

CHAPTER-01 MODULE-I
FUNDAMENTALS OF POWER SYSTEM :- (1)

Power

- * The ability of work done is known as power.
- * The rate of consumption of energy is known as power.

Power In DC Ckt:-

→ Power is product of voltage & current.

$$P = VI$$

Power In AC Ckt:-

- The Rate of cage of energy w.r.t time is in terms of voltage & current.
- In an AC Ckt, the voltage & current are continuously changing. Hence power value changes w.r.t time.
- The average value of varying power is

$$\text{Pav} = \frac{1}{T} \int_0^{2\pi} P \cdot dt$$

$$= \frac{1}{2\pi} \int_0^{\infty} P \cdot dt$$

- The average power is also called as active power or true power.

Euler Identity:-

$$E^{j\theta} = e^{j\theta} = \cos \theta + j \sin \theta$$

$$Z = 5e^{j0} = 5 \angle 0^\circ = 5 + j0$$

- Power system :- (2)
- Generation, transmission & distribution system are the main component of an electrical power system.
 - Generating station & distribution system are connected through a transmission line.
 - Electric power system is the combination of components that transfers other types of energy into electrical energy and transmits this energy to the consumer.

POLAR FORM

$$\rightarrow A \angle \theta$$

$$\underline{\text{Ex}} \quad 10 \angle 30^\circ$$

→ Multiplication & Division.

Addition

$$*(2+j3) + (4+j5) \\ = 6+j8$$

$$*(2+j3) + 3 \angle 30^\circ \\ = 2+j3 + 2.59 \angle j1.5^\circ \\ = 4.59 + j4.5$$

Division

$$*\frac{10 \angle 30^\circ}{2 \angle 50^\circ} \\ = 5 \angle 30^\circ - 50^\circ \\ = 5 \angle -20^\circ$$

$$*(6+j8) + 10 \angle 5^\circ \\ = (10 \angle 53.13) \div (10 \angle 5^\circ) \\ = 1 \angle 53.13 - 5^\circ \\ = 1 \angle 48.13^\circ$$

RECTANGULAR FORM

$$\rightarrow z = R + jX$$

$$\underline{\text{Ex}} \quad 2+j5$$

→ Addition & subtraction.

Multiplication

$$2 \angle 30^\circ \times 5 \angle 40^\circ$$

$$= 10 \angle 30^\circ + 40^\circ \\ = 10 \angle 70^\circ$$

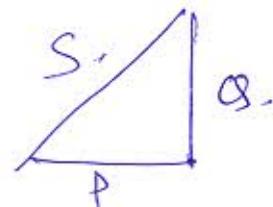
$$*(3-j6) \times 5 \angle 10^\circ \\ = (6.70 \angle -63.43) \times 5 \angle 10^\circ \\ = 33.5 \angle (-63.43 + 10) \\ = 33.5 \angle -53.43^\circ$$

POWER IN BALANCED 3-Q CKT

(3)

- If the magnitude of voltages to neutral V_p for a star connected load is $|V_p| = |V_{an}| = |V_{bn}| = |V_{cn}|$
- If the magnitude of phase current I_p for a star connected load is $|I_p| = |I_{an}| = |I_{bn}| = |I_{cn}|$
- The total 3-Q power is $P = 3|V_p||I_p|\cos\theta_p$ — (1)
- Where, θ_p = phase angle by which phase current I_p lags the phase voltage V_p , i.e. the angle of impedance in each phase.
- If magnitude of $|V_L|$ & $|I_L|$ are the magnitudes of line to line voltage & line to line current respectively.

$$\left\{ \begin{array}{l} |V_p| = \frac{|V_L|}{\sqrt{3}} \\ |I_p| = |I_L| \end{array} \right.$$



