor is

- (a) $\frac{I}{l}$
- (b) $\frac{l}{E}$
- (c) EI
- (d) $\frac{AEI}{I}$

where I = moment of inertia

l = length of member

E = Young's modulus

A = Area of cross-sections

17. The carry over factor for prismatic member with far end fixed is

- (a) $-\frac{1}{2}$
- (b) $\frac{1}{2}$
- (c) $\frac{1}{4}$
- (d) $-\frac{1}{4}$

18. The absolute stiffness of a prismatic member with one end fixed is

- (a) $\frac{2EI}{L}$
- (b) $\frac{4EI}{L}$
- (c) $\frac{3EI}{L}$
- (d) none of the above

19. The absolute stiffness of a prismatic member with one end hinged is

- (a) $\frac{2EI}{L}$
- (b) $\frac{4EI}{L}$
- (c) $\frac{3EI}{L}$
- (d) none

20. In the Fig. shown the degree of external indeterminacy is

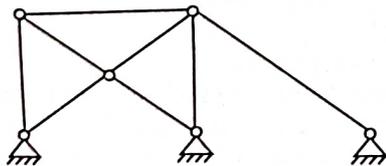


Fig. 10.93.

- (a) 1
- (b) 2
- (c) 3
- (d) 4

21. In the Fig. 10.93, the degree of internal indeterminacy is

- (a) 1
- (b) 2
- (c) 3
- (d) stable and determinate

22. What is the degree of kinetic indeterminacy of the frame shown below if axial deformation is neglected

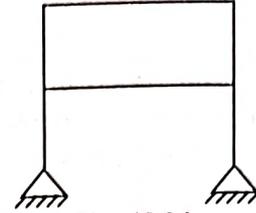


Fig. 10.94.

- (a) 6
- (b) 8
- (c) 10
- (d) 12

23. What is the degree of kinetic indeterminacy of the beam shown below

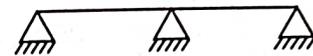


Fig. 10.95.

- (a) 2
- (b) 3
- (c) 4
- (d) 5

24. What is the degree of kinematic indeterminacy of the beam shown in above problem, if the axial deformation is ignored?

- (a) 2
- (b) 3
- (c) 4
- (d) 5

25. The kinematic indeterminacy of the beam is

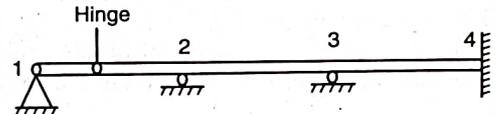


Fig. 10.96.

- (a) 5
- (b) 9
- (c) 14
- (d) 15

26. The kinematic indeterminacy of the frame is:

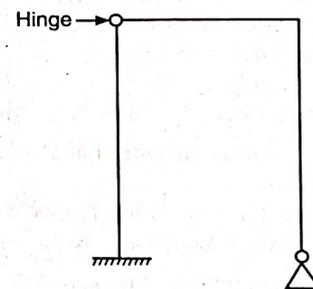


Fig. 10.97.

- (a) 4
- (b) 6
- (c) 8
- (d) 10

27. What is the degree of kinematic indeterminacy of the frame shown below if the axial deformation is ignored?

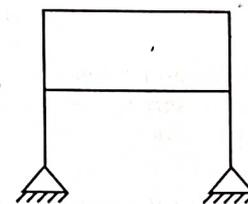


Fig. 10.98.

- (a) 8
- (b) 10
- (c) 12
- (d) 14

28. What is the degree of static indeterminacy of the beam shown below?

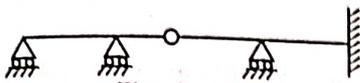


Fig. 10.99.

- (a) 1 (b) 2 (c) 3 (d) 4
29. What is the total degree of indeterminacy in the continuous prismatic beam shown in the figure below?

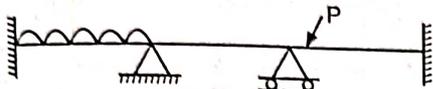


Fig. 10.100.

- (a) 1 (b) 2 (c) 3 (d) 4
30. A suspension bridge with a two-hinged stiffening girder is
- (a) statically determinate
 (b) indeterminate of one degree
 (c) indeterminate of two degrees
 (d) a mechanism

31. What is the kinematic indeterminacy for the frame shown below? (member inextensible)

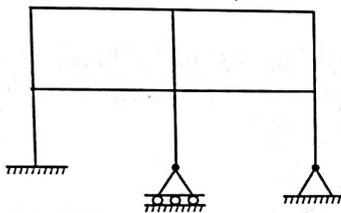


Fig. 10.101.

- (a) 6 (b) 11 (c) 12 (d) 21
32. A suspension bridge with a two-hinged stiffening girder is
- (a) statically determinate
 (b) indeterminate of one degree
 (c) indeterminate of two degrees
 (d) a mechanism
33. What is the statical indeterminacy for the frame shown below?

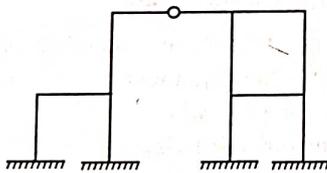


Fig. 10.102.

- (a) 12 (b) 15 (c) 11 (d) 14
34. The portal frame shown below will

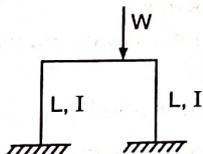


Fig. 10.103.

- (a) not sway (b) sway towards left

- (c) sway towards right
 (d) sway either to left or right

35. The influence line for force in member DC of the truss shown below will be

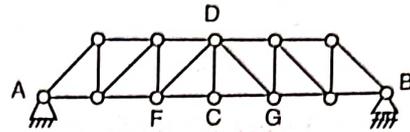


Fig. 10.104.

- (a) A ——— B (b) A ——— B
 F C G F C G
 (c) A ——— B (d) A ——— B
 C C

36. The moment required to rotate the near end of a prismatic beam through unit angle without translation, when the far end is fixed.

- (a) $\frac{EI}{L}$ (b) $\frac{2EI}{L}$ (c) $\frac{3EI}{L}$ (d) $\frac{4EI}{L}$

(IES 2012)

37. A suspension bridge with a two-hinged stiffening girder is statically

- (a) Determinate
 (b) Indeterminate to 1 degree
 (c) Indeterminate to 2 degrees
 (d) Indeterminate to 3 degrees

(IES 2012)

38. The expression given by Castiglianos first theorem to determine the deflection component of any point on structure is

- (a) $\int \frac{M}{EI} \frac{\rho M}{\rho P}$ (b) $\int \frac{\rho M}{\rho P} \frac{dx}{EI}$
 (c) $\int M \left(\frac{\rho M}{\rho P} \right) \frac{dx}{EI}$ (d) None of the above

39. In the moment area method, the difference in slope between any two sections of a loaded flexural member is equal to the

- (a) Area of the $\frac{M}{EI}$ diagram between these two sections
 (b) Moment of the $\frac{M}{EI}$ diagram between these two sections
 (c) $\frac{1}{2} \times$ area of the $\frac{M}{EI}$ diagram between these two sections
 (d) $\frac{1}{2} \times$ moment of the $\frac{M}{EI}$ diagram between these two sections

40. In the moment area method, the deflection of a point A from a tangent at B is equal to the

- (a) Area of $\frac{M}{EI}$ diagram between A and B

(b) Moment of $\frac{M}{EI}$ diagram between A and B about A

(c) Moment of $\frac{M}{EI}$ diagram between A and B about B

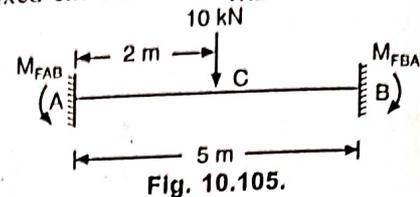
(d) $\frac{1}{2} \times$ area of $\frac{M}{EI}$ diagram between A and B

41. The conjugate beam method falls in the category of
(a) Force method (b) Stiffness method
(c) Displacement method (d) None of the above
42. Bending moment at any section in a conjugate beam gives in the actual beam
(a) Slope (b) Curvature
(c) Deflection (d) None of the above
43. The fixed support in real beam becomes in the conjugate beam is a
(a) Fixed support (b) Hinged support
(c) Roller support (d) Free support
44. The three moments equation is applicable only when
(a) The beam is prismatic
(b) There is no discontinuity such as hinges within the span
(c) The span are equal
(d) There are atleast 2 spans.
45. Which are of the following is true example of statically determinate beam?
(a) One end is fixed and the other end is simply supported
(b) Both the ends are fixed
(c) The beam over hangs over two supports
(d) The beam is supported on three support
(IES 2011)
46. If M is the external moment which rotates the near end of a prismatic beam without translation, the far end being fixed, then the moment induced at the far end is
(a) zero
(b) $\frac{M}{2}$ in the same direction as M
(c) $\frac{M}{2}$ in the opposite direction as M
(d) None of the above
47. The method of moment distribution in structural analysis is
(a) An iterative method (b) An exact method
(c) An approximate method
(d) None of the above
48. The moment distribution method in structural analysis can be treated as
(a) Force method (b) Displacement method
(c) Flexibility method (d) None of the above
(IES 2011)
49. A propped cantilever beam AB of span L is subjected to a moment M at the prop end B. The moment at fixed end A is

- (a) 2 M (b) $\frac{M}{2}$ (c) M (d) $\frac{3M}{4}$

(IES 2010)

50. A fixed beam AB, of constant EI, shown in the figure below, supports a concentrated load of 10 kN. What is the fixed end-moment M_{FAB} at support A?



- (a) 4.8 kN-m (b) 6.0 kN-m
(c) 7.2 kN-m (d) 9.5 kN-m

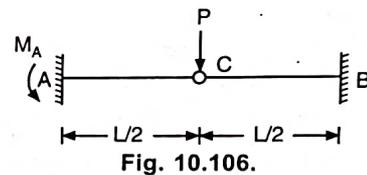
(GATE 2007)

51. If the free end of a cantilever of span L and flexure rigidity EI undergoes a unit displacement (without rotation), what is the bending moment induced at the fixed end?

- (a) $\frac{3EI}{L^2}$ (b) $\frac{4EI}{L^2}$ (c) $\frac{5EI}{L^2}$ (d) $\frac{6EI}{L^2}$

(GATE 2008)

52. The fixed beam AB has a hinge C at mid span. A concentrated load P is applied at C? What is the fixed end moment M_A ?



- (a) PL (b) PL/2 (c) PL/4 (d) PL/B

(IES 2012)

53. The units of flexural stiffness are
(a) Radians per unit rotation
(b) Moment per unit rotation
(c) Force per unit deflection and rotation
(d) Extension per unit force
54. The torsional stiffness of a member can be defined as
(a) Torque for unit moment
(b) Torque for unit twist
(c) Moment for unit twist
(d) Torsion for unit twist
55. The stiffness method in structural analysis is also known as
(a) Unit load method
(b) Consistant deformation method
(c) Force method
(d) Displacement method
(GATE 2006)
56. The flexibility of an element can be defined as
(a) Flexural moment per unit rotation
(b) Rotation for unit moment

- (c) Flexibility for unit translation
- (d) None of the above

57. The elements of flexibility matrix of a structure
- (a) are independent of the choice of coordinates
 - (b) are dependent on the choice of coordinates
 - (c) are always dimensionally homogeneous
 - (d) both (a) and (c)
58. An increase in temperature on the top fibre of a simply supported beam will cause
- (a) Downward deflection
 - (b) Upward deflection
 - (c) No deflection
 - (d) Angular rotation about neutral axis

59. A fixed beam with central point load undergoes a slight settlement at one end. Select suitable answer from the following:
- (a) Moment induced at both ends will be same
 - (b) Moment induced at the end that has undergone settlement will be maximum
 - (c) Moment induced will be maximum at the end having no settlement
 - (d) Zero moment at the end that has settled
- (IES 2011)

60. A uniformly distributed load (w) of length shorter than the span crosses a girder. The bending moment at a section in girder will be maximum when
- (a) Head of the load is at the section
 - (b) Tail of the load is at the section
 - (c) Section divides the load in the same ratio as it divides the span
 - (d) Section divides the load in two equal lengths
- (IES 2011)

61. Consider the following statements relating to structural analysis:
1. Flexibility matrix and its transpose are equal
 2. Elements of main diagonal of stiffness matrix are always positive
 3. For unstable structures, coefficients in leading diagonal matrix can be negative
- Which of these statements is/are correct?
- (a) 1, 2 and 3
 - (b) 1 and 2 only
 - (c) 2 and 3 only
 - (d) 3 only
- (IES 2011)

62. Flexibility matrix for a beam element is $[F] =$
- $$\frac{1}{EI} \begin{bmatrix} 36 & 9 \\ 9 & 4 \end{bmatrix}$$

What is the corresponding stiffness matrix $[S]$?

(a) $[S] = \frac{EI}{63} \begin{bmatrix} 36 & -9 \\ -9 & 4 \end{bmatrix}$ (b) $[S] = \frac{EI}{63} \begin{bmatrix} 36 & 9 \\ 9 & 4 \end{bmatrix}$

(c) $[S] = \frac{EI}{63} \begin{bmatrix} 4 & -9 \\ -9 & 36 \end{bmatrix}$ (d) $[S] = \frac{EI}{63} \begin{bmatrix} 4 & 9 \\ 9 & 36 \end{bmatrix}$

(IES 2010)

63. Flexibility matrix of the beam shown below is:

$$\delta = \frac{1}{3EI} \begin{bmatrix} 1 & 2 \\ 2 & 8 \end{bmatrix}$$

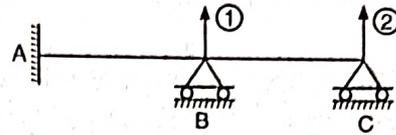


Fig. 10.107.

- If support B settles by $\frac{\Delta}{EI}$ units, what is the reaction at B
- (a) 0.75Δ
 - (b) 3.0Δ
 - (c) 6.0Δ
 - (d) 24.0Δ
- (IES 2010)

64. What is the value of flexibility coefficient f_{12} for the continuous beam shown in figure below?

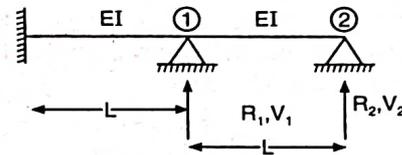


Fig. 10.108.

- (a) $\frac{L^3}{3EI}$
 - (b) $\frac{L^3}{2EI}$
 - (c) $\frac{L^3}{8EI}$
 - (d) $\frac{L^3}{1.2EI}$
- (IES 2010)

65. Euler critical load of a column restrained against rotation and translation at both end is

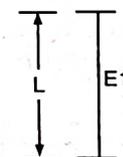


Fig. 10.109.

- (a) $\frac{EI}{L^2}$
- (b) $-\frac{EI}{L^2}$
- (c) $\frac{1.33 \pi^2 EI}{L^2}$
- (d) $\frac{2.02 \pi^2 EI}{L^2}$

66. The pin jointed plane truss $abcd$ supported by means of links as shown in Fig. 10.110.

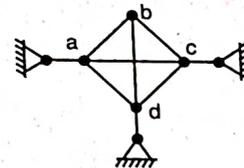


Fig. 10.110.

- (a) Stable and determinate
- (b) Stable and indeterminate
- (c) Geometrical unstable and has the internal indeterminacy
- (d) A mechanism

67. Which one of the following statements is correct?
- (a) In slope-deflection method, the force are taken as unknowns

- (b) In slope-deflection method, the joint rotations are taken as unknowns
- (c) Slope-deflection method is not applicable for beams and frames having settlement at the supports
- (d) Slope deflection method is also known as force method

68. The deflection at the free end F of cantilever beam HF having uniform flexural rigidity EI is

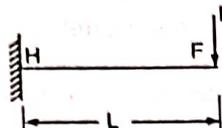


Fig. 10.111.

- (a) $\frac{PL^2}{3EI}$ (b) $\frac{PL^2}{2EI}$ (c) $\frac{5PL^2}{38EI}$ (d) $\frac{PL^2}{48EI}$

(GATE 2008)

69. The Euler's critical buckling load for a column restrained against rotation and translation at one end and against translation at the other end is

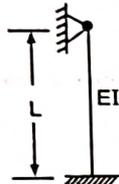


Fig. 10.112.

- (a) $\pi^2 EI/L^2$ (b) $4 \pi^2 EI/L^2$
 (c) $\pi^2 EI/4L^2$ (d) $2 \pi^2 EI/L^2$

70. For maximum negative bending moment at support B of continuous beam $ABCDE$ the live load should be placed in the spans

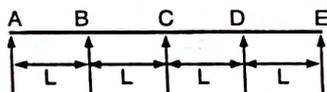


Fig. 10.113.

- (a) AB and CD (b) AD , BC and DE
 (c) BC and DE (d) AB , BC , CD and DE

71. What is the value of vertical reaction at A for the frame shown in figure below?

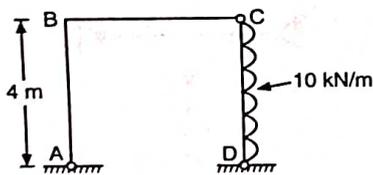


Fig. 10.114.

- (a) 0 (b) 10 kN (c) 16 kN (d) 20 kN

(IES 2009)

72. The reactions R_1 and R_2 of the beam simply supported on springs having stiffness K and $3K$ are

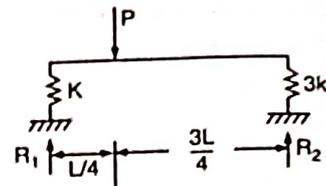


Fig. 10.115.

- R_1 R_2
 (a) $P/2$ $P/2$
 (b) $P/4$ $3P/4$
 (c) $3P/4$ $P/4$
 (d) $3P/8$ $P/8$

73. The bending moment at b of column ab by the portal method of analysis

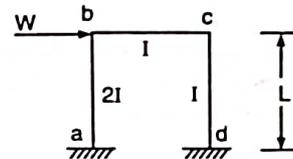


Fig. 10.116.

- (a) $\frac{WL}{4}$ (b) $\frac{WL}{2}$ (c) WL (d) $\frac{WL}{3}$

74. The bar force bd in the truss beam bracket $abcd$ is

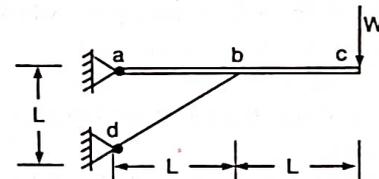


Fig. 10.117.

- (a) $\frac{W}{\sqrt{2}}$ (comp.) (b) $W\sqrt{2}$ (comp.)
 (c) W (tensile) (d) $W\sqrt{2}$ (comp.)

75. What is the force in member AB of the pin jointed frame as shown below?

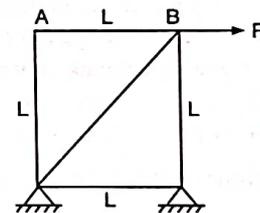


Fig. 10.118.

- (a) P (tension) (b) P (compression)
 (c) $\frac{P}{\sqrt{2}}$ (compression) (d) zero (IES 2006)

76. If members are axially rigid the number of independent degrees of freedom of joints of the rigid frame abc has is

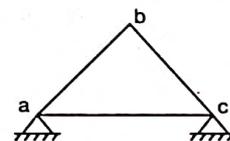


Fig. 10.119.

ANSWERS

1. (a)	2. (a)	3. (a)	4. (c)	5. (b)	6. (b)	7. (a)	8. (c)	9. (a)	10. (d)
11. (c)	12. (a)	13. (c)	14. (d)	15. (c)	16. (a)	17. (b)	18. (b)	19. (c)	20. (b)
21. (d)	22. (d)	23. (b)	24. (b)	25. (c)	26. (c)	27. (a)	28. (b)	29. (d)	30. (b)
31. (b)	32. (b)	33. (c)	34. (c)	35. (b)	36. (d)	37. (c)	38. (c)	39. (a)	40. (b)
41. (a)	42. (c)	43. (d)	44. (b)	45. (c)	46. (b)	47. (a)	48. (b)	49. (b)	50. (c)
51. (d)	52. (c)	53. (b)	54. (b)	55. (d)	56. (b)	57. (b)	58. (b)	59. (c)	60. (c)
61. (b)	62. (c)	63. (c)	64. (d)	65. (b)	66. (c)	67. (b)	68. (b)	69. (d)	70. (b)
71. (c)	72. (c)	73. (a)	74. (c)	75. (d)	76. (d)	77. (d)	78. (d)	79. (b)	80. (c)

Q. A two hinged parabolic arch of span 25m & rise 5m carries a udl of 40 kN/m over the half of the span & a concentrated load of 100 kN at the crown. Find the horizontal thrust at each support.

Solⁿ

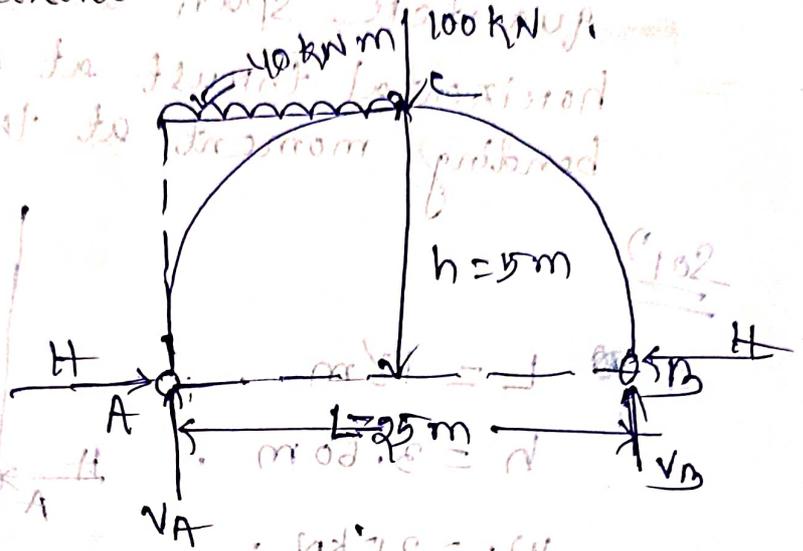
$$L = 25 \text{ m}$$

$$h = 5 \text{ m}$$

$$W_1 = 40 \text{ kN/m}$$

$$W_2 = 100 \text{ kN}$$

(Parabolic arch)



Horizontal thrust due to udl on the left half of the span,

$$H_1 = \frac{wl^2}{16h} = \frac{40 \times 25^2}{16 \times 5} = 312.5 \text{ kN}$$

Horizontal load due to point load on the crown.