Substituting this value in eq (1) we get

$$P = \sqrt{3}|V_L||I_L|\cos\theta_p — (3)$$

→ The total power VAR's are

$$Q = 3|V_p||I_p|\sin\theta_p \quad \} — (4)$$

$$Q = \sqrt{3}|V_L||I_L|\sin\theta_p$$

→ The volt ampers of the load are

$$|S| = \sqrt{P^2 + Q^2} = \sqrt{3}|V_L||I_L| — (5)$$

→ If the load is connected delta the voltage across each impedance is line to line $= \frac{|I_L|}{\sqrt{3}}$.

$$\left. \begin{array}{l} |V_p| = |V_L| \\ |I_p| = \frac{|I_L|}{\sqrt{3}} \end{array} \right\} — (6)$$

$$\text{The total power is } P = 3|V_p||I_p|\cos\theta_p = \sqrt{3}|V_L||I_L|\cos\theta_p$$

(4)

$$= \frac{|I_L|}{\sqrt{3}}$$

$$\begin{aligned} |V_p| &= |V_L| \\ |I_p| &= \frac{|I_L|}{\sqrt{3}} \end{aligned} \quad \left. \right\} \quad \text{--- (6)}$$

→ The total power is

$$\begin{aligned} P &= 3 |V_p| |I_p| \cos \phi_p \\ &= \sqrt{3} |V_L| |I_L| \cos \phi_p \end{aligned}$$

PER UNIT QUANTITY

- The per unit value of any quantity is defined as the ratio of the quantity to its base expressed as a decimal.
- The ratio in percent is 100 times of the value in per unit.
- For 1-Q system or 3-Q systems where the term current refers to the line current where the term voltage refers to the voltage to neutral. Where the term kva per phase refers to the kva per phase the following formulas relate quantities.
- Per unit value of any quantity =
- $$= \frac{\text{actual quantity}}{\text{Base quantity}}$$

$$I_{pu} = \frac{I_A}{I_B}$$

$$Z_{pu} = \frac{Z_A}{Z_B}$$

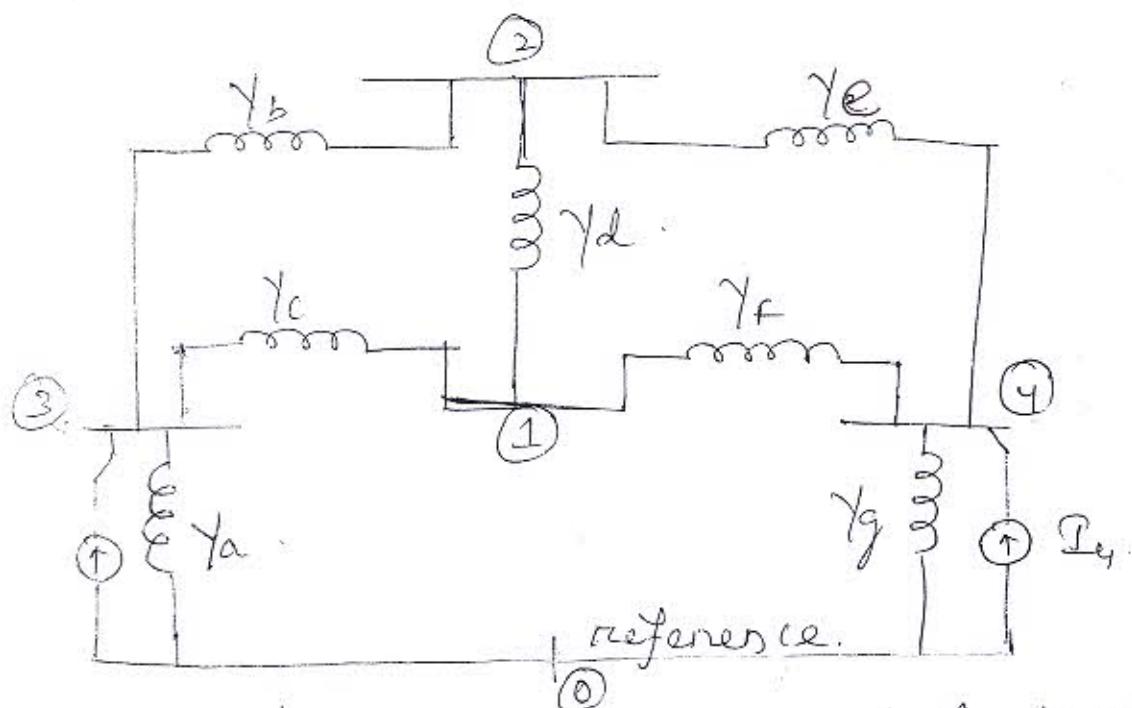
en The reactance of a generator (6) as x'' , which is given as 0.25 per unit base of the generators name plate rating of 18kV, 200MVA. The base for calculation is 20kV, 100MVA. Determine x'' on the new base.