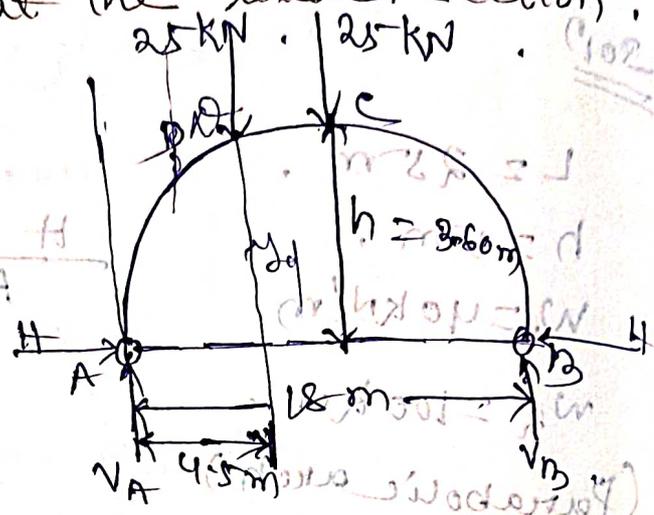
$$H_2 = \frac{25^2}{128} \frac{wl}{h} = \frac{25^2}{128} \times \frac{100 \times 25}{5} = 97.66 \text{ kN}$$

$$\begin{aligned} \text{Total horizontal thrust, } H &= H_1 + H_2 \\ &= 312.5 + 97.66 \\ &= 410.16 \text{ kN} \end{aligned}$$

Q1 A two hinged parabolic arch of span 18m & rise 3.60m carries two concentrated load 25kN each at the crown of left quarter span section. Find the horizontal thrust at each support & the bending moment at the loaded section.

Solⁿ

$L = 18\text{m}$
 $h = 3.60\text{m}$
 $W_1 = 25\text{kN}$
 $W_2 = 25\text{kN}$



Horizontal thrust due to load at Q,

$$H_1 = \frac{5W}{8hL^3} a(l-a)(l^2 + la - a^2)$$

$$= \frac{5}{8} \times \frac{25}{3.60 \times 18^3} \times 4.5(18 - 4.5)(18^2 + 18 \times 4.5 - 4.5^2)$$

$$= 17.395 \text{ kN}$$

Horizontal thrust due to load at the crown

$$H_2 = \frac{25}{128} \frac{Wl}{h} = \frac{25}{128} \times \frac{25 \times 18}{3.60}$$

$$= 24.414 \text{ kN}$$

Total horizontal thrust, $H = H_1 + H_2$

$$= 17.395 + 24.414$$

$$= 41.809 \text{ kN}$$

Date = 06/02/2019

Taking moment about point 'A' left support 'A'

$$\sum M_A = 0$$

$$\Rightarrow V_B \times 18 = (25 \times 9) + (25 \times 4.5)$$

$$\Rightarrow V_B = 18.75 \text{ kN}$$

$$V_A + V_B = 25 + 25$$

$$\Rightarrow V_A = 50 - 18.75 = 31.25 \text{ kN}$$

ordinate of the arch at 'D'

$$y_d = \frac{4h}{l^2} n(l-n)$$

$$= \frac{4 \times 3.6}{18^2} \times 4.50 (18 - 4.50)$$

$$= 2.70 \text{ kN}$$

$$\text{B.M. at 'D'} = 31.25 \times 4.5 - 41.809 \times 2.70$$

$$= 27.741 \text{ kNm}$$

$$\text{B.M. at 'c'} = 18.75 \times 9 - 41.809 \times 3.60$$

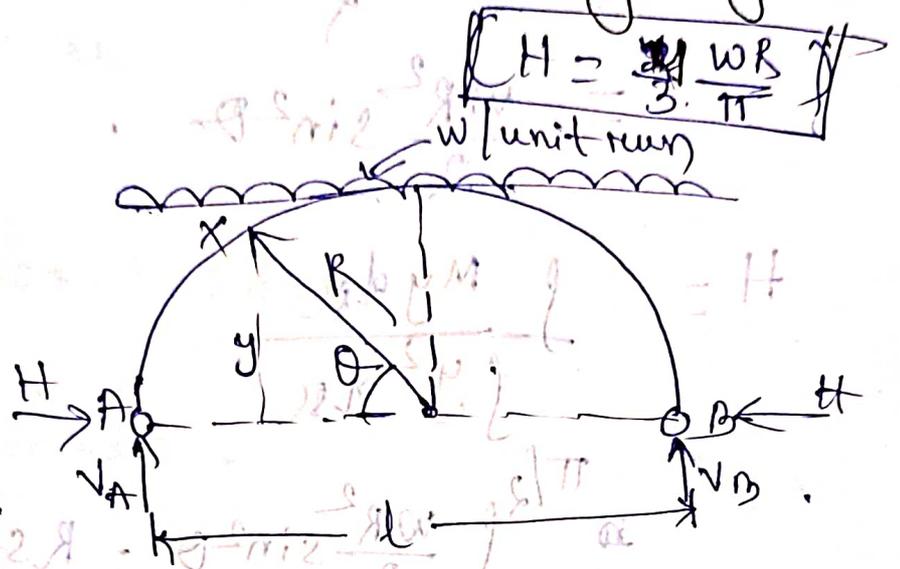
$$= 18.24 \text{ kNm}$$

Q: A two hinged semicircular arch of radius 'R' carries a udl of w /unit run over the whole span. Determine the horizontal thrust at each support.

Assume uniform flexural rigidity.

Solⁿ

$$\begin{aligned} V_a &= V_b = \frac{wL}{2} \\ &= \frac{w \times 2R}{2} \\ &= wR \end{aligned}$$



Each vertical reaction = $w \cdot R$

The beam moment at any section,

$$\begin{aligned} M &= wR \cdot R - R \cos \theta - \frac{wR - R \cos \theta \cdot R - R \cos \theta}{2} \\ &= wR \cdot R (1 - \cos \theta) - w \cdot R (1 - \cos \theta) \cdot \frac{R(1 - \cos \theta)}{2} \\ &= \frac{wR^2 (1 - \cos \theta)}{2} - \frac{wR^2 (1 - \cos \theta)^2}{2} \\ &= \frac{2wR^2 \cdot 2 \cos \theta - wR^2}{2} \\ &= \frac{wR^2 - wR^2 \cos \theta - wR^2 (1 - 2 \cos \theta + \cos^2 \theta)}{2} \\ &= \frac{2wR^2 - 2wR^2 \cos \theta - wR^2 + 2wR^2 \cos \theta - wR^2 \cos^2 \theta}{2} \end{aligned}$$

$$WR^2 - WR^2 \cos^2 \theta$$

$$WR^2 (1 - \cos^2 \theta)$$

$$= \frac{WR^2}{2} \sin^2 \theta$$

$$H = \frac{\int My dx}{\int y^2 dx}$$

$$= \int_0^{\pi/2} \frac{WR^2}{2} \sin^2 \theta \cdot R \sin \theta d\theta$$

$$= \int_0^{\pi/2} R^2 \sin^2 \theta d\theta$$

$$= \frac{WR}{2} \int_0^{\pi/2} \sin^2 \theta d\theta$$

$$= \frac{WR}{2} \int_0^{\pi/2} \frac{1}{2} (1 - \cos 2\theta) d\theta$$

$$= \frac{WR}{4} \int_0^{\pi/2} (1 - \cos 2\theta) d\theta$$

$$= \frac{WR}{4} \times \frac{1}{2} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi/2}$$

$$= \frac{1}{8} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi/2}$$

$\frac{dW}{d\theta} = dV = dU$
 $\frac{d}{d\theta} (WR^2 \sin^2 \theta) = \frac{d}{d\theta} (WR^2 \cos^2 \theta)$
 $2WR^2 \sin \theta \cos \theta = -2WR^2 \cos \theta \sin \theta$
 $2WR^2 \sin \theta \cos \theta = -2WR^2 \sin \theta \cos \theta$
 $4WR^2 \sin \theta \cos \theta = 0$
 $2WR^2 \sin 2\theta = 0$
 $\sin 2\theta = 0$
 $2\theta = 0, \pi$
 $\theta = 0, \frac{\pi}{2}$

$$= \frac{\int_0^{\pi/2} \frac{WR}{2} \sin^2 \theta \cdot \sin \theta \, d\theta}{\int_0^{\pi/2} \sin^2 \theta \, d\theta}$$

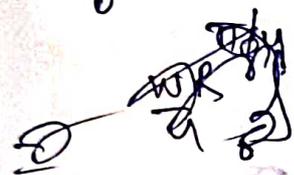
~~Let $\sin^2 \theta = u$~~

~~$\sin \theta \, d\theta = \frac{1}{2} du$~~

$$= \frac{\int_0^{\pi/2} \frac{WR}{2} \cdot \frac{1}{2} (1 - \cos 2\theta) \sin \theta \, d\theta}{\int_0^{\pi/2} \sin^2 \theta \, d\theta}$$

$$= \frac{\int_0^{\pi/2} \frac{WR}{4} (1 - \cos 2\theta) \sin \theta \, d\theta}{\int_0^{\pi/2} \sin^2 \theta \, d\theta}$$

$$= \frac{\int_0^{\pi/2} \frac{WR}{4} \sin^2 \theta \, d\theta}{\int_0^{\pi/2} \sin^2 \theta \, d\theta}$$



Let $1 - \cos 2\theta = \frac{2 \times WR \times \sin^2 \theta}{4}$

$\sin \theta \, d\theta = du$

$$\int_0^{\pi/2} \frac{1}{2} u \, du = \frac{1}{2} \left[\frac{u^2}{2} \right]_0^{\pi/2}$$

$$= \frac{WR}{2} \times \frac{1}{2} \left[\frac{\cos(1 - \cos 2\theta)^2}{2} \right]_0^{\pi/2}$$

$$= \frac{WR}{4} \left[\frac{(1+1)^2}{2} - \frac{(1-0)^2}{2} \right]$$

$$= \frac{WR}{4} \left[\frac{4}{2} - \frac{1}{2} \right]$$

$$= \frac{WR}{4} \times \frac{3}{2} = \frac{3WR}{8}$$

$\frac{3WR}{8}$

$$\int_0^{\pi/2} \sin \theta d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2} (1 - \cos 2\theta) d\theta$$

$$= \frac{1}{2} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi/2}$$

$$= \frac{1}{2} \left[\left(\frac{\pi}{2} - \frac{1}{2} \sin 2 \times \frac{\pi}{2} \right) - \left(0 - \frac{1}{2} \times \sin 2 \times 0 \right) \right]$$

$$= \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$$

$$= \frac{3WR}{8} \times \frac{4}{\pi} = \frac{3WR}{2\pi}$$

=

$$\frac{3WR}{2\pi}$$

$$\left[\frac{3}{8} \right] \frac{1}{2} = \frac{3WR}{4}$$

$$\int_0^{\pi/2} \left[\frac{3}{8} (\cos 2\theta - 1) \cos \theta \right] \frac{1}{2} \times \frac{4WR}{8} = \frac{3WR}{8} \times \frac{4}{\pi}$$

$$\left[\frac{3}{8} \times \frac{1}{2} \times \frac{4WR}{8} \right] \frac{1}{2} = \frac{3WR}{2\pi}$$

Ans: $\frac{3WR}{2\pi}$

Q. → A two hinged parabolic arch of span 25 m & rise 5 m carries a udl of 40 kN/m over the half of the span & a concentrated load of 100 kN at the crown. Find the horizontal thrust at each support.

Solⁿ

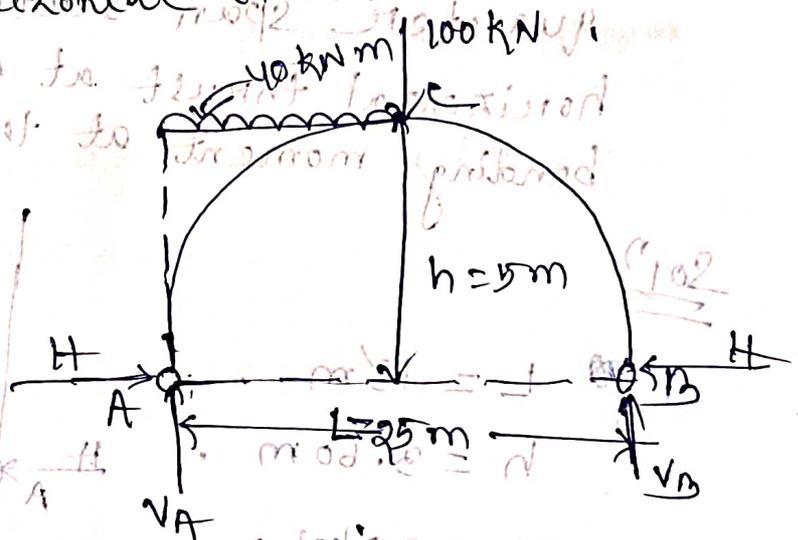
$$L = 25 \text{ m}$$

$$h = 5 \text{ m}$$

$$W_1 = 40 \text{ kN/m}$$

$$W_2 = 100 \text{ kN}$$

(Parabolic arch)



Horizontal thrust due to udl on the left half of the span,

$$H_1 = \frac{wl^2}{16h} = \frac{40 \times 25^2}{16 \times 5} = 312.5 \text{ kN}$$

Horizontal load due to point load on the crown.

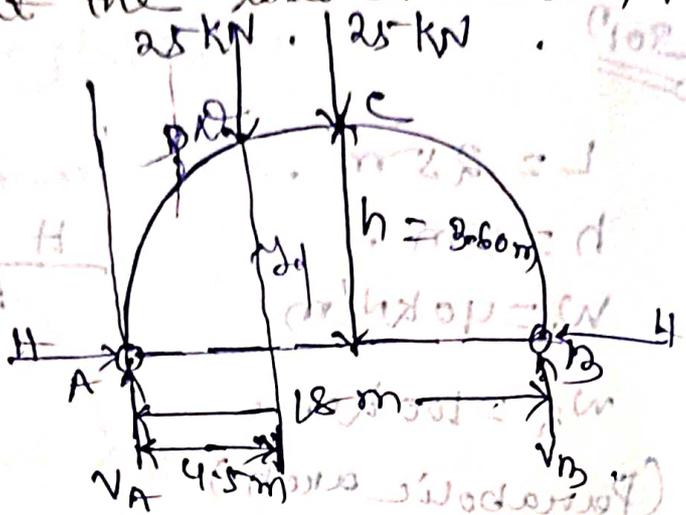
$$H_2 = \frac{25^2}{128} \frac{wl}{h} = \frac{25^2}{128} \times \frac{100 \times 25}{5} = 97.66 \text{ kN}$$

$$\begin{aligned} \text{Total horizontal thrust, } H &= H_1 + H_2 \\ &= 312.5 + 97.66 \\ &= 410.16 \text{ kN} \end{aligned}$$

Q1 A two hinged parabolic arch of span 18m & rise 3.60m carries two concentrated load 25 kN each at the crown & left quarter span section. Find the horizontal thrust at each support & the bending moment at the loaded section.

Solⁿ

- $L = 18\text{ m}$
- $h = 3.60\text{ m}$
- $W_1 = 25\text{ kN}$
- $W_2 = 25\text{ kN}$



Horizontal thrust due to load at P,

$$H_1 = \frac{5W}{8h} a(l-a) (l^2 + la - a^2)$$

$$= \frac{5}{8} \times \frac{25}{3.60 \times 18} \times 4.5 (18 - 4.5) (18^2 + 18 \times 4.5 - 4.5^2)$$

$$= 17.395\text{ kN}$$

Horizontal thrust due to load at the crown

$$H_2 = \frac{25}{128} \frac{Wl}{h} = \frac{25}{128} \times \frac{25 \times 18}{3.60}$$

$$= 24.414\text{ kN}$$

Total horizontal thrust, $H = H_1 + H_2$

$$= 17.395 + 24.414$$

$$= 41.809 \text{ kN}$$

Date ~~06/08/2019~~

Taking moment about point 'A' left support 'A'

$$\sum M_A = 0$$

$$\Rightarrow V_B \times 18 = (25 \times 9) + (25 \times 4.5)$$

$$\Rightarrow V_B = 18.75 \text{ kN}$$

$$V_A + V_B = 25 + 25$$

$$\Rightarrow V_A = 50 - 18.75 = 31.25 \text{ kN}$$

ordinate of the arch at 'D'

$$y_d = \frac{4h}{l^2} x(l-x)$$

$$= \frac{4 \times 3.6}{18^2} \times 4.50 (18 - 4.50)$$

$$= 2.70 \text{ kN}$$

$$\text{B.M. at 'D'} = 31.25 \times 4.5 - 41.809 \times 2.70$$

$$= 27.741 \text{ kNm}$$

$$\text{B.M. at 'c'} = 18.75 \times 9 - 41.809 \times 3.60$$

$$= 18.24 \text{ kNm}$$


 A cable of span 150 m & dip 15 m carries a load 6 kN/m run of horizontal span. Find the maximum tension for the cable & the inclination of the cable at the support. Find the forces transmitted to the supporting pier.

(i) if the cable is passed over smooth rollers on the top of the pier.

(ii) if the cable is clamped to a saddle with smooth rollers resting on the top of the pier.


 For each of the above cases the anchor cable is 30° to the horizontal at the supporting pier is 20 m high. Find the maximum B.M. for the pier.

Solⁿ

Given data

$$w = 6\text{ kN/m}$$

$$L = 150\text{ m}$$

$$h = 15\text{ m}$$

Vertical reaction at each end of the cable

$$V_a = \frac{wL}{2} = \frac{6 \times 150}{2} = 450\text{ kN}$$

Vertical reaction at each end of the suspension cable,

$$H = \frac{wl^2}{8h} = \frac{6 \times 150^2}{8 \times 15} = 1125 \text{ kN}$$

Maximum Tension in the suspension cable

$$\begin{aligned} T &= \sqrt{V_a^2 + H^2} \\ &= \sqrt{(450)^2 + (1125)^2} \\ &= 1211.7 \text{ kN} \end{aligned}$$

Inclination β of the suspension cable at the support with the horizontal,

$$\begin{aligned} \tan \beta &= \frac{V_a}{H} \\ \Rightarrow \beta &= \tan^{-1} \frac{V_a}{H} = \tan^{-1} \frac{450}{1125} \\ &= 21'48'' \end{aligned}$$

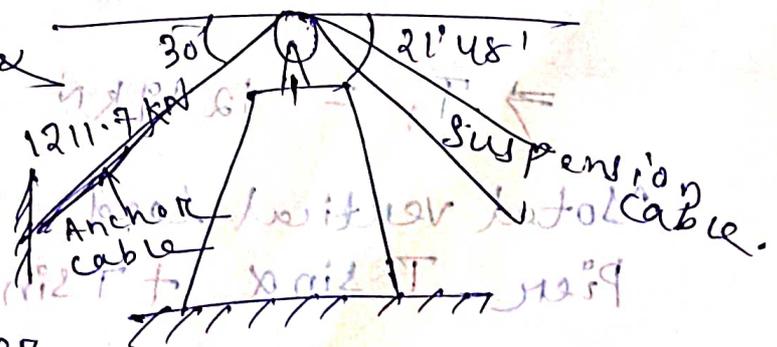
(i) When the cable is passed over smooth roller on the top of the pier

For this case tension in the anchor cable & suspension cable are

same

Total vertical load transmitted to the pier

$$\begin{aligned} &= T \sin 30^\circ + T \sin 21'48'' \\ &= 1055.85 \text{ kN} \end{aligned}$$



Net horizontal load transmitted to the pier

$$T \cos 21.48^\circ - T \cos 30^\circ$$

$$= 1211.7 (\cos 21.48^\circ - \cos 30^\circ) = H$$

$$= 75.7 \text{ kN}$$

Maximum B.M. for the pier,

$$= 75.7 \times 20 = 1514 \text{ kNm}$$

Date - 08/08/2019

(ii) When the cable is clamped to a saddle with roller resting on the pier

In this case let T_1 be the tension in the anchor cable. Resolving the forces on the saddle horizontally.



$$T_1 \cos 30^\circ = T \cos 21.48^\circ$$

$$\Rightarrow T_1 = \frac{1211.7 \cos 21.48^\circ}{\cos 30^\circ}$$

$$\Rightarrow T_1 = 1299 \text{ kN}$$

Total vertical load transmitted to the pier $T_1 \sin \alpha + T \sin \beta$

$$= 1299 \sin 30^\circ + 1211.7 \sin 21.48^\circ$$

$$= 1099.5 \text{ kN}$$

For this case horizontal components T_1 & T balance.

Hence there will be no bending moment on the pier.

Q. Long.
 A two hinged parabolic arch of span 40m & rise 5m carrying a udl of 5 kN/m on the left half of the span & also a concentrated load 40 kN at the crown. Determine the horizontal thrust at the support & the maximum B.M. for the arch. Assuming secant variation of moment of inertia of the arch section.