Ans
$$x'' = (0.25) \times \left(\frac{18}{20}\right)^2 \times \left(\frac{100}{500}\right)$$

$$= 0.0405$$

NODE EQUATIONS

Consider a 4bus system as shown in figure.



- Current sources are connected at node 3 and all other elements are represented as admittances.
- single subscript notation is used to the voltage of each node w.r.t reference node
- Applying KCL at node 1.

$$(V_1 - V_3)Y_c + (V_1 - V_2)Y_d + (V_1 - V_4)Y_f = 0 \quad (1)$$

- Applying KCL at node 2. ⑥
- $$(V_2 - V_3)\gamma_b + (V_2 - V_1)\gamma_d + (V_2 - V_4)\gamma_e = 0 \quad \text{--- } ②$$
- Applying KCL at node 3.
- $$(V_3 - V_2)\gamma_b + (V_3 - V_1)\gamma_c + V_3\gamma_a = I_3 \quad \text{--- } ③$$
- Applying KCL at node 4
- $$(V_4 - V_2)\gamma_e + (V_4 - V_1)\gamma_f + V_4\gamma_g = I_4 \quad \text{--- } ④$$

$$\begin{matrix} & \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \\ \textcircled{1} & \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \textcircled{2} & \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} \\ \textcircled{3} & \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} \\ \textcircled{4} & \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} \end{matrix}_{4 \times 4} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

- The γ matrix is designated as γ_{BOS} and called the bus admittance matrix.
- The usual rules for forming the typical γ_{BOS} are:

1. The diagonal elements γ_{ii} = sum of the admittances directly connected to the node i .
2. The diagonal elements γ_{ij} = negative of the Net admittance connected bet' nodes i and j .
3. The diagonal admittances are called the self admittances of the nodes & the off diagonal admittances are called mutual admittances of the nodes.

①

②

③

④

⑦

$$\begin{bmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \\ \textcircled{1} & Y_c + Y_d + Y_f & -Y_d & -Y_e & -Y_f \\ \textcircled{2} & -Y_d & Y_b + Y_d + Y_e & -Y_b & -Y_e \\ \textcircled{3} & -Y_c & -Y_b & Y_a + Y_b + Y_c & 0 \\ \textcircled{4} & -Y_f & -Y_e & 0 & Y_e + Y_f + Y_g \end{bmatrix}$$

4. separating out the entries anyone of the admittances suppose (Y_c) then we can obtain

$$Y_{bus} = \begin{bmatrix} (Y_a + Y_f) & -Y_d & 0 & -Y_f \\ -Y_d & Y_b + Y_d + Y_e & -Y_b & -Y_e \\ 0 & -Y_b & (Y_a + Y_b) & 0 \\ -Y_f & -Y_e & 0 & Y_e + Y_f + Y_g \end{bmatrix} +$$

$$\begin{bmatrix} Y_c & 0 & -Y_c & 0 \\ 0 & 0 & 0 & 0 \\ -Y_c & 0 & Y_c & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