Solⁿ

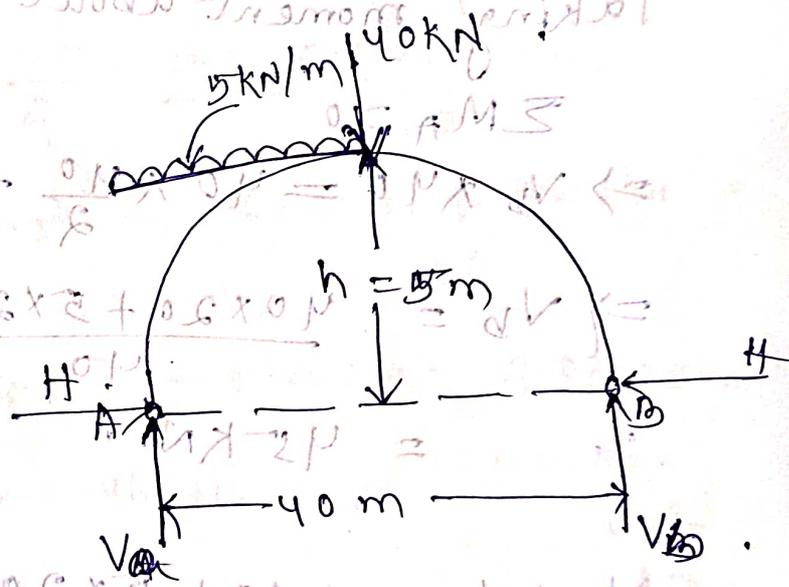
Given data

$L = 40\text{m}$

$h = 5\text{m}$

$w_1 = 5\text{ kN/m}$

$w_2 = 40\text{ kN}$



Horizontal thrust due to udl on the left half of the span,

$$H_1 = \frac{1}{2} \times \frac{wL^2}{8h} = \frac{5 \times 40^2}{16 \times 5} = 100\text{ kN}$$

Horizontal thrust due to point load on the crown,

$$H_2 = \frac{25}{128} \times \frac{wl}{h} = \frac{25}{128} \times \frac{40 \times 40}{5} = 62.5\text{ kN}$$

Total horizontal thrust = $H_1 + H_2$

$$= 100 + 62.5$$

$$= 162.5 \text{ kN}$$

Date - 09/08/2019

Let V_a & V_b are the vertical reaction at the left & right support.

Let H be the horizontal thrust at each support.

Taking moment about left support 'A'.

$$\sum M_A = 0$$

$$\Rightarrow V_b \times 40 = 40 \times \frac{40}{2} + 5 \times \frac{40^2}{2} \times \frac{40}{4}$$

$$\Rightarrow V_b = \frac{40 \times 20 + 5 \times 20 \times 40}{40}$$

$$= 45 \text{ kN}$$

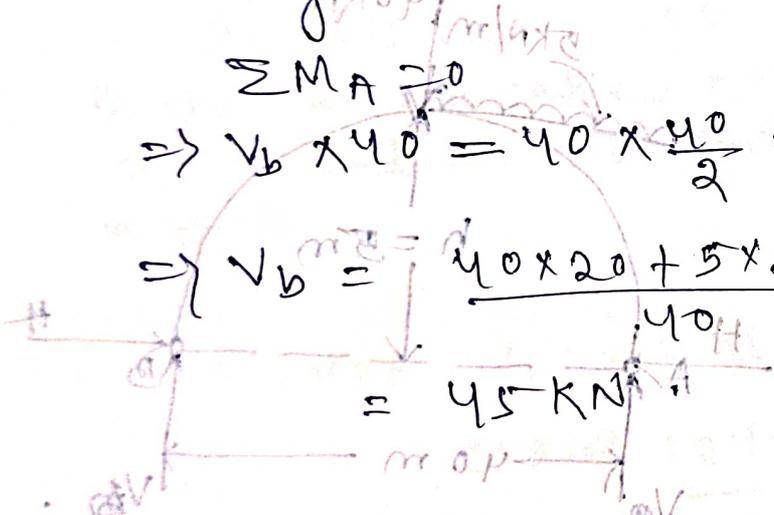
$$V_a + V_b = 40 + 5 \times 20$$

$$\Rightarrow V_a = 140 - 45$$

$$= 95 \text{ kN}$$

$$H^2 = \frac{158}{18} \times \frac{32}{18} = \frac{32}{18} \times \frac{32}{18} \times \frac{18}{18}$$

$$= 68.2 \text{ kN}$$



Maximum negative bending moment,

$$-M_{\max} = -36.25x + 2.03125x^2$$

$$= -36.25 \times 8.92 + 2.03125 \times (8.92)^2$$

$$= -161.73 \text{ kNm}$$

Q:- A two hinged parabolic arch of span 40m & rise 8m carries a point load of 80 kN at a distance of 10m from the left support. Find the horizontal thrust at each support. Find also the maximum bending moment.

Solⁿ

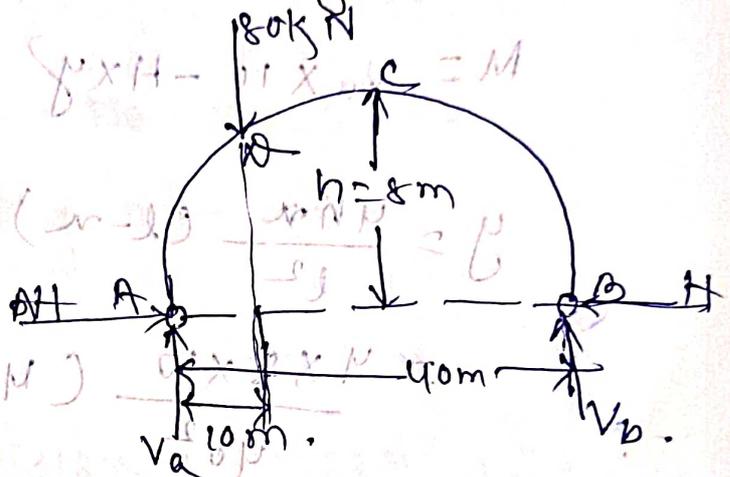
Given data,

$$L = 40 \text{ m}$$

$$h = 8 \text{ m}$$

$$W = 80 \text{ kN}$$

$$a = 10 \text{ m}$$



Let V_a & V_b are the vertical reaction at the left & right support.

Let H be the horizontal thrust at each support.

Taking moment about A = 0

$$\sum M_A = 0$$

$$\Rightarrow V_b \times 40 - 80 \times 10 = 0$$

$$\Rightarrow V_b = \frac{800}{40}$$

$$= 20 \text{ kN}$$

$$V_a + V_b = 80$$

$$\Rightarrow V_a = 80 - 20 = 60 \text{ kN}$$

Horizontal thrust, $H = \frac{5}{8} \frac{w}{h \times l^3} a(l-a) (l^2 + la - a^2)$

$\Rightarrow H = \frac{5}{8} \times \frac{80}{8 \times 40^3} \times 10 (40-10) (40^2 + 40 \times 10 - 10^2)$

$\Rightarrow H = 55.664 \text{ kN}$

Maximum positive b.m. will occur ^{under} the load 80 kN at $x = 10 \text{ m}$

$M = V_a x 10 - H x y$

$y = \frac{4h^2 x}{l^2} (l-x)$

$= \frac{4 \times 8 \times 10}{40^2} (40-10)$

$M_{\text{max}} = 80 \times 10 - 55.664 \times 6 = 266.016 \text{ kN}$

Maximum & negative bending moment will occur at a section of the right half of the span

b.m. at any section bc distance x from B.

$M = 20x - Hy$

$\frac{dM}{dx} = 20 - 80 = 0$
 $\Rightarrow 80 = 20 + 80 = 100 \text{ kN}$

$$y = \frac{4bn}{l^2} (l-n) \quad (-135.1356 \text{ KNm})$$

$$= \frac{4 \times 8n}{40^2} (40-n)$$

$$= 0.8n - 0.02n^2$$

$$M = 20n - 55.654(0.8n - 0.02n^2)$$

$$= 20n - 44.5312n + 1.11328n^2$$

$$= -24.5312n + 1.11328n^2$$

$$= -24.5312n + 1.11328n^2$$

Maximum bending moment

$$\frac{dM}{dn} = 0$$

$$\Rightarrow \frac{d}{dn} (-24.5312n + 1.11328n^2) = 0$$

$$\Rightarrow -24.5312 + 2.22656n = 0$$

$$\Rightarrow n = \frac{24.5312}{2.22656}$$

$$= 11.0175 \text{ m}$$

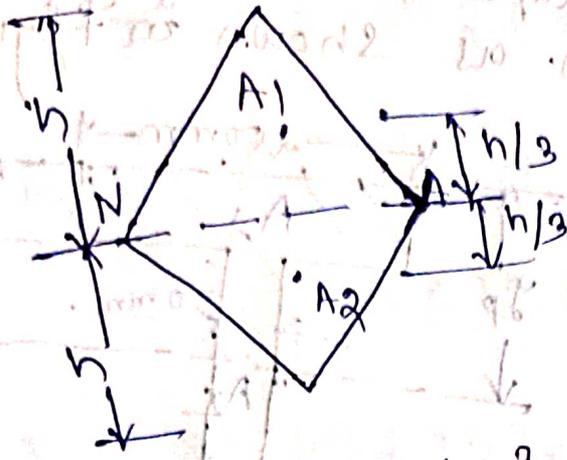
Maximum negative bending moment,

$$+ M_{\text{max}} = -24.5312n + 1.11328n^2$$

$$= -24.5312 \times 11.0175 + 1.11328 \times (11.0175)^2$$

$$= -135.14 \text{ KNm}$$

Shape Factor for the Diamond Section:



$$I = 2 \times \frac{bh^3}{12} = \frac{bh^3}{6}$$

$$M_y = f_y \frac{I}{y_{max}} = f_y \times \frac{\frac{1}{6} \times bh^3}{h}$$

$$M_y = f_y \frac{bh^2}{6}$$

$$A_1 = A_2 = \frac{1}{2} \times b \times h$$

$$y_1 = \frac{1}{3}h, \quad y_2 = \frac{1}{3}h$$

$$M_p = f_y [A_1 y_1 + A_2 y_2]$$

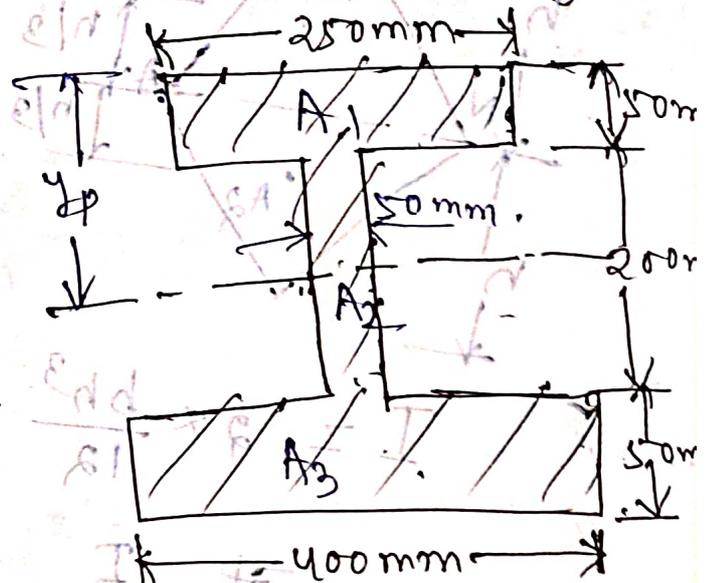
$$= f_y \left[\frac{1}{2} \times b \times h \times \frac{h}{3} \right] \times 2$$

$$= f_y \frac{bh^2}{3}$$

$$S = \frac{f_y \frac{bh^2}{3}}{f_y \frac{bh^2}{6}}$$

$$S = 2$$

Q: Determine the shape factor of unequal I-section as shown in figure



Solⁿ

Elastic neutral axis from top fibre

$$y_{it} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(250 \times 50 \times \frac{50}{2}) + \{200 \times 50 \times (\frac{50}{2} + \frac{200}{2})\} + \{400 \times 50 \times (\frac{50}{2} + 200 + \frac{50}{2})\}}{250 \times 50 + 200 \times 50 + 400 \times 50}$$

$$= 172.06 \text{ mm}$$

$$I_x = \frac{250 \times 50^3}{12} + (250 \times 50) \cdot (172.06 - 25)^2$$

$$= 272937211.7 \text{ mm}^4$$

$$I_x = I_g + ab^2$$

$$I_2 = \frac{50 \times 200^3}{12} + (200 \times 50) (172.06 - 150)^2$$

$$= 38199769.33 \text{ mm}^4$$

$$I_3 = \frac{400 \times 50^3}{12} + (400 \times 50) (275 - 172.06)^2$$

$$= 216099538.7 \text{ mm}^4$$

$$I = 272937211.7 + 38199769.33 + 216099538.7$$

$$= 527236519.7 \text{ mm}^4$$

$$y_{\text{max}} = 172.06 \text{ mm}$$

$$M_y = f_y \times \frac{I}{y_{\text{max}}}$$

$$= f_y \times \frac{527236519.7}{172.06}$$

$$= 3064259.67 f_y \text{ Nmm}$$

Plastic moment capacity (M_p) (MP)

Let, plastic neutral axis p at distance y_p

calculate y_p ,

$$(250 \times 50) + 50 (y_p - 50) = A/2$$

$$\Rightarrow 12500 + 50 y_p - 2500 = \frac{(250 \times 50) + (250 \times 50) + (400 \times 50)}{2}$$

$$\Rightarrow y_p = 225 \text{ mm}$$

Dividing the total area into four rectangles two is compression & two is tension zone.

$$M_p = f_y \sum A y$$

$$= f_y \left[(250 \times 50) (225 - 25) + \right.$$

$$\left. 50 (225 - 50) \left(\frac{225 - 50}{2} \right) + \right.$$

$$\left. + 50 (250 - 225) \left(\frac{250 - 225}{2} \right) + \right.$$

$$\left. 50 \times 400 (275 - 225) \right]$$

$$= f_y (2500000 + 765625 + 15625 + 1000000)$$

$$= 1841250 f_y \text{ Nmm}$$

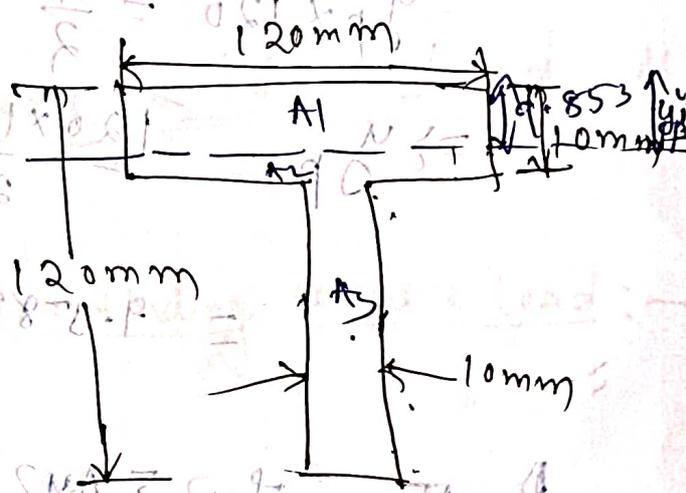
$$= 4281250 f_y \text{ Nmm}$$

Shape factor =

$$S = \frac{M_p}{M_y} = \frac{4281250 \text{ Nmm}}{3064259.67 \text{ Nmm}}$$

$$= 1.397$$

Q:- Determine the shape factor for T-section



soln

$$\frac{M_{y,p}}{M_y} = \frac{F_y \times I}{I_{\text{main}}}$$

$$\bar{y} = \frac{(120 \times 10) \times \frac{10}{2} + (120 \times 10) \times (10 + \frac{110}{2})}{(120 \times 10) + (110 \times 10)}$$

$$= \frac{600 + (110 \times 10) \times 65}{1200 + 1100}$$

$$= \frac{600 + 7150}{2300} = 33.70 \text{ mm}$$

$$I = \left\{ \frac{120 \times 10^3}{12} + (120 \times 10) (33.70 - 5)^2 \right\} + \left\{ \frac{10 \times 110^3}{12} + (10 \times 110) (65 - 33.70)^2 \right\}$$

$$= 3185253.67 \text{ mm}^4$$

$$y_{max} = 120 - 33.70 = 86.3 \text{ mm}$$

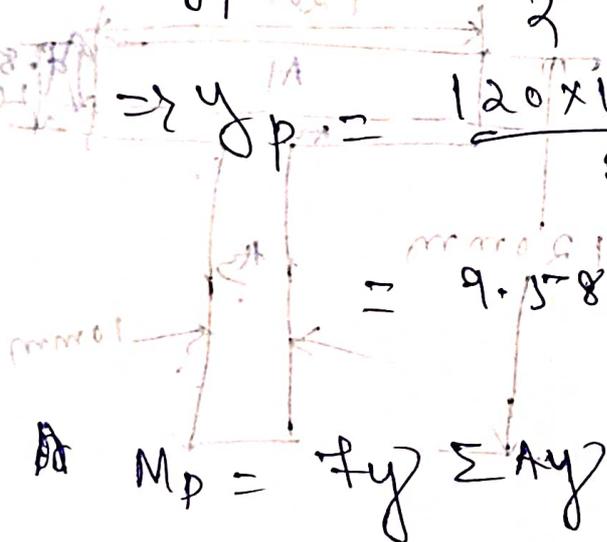
$$M_y = f_y \times \frac{I}{y_{max}} = 36909.08 f_y \text{ Nmm}$$

Assuming plastic N.A. lies in the flange its distance from top fibre y_p

$$y_p \times 120 = \frac{A}{2} (120 \times 10 + 110 \times 10)$$

$$\Rightarrow y_p = \frac{120 \times 10 + 110 \times 10}{2 \times 120}$$

$$= 9.583 \text{ mm}$$



$$M_p = f_y \sum Ay$$

$$= f_y \left[120 \times 9.583 \times \frac{9.583}{2} \right.$$

$$+ 120 \times (10 - 9.583) \times \frac{10 - 9.583}{2}$$

$$\left. + 110 \times 10 \times \left((10 - 9.583) + \frac{110}{2} \right) \right]$$

$$= f_y [5570.4 + 10.433 + 60958.7]$$

$$= f_y [66479.133 \text{ Nmm}] = I$$

$$\left\{ \int (0.7 - 0.33 - 0.33) (0.1 \times 0.1) + \frac{(0.1 \times 0.1)}{12} \right\}$$

$$= M_{max} = 218222.3 \text{ Nmm}$$

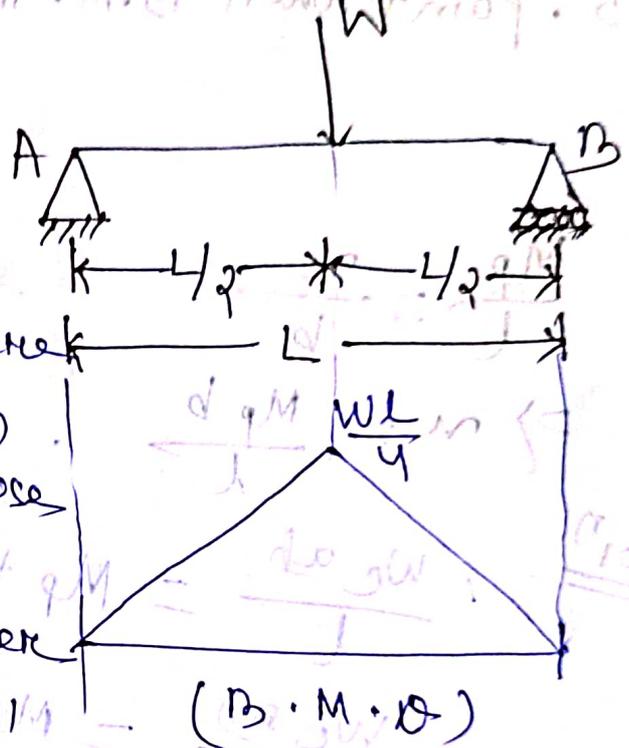
$$S = \frac{M_p}{M_y} = \frac{66479.133}{36909.08} = 1.801$$

Ex: Determine the collapse load (W_c) in the S/S beam.

Sol:

Note: - Since S/S beam

is determinate structure formation of one hinge in the beam creates collapse mechanism. Since the moment is maximum under the load. The hinge will form at that place.



By using lower bound theorem

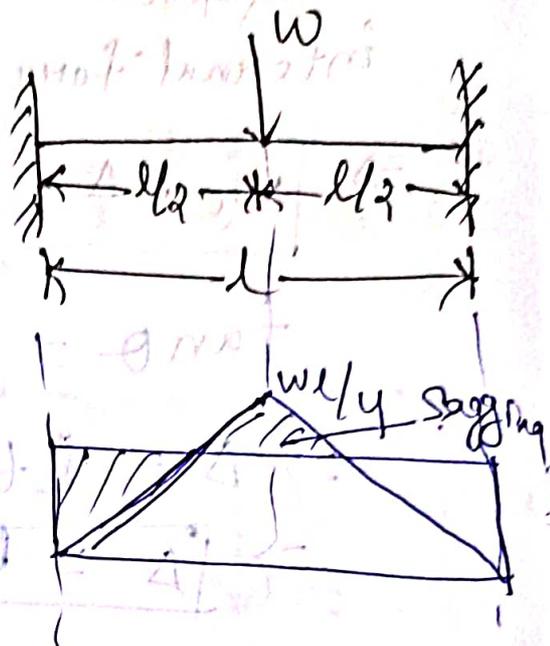
$$\frac{Wl}{4} \geq M_p$$

$$\Rightarrow \frac{W_c \cdot l}{4} = M_p$$

$$\Rightarrow W_c = \frac{4M_p}{l}$$

Q: Determine the collapse load in case of fixed beam as shown in figure.

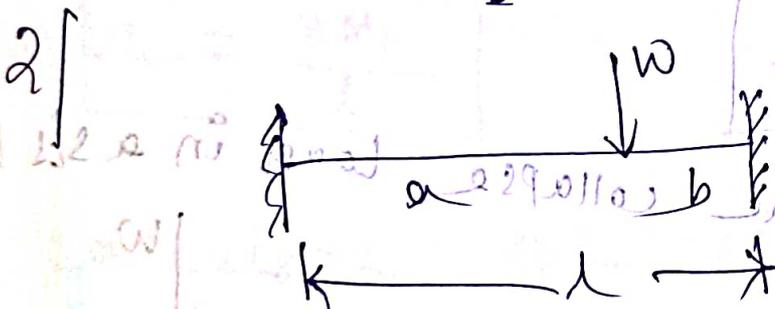
Solⁿ using ~~lower~~ bound theorem



LBT

$$\frac{Wcl}{4} \geq M_p + M_p$$

$$\Rightarrow Wc \geq \frac{8M_p}{l}$$



$$\frac{Wcab}{l} = M_p + M_p$$

$$\Rightarrow Wc = \frac{2Mpl}{ab}$$

Q:- Determine the collapse load in a SLS beam.