→ The matrix for (Y_c) can be written as shown above for more compactly as follows:

$$Y_c = \textcircled{1} \begin{bmatrix} Y_c & -Y_c \\ -Y_c & Y_c \end{bmatrix}$$

POWER FLOW SOLUTIONS

Power flow problem

- Some points about the load flow study:
in a power system constituents the electrical performance of power flows (real & reactive) for specified condition when the system is operating under the steady state.
- The load flow study provides information about the line & transformer loads (as well as losses). Through out the system and voltages at the different points at the system for evaluation of regulating of the performance of the power system.
- It is a steady state analysis of an interconnected connecting during normal operation. Load flow study is necessary for planning, operation & economics scheduling of exchange of power b/w the utilities.
- The load flow problem consist of determining the magnitude & phase angle of voltages at each bus and also the active & reactive power flow in each line.

BUS

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- It is a node at which one or more lines, one or many loads, generators are connected.
- In a power system a bus or node is associated with 4 quantities.
- 1. voltage magnitude ($|V|$) .
- 2. voltage phase angle (δ)
- 3. Real power (P) .
- 4. Reactive power (Q) .
- In a local flow solⁿ, two quantities out of the above 4 quantities are specified & remaining two quantities are obtained through the solⁿ of eqⁿ.
- The system buses are generally classified into 3 categories.
- 1. Local bus / PQ bus .
- 2. Generator bus / voltage controlled bus / PV bus / Regulated bus .
- 3. Slack bus / Reference bus / swing bus .

Load bus / PQ bus .

- At these buses active power & reactive power are specified. The voltage magnitude ($|V|$) & phase angle (δ) are required to be calculated.

→ At the local bus voltage can be allowed to vary within the permissible value s.t. 10

2. Generator Bus / PV Bus

→ At these buses active power 'P' & voltage magnitude (V) are specified. The phase angle 'δ' & the reactive power 'Q' are required to be calculated.

3. slack bus / swing bus

→ At these buses voltage magnitude (V) & phase angle 'δ' are specified.
→ Real power 'P' & reactive power 'Q' are to be determined through the local flow solution.

→ This bus makes up the difference between the scheduled loads & generated power that are caused by the losses in the network.

THE SYSTEM MODEL (static local flow equation (SLFE))

→ We know that,

$$[I_{bus}] = [\gamma_{bus}] [V_{bus}]$$

→ The current entering to the ^{city} bus of an ^{'n'} bus system given by.

$$I_i = \gamma_{i1} v_1 + \gamma_{i2} v_2 + \gamma_{i3} v_3 + \dots + \gamma_{in} v_n$$

$$\Rightarrow I_i = \sum_{p=1}^n \gamma_{ip} v_p \quad \text{--- (1)}$$

$$\Rightarrow P_i = |V_i| \sum_{p=1}^2 |\gamma_{ip}| |V_p| \cos(\delta_i - \delta_p) \quad \text{--- (1)}$$

$$Q_i = |V_i| \sum_{p=1}^2 |\gamma_{ip}| |V_p| \sin(\delta_i - \delta_p - 90^\circ)$$

$$\Rightarrow Q_i = |V_i| \sum_{p=1}^2 |\gamma_{ip}| |V_p| \sin(\delta_i - \delta_p - 90^\circ)$$

$\boxed{\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B}$

$$\Rightarrow Q_i = |V_i| \sum_{p=1}^2 |\gamma_{ip}| |V_p| \left[\begin{array}{l} \sin(\delta_i - \delta_p) \cos 90^\circ - \cos(\delta_i - \delta_p) \sin 90^\circ \end{array} \right]$$

$$\Rightarrow Q_i = |V_i| \sum_{p=1}^2 |\gamma_{ip}| |V_p| \left\{ -\cos(\delta_i - \delta_p) \right\}$$

$$\Rightarrow Q_i = |V_i| \sum_{p=1}^2 |\gamma_{ip}| |V_p| \cos(\delta_p - \delta_i) \quad \text{--- (2)}$$

Study of load flow problem.