Solⁿ
 External work done = $w_c \cdot \Delta$

$$\theta_1 = \theta_2 = \theta$$

$$\Delta = \frac{1}{2} \theta_1 = \frac{1}{2} \theta_2 = \frac{l}{2} \theta$$

Internal work done

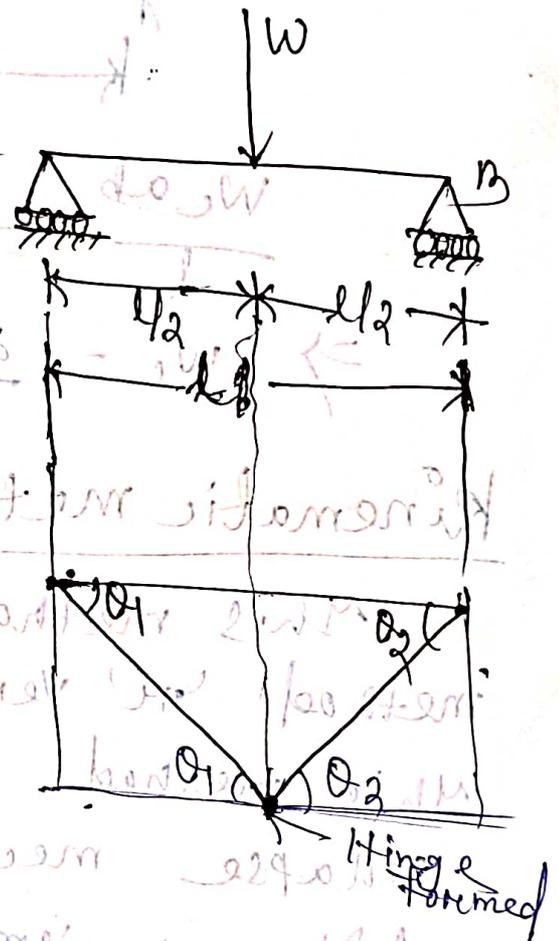
$$= M_p \theta_1 + M_p \theta_2$$

$$= 2M_p \theta$$

$$E \cdot W = I \cdot W$$

$$\Rightarrow w_c \cdot \frac{l}{2} \theta = 2M_p \theta$$

$$\boxed{w_c = \frac{4M_p}{l}}$$



Q:- calculate the collapse load w_c in fixed beam by using upper bound theorem (UBT)

Soln

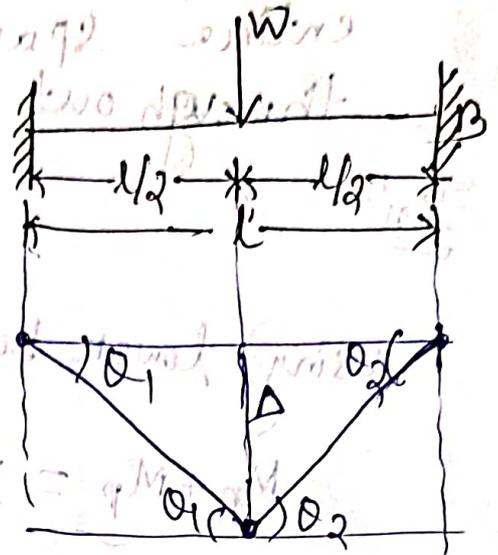
$$\begin{aligned} \text{External work done} &= w_c \cdot \Delta \\ &= w_c \cdot \frac{l}{2} \theta \end{aligned}$$

$$\begin{aligned} \text{Internal work done} &= M_p \theta_1 + M_p \theta_2 + M_p \theta_1 + M_p \theta_2 \\ &= 4 M_p \theta \end{aligned}$$

$$\text{External work done} = \text{Internal work done}$$

$$w_c \frac{l}{2} \theta = 4 M_p \theta$$

$$\Rightarrow \boxed{w_c = \frac{8 M_p}{l}}$$



$$\theta_1 = \theta_2 = \theta$$

$$\Delta = \frac{l}{2} \theta$$

Q:- calculate the collapse load for propped cantilever beam.

Soln

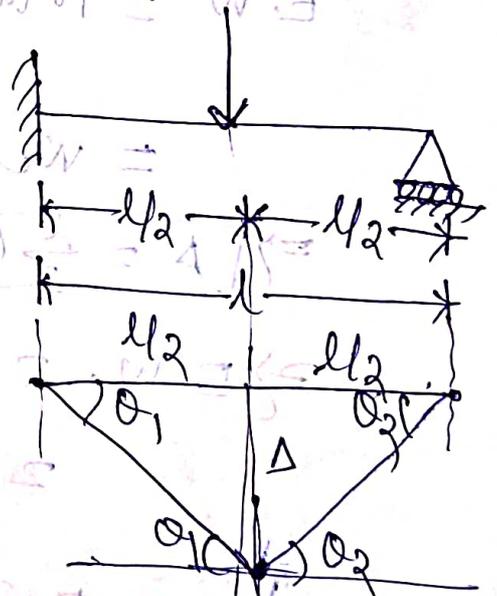
$$\begin{aligned} E \cdot w &= w_c \Delta \\ &= w_c \frac{l}{2} \theta \end{aligned}$$

$$\begin{aligned} I \cdot w &= M_p \theta_1 + M_p \theta_1 + M_p \theta_2 \\ &= 3 M_p \theta \end{aligned}$$

$$E \cdot w = I \cdot w$$

$$\Rightarrow w_c \frac{l}{2} \theta = 3 M_p \theta$$

$$\Rightarrow \boxed{w_c = \frac{6 M_p}{l}}$$



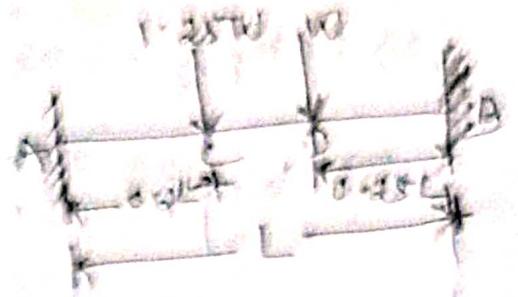
$$\theta_1 = \theta_2 = \theta$$

$$\Delta = \frac{l}{2} \theta$$

Q. Determine the collapse load in a fixed beam shown in the figure.

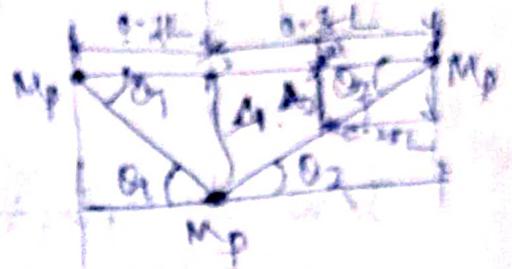
Note -
Real mechanism -

The real mechanism is the one in which internal work done is less & external work done is more. So that least collapse maximum plastic moment capacity is required.



Mechanism - 1

Interior hinge under the load $1.25w$.



$$\Delta_1 = 0.2\theta_1 = 0.8\theta_2$$

$$\Rightarrow \theta_1 = \frac{0.8}{0.2} \theta_2$$

$$\Rightarrow \boxed{\theta_1 = 4\theta_2}$$

$$\frac{0.8L}{\Delta_1} = \frac{0.25L}{\Delta_2}$$

$$\Rightarrow 0.8L\Delta_2 = 0.25L\Delta_1$$

$$\Rightarrow \Delta_2 = \frac{0.25}{0.8} \Delta_1$$

$$\checkmark \text{ I. W. } = M_p\theta_1 + M_p\theta_2 + M_p\theta_3 + M_p\theta_2$$

$$= M_p4\theta_2 + M_p\theta_2$$

$$+ M_p4\theta_2 + M_p\theta_2$$

$$= 10M_p\theta_2$$

$$\text{E. W. } = 1.25w_e A_1 + w_e A_2$$

$$= 1.25w_e A_1 + w_e \frac{0.25}{0.8} A_1$$

$$\geq w_e \left[1.25 + \frac{0.25}{0.8} \right] 0.8L\theta_2$$

$$= w_e 1.25L\theta_2$$

$$\left. \begin{aligned} &1.25w_e \times 0.8L\theta_2 \\ &+ w_e \times 0.25L\theta_2 \\ &= w_e L\theta_2 + 0.25w_e L\theta_2 \\ &= 1.25w_e L\theta_2 \end{aligned} \right\}$$

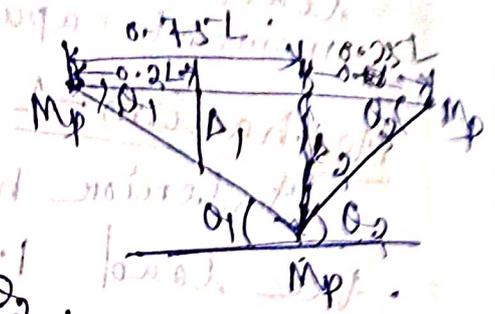
$$I \cdot w = E \cdot w$$

$$\Rightarrow 10 M_p \theta_2 = w_c l \cdot 25 L \theta_2$$

$$\Rightarrow \boxed{w_c = \frac{8 M_p}{L}}$$

Mechanism - 3

Internal hinged under load w .



$$I \cdot w = M_p \theta_1 + M_p (\theta_1 + \theta_2) + M_p \theta_3$$

$$= M_p \theta_1 + M_p 4 \theta_1 + 3 M_p \theta_2$$

$$= 8 M_p \theta_1$$

$$E \cdot w = 1.25 w_c A_1 + w_c A_2$$

$$= 1.25 w_c \times \frac{0.2}{0.75} A_2 + w_c A_2$$

$$= w_c L \theta_1$$

$$I \cdot w = E \cdot w$$

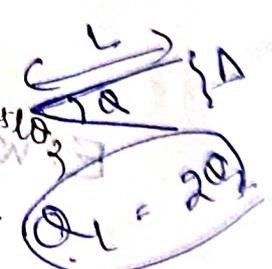
$$\Rightarrow 8 M_p \theta_1 = w_c L \theta_1$$

$$\Rightarrow \boxed{w_c = \frac{8 M_p}{L}}$$

$$\Rightarrow A_1 = \frac{0.2L}{0.75L} A_2$$

$$\Rightarrow \theta_2 = \frac{0.75}{0.25} \theta_1 = 3 \theta_1$$

$$4/3 \theta_1 = 2/3 L \theta_2 = \Delta$$



$$\Rightarrow \theta_2 = \frac{0.75}{0.25} \theta_1$$

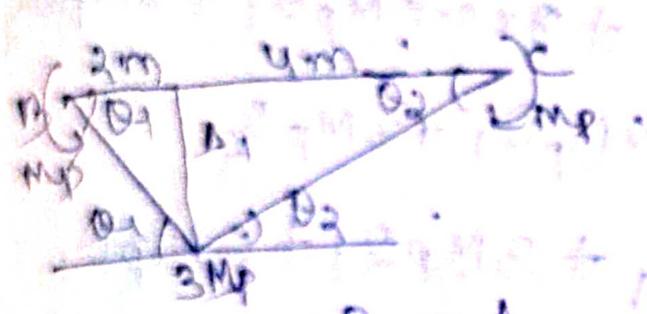
$$\theta_1 = 2 \theta_2$$

$$w_c L \theta_1 = 1.25 w_c \left[\frac{0.2}{0.75} + 1 \right] A_2 \theta_2$$

$$w_c L \theta_1 = 1.25 w_c L \theta_2$$

g) Determine the collapse load w_c for the frame shown in figure.