To study the load flow problem two most commonly methods are follows -

1. Gauss - Seidel method.

2. Newton - Raphson method.

1. Gauss - Seidel method

→ For Gauss - Seidel method two cases arises

These are

a. Gauss - Seidel method when PV bus are present -

b. Gauss - Seidel method when PV bus are absent -

a. Gauss-Seidel method when PV bus is absent.

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→ consider Gauss-Seidel algorithm for a system in which voltage controlled buses are absent.

→ out of 'n' buses one bus is slack bus, $(n-1)$ no of PQ bus are present.

→ initially assume the magnitude of angle of voltage at $(n-1)$ buses then these voltages at every stage of iteration.

→ we know that,

$$\mathcal{I}_i = \gamma_{cc} V_i + \sum_{\substack{p=1 \\ p \neq i}}^n \gamma_{cp} V_p$$

$$\Rightarrow \gamma_{cc} V_i = \mathcal{I}_i - \sum_{\substack{p=1 \\ p \neq i}}^n \gamma_{cp} V_p$$

$$\Rightarrow V_i = \frac{1}{\gamma_{cc}} \left[\mathcal{I}_i - \sum_{\substack{p=1 \\ p \neq i}}^n \gamma_{cp} V_p \right] \quad \text{--- (1)}$$

Again we know

$$P_c - jQ_c = V_i * \mathcal{I}_i$$

$$\Rightarrow \mathcal{I}_i = \frac{P_c - jQ_c}{V_i *} \quad \text{--- (2)}$$

Substituting the value of \mathcal{I}_i from eqn (2) in eqn (1) we get,

$$V_i = \frac{1}{\gamma_{cc}} \left[\frac{P_c - jQ_c}{V_i *} - \sum_{\substack{p=1 \\ p \neq i}}^n \gamma_{cp} V_p \right] \quad \text{--- (3)}$$

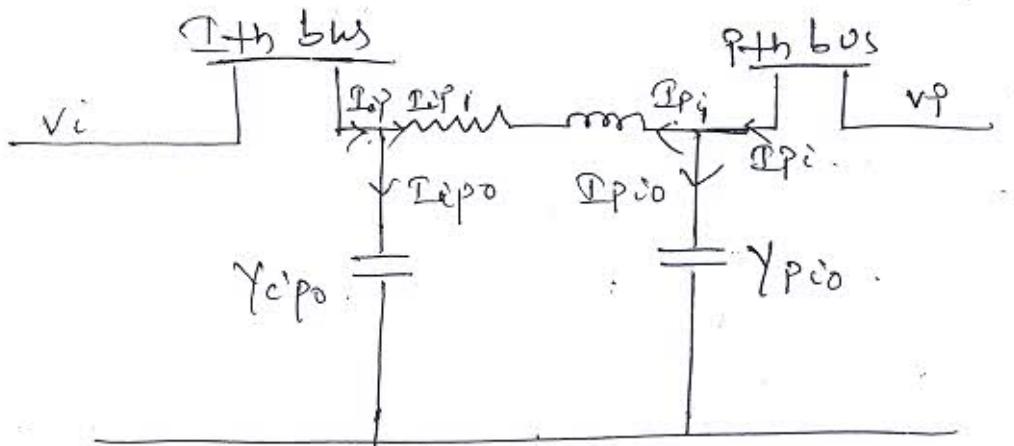
Since the Bus 1 is the slack bus, eq^③ (j3) represents a set of $n-1$ eq^② here which are to be solved simultaneously voltages $v_1, v_2, v_3, \dots, v_n$

$$\Rightarrow v_i = \frac{k^i}{v_i^*} - \sum_{\substack{p=1 \\ p \neq i}}^n L_{ip} v_p \quad \text{--- (4)}$$

$$\text{Where } k^i = \frac{p_i - j Q_i}{Y_{cii}} \quad (\text{for } i = 2, 3, 4, \dots, n)$$

$$L_{ip} = \frac{Y_{ip}}{Y_{cii}} \quad (\text{for } i = 2, 3, 4, \dots, n) \quad \text{& } p = 1, 2, 3, 4, \dots, n \quad p \neq i$$

Consider a line connecting i^{th} bus & p^{th} bus as shown in figure.