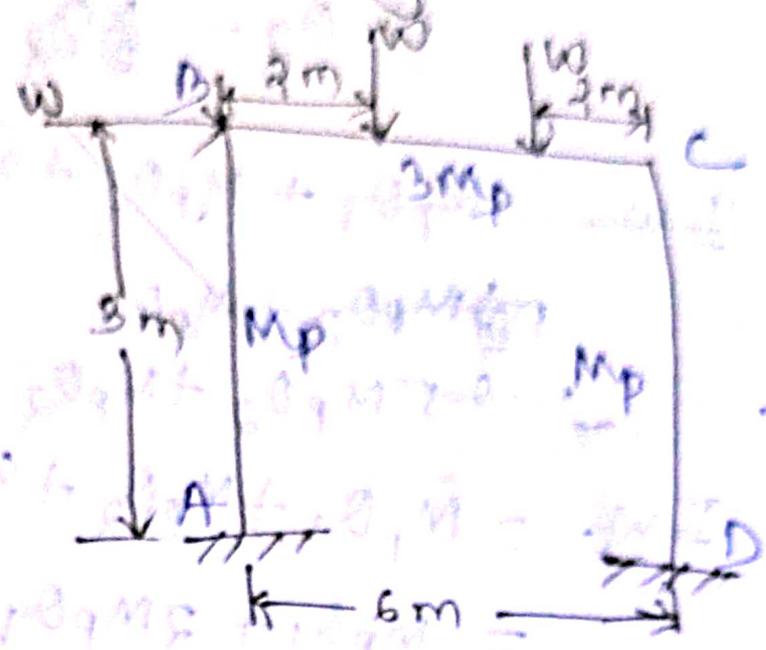
2017
Beam Mechanism (M)
 in M.C



$$2\theta_1 = 4\theta_2 = \Delta_1$$

$$\Rightarrow \theta_1 = \frac{4}{2}\theta_2$$

$$\boxed{\theta_1 = 2\theta_2}$$

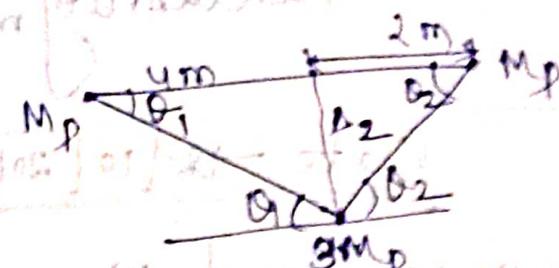


$$\begin{aligned} I.W. &= M_p\theta_1 + M_p\theta_2 + 3M_p\theta_1 + 3M_p\theta_2 \\ &= 2M_p\theta_2 + M_p\theta_2 + 6M_p\theta_2 + 3M_p\theta_2 \\ &= 12M_p\theta_2 \end{aligned}$$

$$\begin{aligned} E.W. &= w_c \Delta_1 + w_c \Delta_2 \\ &= w_c \times 4\theta_2 + w_c \times 2\theta_2 \\ &= 6w_c\theta_2 \end{aligned}$$

$$\begin{aligned} I.W. &= E.W. \\ \Rightarrow 12M_p\theta_2 &= 6w_c\theta_2 \\ \Rightarrow w_c &= 2M_p \end{aligned}$$

beam mechanism (1) in BC



~~4Mp~~ $4\theta_1 = 2\theta_2 = \Delta$
 $\Rightarrow \theta_2 = \frac{2\theta_1}{0.5}$

~~I.W. = Mp\theta_1 + Mp\theta_2 + 3Mp\theta_1 + 3Mp\theta_2~~

~~$\Rightarrow 0.5Mp\theta_2 + Mp\theta_2 + 3 \times 0.5Mp\theta_1 + 3Mp\theta_2$~~

I.W. = $Mp\theta_1 + Mp\theta_2 + 3Mp\theta_1 + 3Mp\theta_2$
 $= Mp\theta_1 + 2Mp\theta_1 + 3Mp\theta_1 + 6Mp\theta_1$
 $= 12Mp\theta_1$

E.W. = $w_c A_1 + w_c A_2$
 $= w_c 2\theta_1 + w_c 4\theta_1$

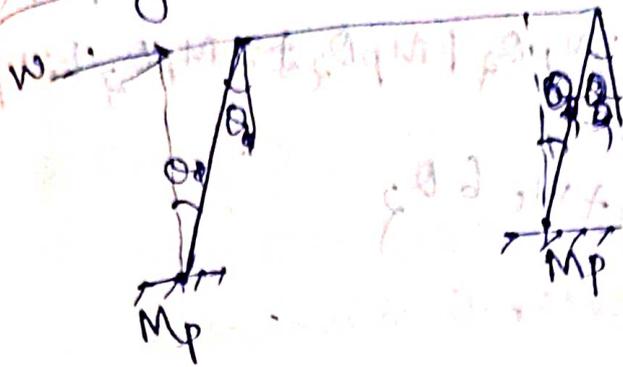
$= 6w_c\theta_1$

I.W. = E.I. \Delta

$\Rightarrow 12Mp\theta_1 = 6w_c\theta_1$

$\Rightarrow w_c = 2Mp$

(iii) sway mechanism



$$I.W. = M_p \theta + M_p \theta + M_p \theta + M_p \theta = 4M_p \theta$$

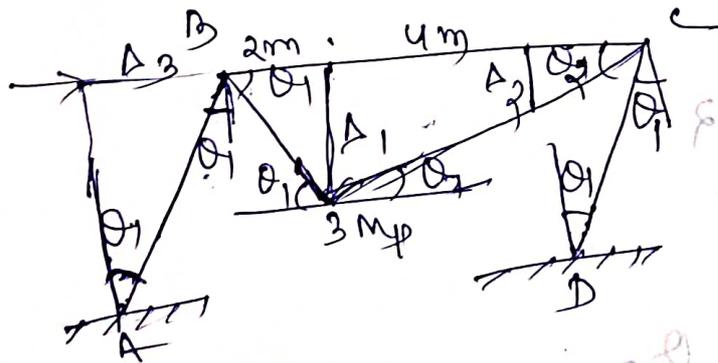
$$E.W. = W_c \Delta = W_c \times 3\theta$$

$$I.W. = E.W.$$

$$\Rightarrow 4M_p \theta = W_c \times 3\theta$$

$$\Rightarrow W_c = \frac{4}{3} M_p = 1.33 M_p$$

(iv) Combined Mechanism (E)



$$2\theta_1 = 4\theta_2 = \Delta$$

$$\Rightarrow \theta_1 = 2\theta_2$$

$$\Delta_2 = 2\theta_1 = \theta_2$$

$$3\theta_1 = 6\theta_2 = \Delta_3$$

$$I.W. = M_p \theta_1 + M_p \theta_2 + M_p \theta_3 + M_p \theta_4 = W_c \Delta$$

$$= 10M_p \theta_1 + 2M_p \theta_1 + 10M_p \theta_1 + 10M_p \theta_1 = W_c \Delta$$

$$= 32M_p \theta_1 = W_c \Delta$$

$$\Rightarrow W_c = \frac{32M_p \theta_1}{\Delta} = \frac{32M_p \theta_1}{3\theta_1} = 10.67 M_p$$

$$\begin{aligned}
 I.W. &= M_p \theta_1 + 3M_p(\theta_1 + \theta_2) + M_p \theta_2 + M_p \theta_1 + M_p \theta_1 \\
 &= 2M_p \theta_2 + 6M_p \theta_2 + 3M_p \theta_2 + M_p \theta_2 + 2M_p \theta_2 + 2M_p \theta_2 \\
 &= 16M_p \theta_2
 \end{aligned}$$

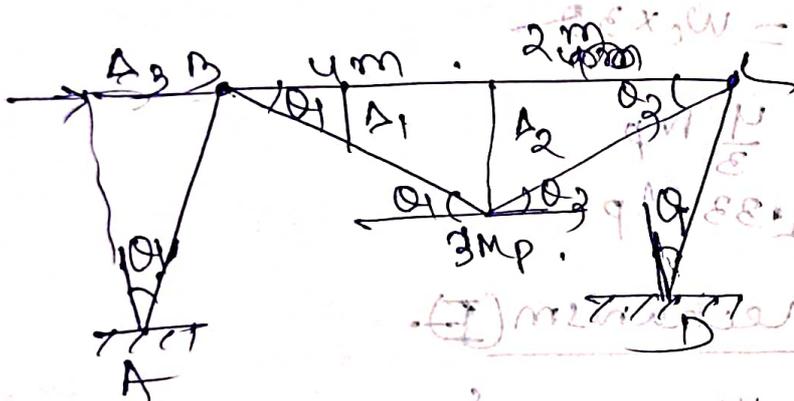
$$\begin{aligned}
 E.W. &= w_c \times 4\theta_2 + w_c \times 2\theta_2 + w_c \times 6\theta_2 \\
 &= 12w_c \theta_2
 \end{aligned}$$

$$I.W. = E.W.$$

$$\Rightarrow 16M_p \theta_2 = 12w_c \theta_2$$

$$\Rightarrow w_c = \frac{16}{12} M_p = 1.33 M_p$$

(V) combined mechanism (II)



$$4\theta_1 = \Delta_2 = 2\theta_2$$

$$\Rightarrow \theta_2 = 2\theta_1$$

$$\Delta_1 = 2\theta_2$$

$$\Delta_3 = 3\theta_1 = 6\theta_2$$

$$\begin{aligned}
 I.W. &= M_p \theta_1 + 3M_p(\theta_1 + \theta_2) + M_p \theta_2 + M_p \theta_1 + M_p \theta_1 \\
 &= 16M_p \theta_1
 \end{aligned}$$

$$\begin{aligned}
 E.W. &= w_c \Delta_1 + w_c \Delta_2 + w_c \Delta_3 \\
 &= w_c 2\theta_1 + w_c 4\theta_1 + w_c 6\theta_1 \\
 &= 12w_c \theta_1
 \end{aligned}$$

$$16 \text{ Mp } \theta_1 = 12 w_c \theta_1$$
$$\Rightarrow w_c = 1.33 \text{ Mp}$$

From Eqⁿ (5) \rightarrow (5) we conclude that $w_c = 1.333 \text{ Mp}$ is a way of combined mechanism developed simultaneously.

Stiffness matrix method

1. The systematic development of slope deflection method in the matrix form has given rise to stiffness matrix method.

2. In this method the basic unknowns are displacement.

3. In this method the equations of equilibrium are formed & solved to get slope & deflection at the joint. By using these moment & shear forces are calculated.

Flexibility matrix method

1. The systematic development of consistent deformation method in a matrix form has led to flexibility matrix method.

2. In this method basic unknowns are redundant forces.

3. In this analysis first identify basic determinate structure & thereby identify redundant forces.

The number of redundant forces are equal to the degree of static indeterminacy.