Current in betⁿ two buses.

- The line is represented by a T ckt.
- The current I_{ip} & complex power S_{ip} are fed at the i^{th} bus.

$$I_{ip} = I_{ip1} + I_{ip0}$$

$$\Rightarrow I_{ip} = (v_i - v_p) Y_{ip} + v_i Y_{ip0} \quad \text{--- (5)}$$

$$S_{op} = P_{op} + j Q_{op}$$

$$\Rightarrow S_{op} = V_i T_{op}^* \quad \text{--- (6)}$$

from eqⁿ (5)

$$T_{op}^* = (V_i^* - V_p^*) Y_{op}^* + V_c^* Y_{op0}^* \quad \text{--- (7)}$$

from eqⁿ (7) the value of T_{op}^* is put in eqⁿ (6)
and we get,

$$\Rightarrow S_{op} = V_i [(V_i^* - V_p^*) Y_{op}^* + V_c^* Y_{op0}^*]$$

$$\Rightarrow S_{op} = V_i (V_i^* - V_p^*) Y_{op}^* + V_i V_c^* Y_{op0}^* \quad \text{--- (8)}$$

similarly the complex power

$$S_{pi} = P_{pi} + j Q_{pi}$$

$$S_{pi} = V_p [(V_p^* - V_i^*) Y_{pi}^* + V_p^* Y_{pi0}^*] \quad \text{--- (9)}$$

→ The total transmission loss is equal to the sum of the line losses over all the lines.

b. Gauss-Seidel method when PV buses
are present.

~~Assumptions~~

$i=1 \rightarrow$ slack bus / reference bus.

$i=2, 3, 4, \dots \rightarrow$ (PV bus)

$i=m+1, m+2, \dots \rightarrow$ (PQ bus)

→ The voltage controlled buses the bus voltage must be equal to the specified voltage i.e. $|V_i|_{\text{specified}}$. The maxⁿ & minⁿ reactive power at these buses are also specified.
of the value of ' Q_i ' ($i=2, 3, \dots, n$) must be lies betⁿ this limit.

Newton-Raphson Method. (15)
 → for N-R method two cases arises there
 are

1. N-R method using rectangular co-ordinates.
2. N-R method using polar co-ordinates.

1. N-R method using rectangular co-ordinates.
 → In this method all the quantities are
 expressed in rectangular form.

$$\text{Let } V_p = (V_p) \angle \delta_p$$

$$= |V_p|(\cos \delta + j \sin \delta)$$

$$= e_p + j f_p$$

$$\rightarrow \text{where } e_p = |V_p| \cos \delta$$

$$f_p = |V_p| \sin \delta$$

→ The active & reactive power at each bus are the function of e & f .

$$P_i = v_1(e, f)$$

$$Q_i = v_2(e, f)$$

→ For a system of N buses if b_{N-1} is designated as slack bus, the different eq relating to the change in active and reactive powers due to the changes in e_p and f_p 's given by

$$\Delta P_i = \sum_{P=i}^N \frac{\partial P_i}{\partial e_p} \Delta e_p + \sum_{P=i}^N \frac{\partial P_i}{\partial f_p} \Delta f_p \quad \text{--- (1)}$$

$$\rightarrow \text{similarly } \Delta Q_i = \sum_{P=i}^N \frac{\partial Q_i}{\partial e_p} \Delta e_p + \sum_{P=i}^N \frac{\partial Q_i}{\partial f_p} \Delta f_p \quad \text{--- (2)}$$

Where $\Delta P_i \rightarrow$ difference bet' the specified and calculated values of P_i .

$\Delta Q_i \rightarrow$ Difference bet' the specified and calculated values of Q_i . (16)

There are two eq's for each of $(n-1)$ buses.
In matrix form these eq's can be written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} \quad \text{--- (3)}$$

where $J_1, J_2, J_3, J_4 \rightarrow$ the elements of

→ Eq (3) can be used to determine the components of bus voltage any of value through an interactive procedure.

→ For PV bus, reactive power ' Q_i ' is not specified but the voltage magnitude $|V_i|$ is specified.

Hence, $V_i = e_i + j f_i$

$$|V_i|^2 = e_i^2 + f_i^2$$

for each PV bus eq (2) can be written as

$$\Delta |V_i|^2 = \frac{\partial |V_i|^2}{\partial e_i} \Delta e_i + \frac{\partial |V_i|^2}{\partial f_i} \Delta f_i$$

→ The total no. of eq's is $2(n-1)$. If Polar form is used instead of rectangular form the no of eq's will be smaller. Therefore, N-R method in rectangular co-ordinate's rarely used in actual practice Polar form is used.

N-R method using polar co-ordinate's—

→ This method is applied for load flow study when the bus voltages are expressed in polar form.

→ In active power and reactive power at each bus are the function of magnitude and phase angle of the bus voltages, therefore,

Power flow studies in system design and operation

(17)

Base - case

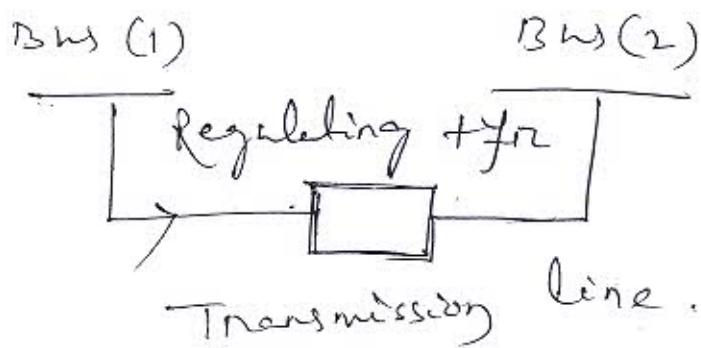
- A power flow study for a system operating under actual or projected normal operating condition is called as Base-case.
- The transmission planning Engineer can discover system weaknesses such as low voltages, line overloads, these weaknesses can be removed by making design studies involving changes on conditions to the Base-case system.
- Integration bet' the system designer & the computer based power flow programme continues until the system performance satisfies the layout, planning & operating criteria.

Regulating Transformer

- The active power & reactive power flow along a transmission line can be achieved by using Control Tfr.
- The real power can be controlled by means of shifting the phase angle of the voltage & reactive power can be control by changing the magnitude of the voltage.

→ voltage magnitude or phase angle (18) through small values can be changed by regulating γ_{fr} .

→ The presence of regulating γ_{fr} in t.l. modifies the bus admittance matrix i.e. $[Y_{bus}]$ matrix. There by modifying the load flow solution.



→ Sparsity

* Sparsity

→ The sparsity of an $n \times n$ matrix is defined as the ratio of the total no of zero elements to the square of the order of the $[Y_{bus}]$ matrix.

→ sparsity = $\frac{\text{Total no of zero elements}}{\text{square of order of the bus admittance matrix}} \times 100$

$$= \frac{\text{Total no of zero elements}}{\pi \rho \eta^2} \times 100$$

Where, γ^2 = square of order of bus admittance matrix.

* In large power system, sparsity is as high as 97%.

$$\text{ex} \quad \begin{bmatrix} \gamma_{11} & 0 & 0 \\ 0 & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & 0 \end{bmatrix} \quad 3 \times 3.$$

$$\text{sparsity} = \frac{4}{9} = \frac{4}{9} \times 100 = 44.4\%.$$

Decoupled method

- By taking into account the physical properties of the system, the load flow studies can ~~make~~ faster & more efficient.
- The properties of the system are

1. Network security.

2. Loose physical interactions b/w MW & MVAR flows in a power system.

X100. 1. Network security.

→ since each bus is connected to only a small no (usually 2 to 3) of other buses, γ_{bus} of a large network is very sparse i.e. it has a large no of zero elements.