

Two Dimensional Transformation

It is the process of manipulating the object. Manipulation is required because when several objects described in their own co-ordinate system, need to be described in one master co-ordinate system.

→ If it is divided into 2 type

1) Geometric transformation

2) Co-ordinate "

Geometric Transformation:

Here the objects itself is transformed and lead to a stationary (fixed) co-ordinate system or background.

Co-ordinate Transformation:

Here the co-ordinate need to be transformed with a stationary point.

operation on geometric transformation are

1 → Translation

2 → Rotation

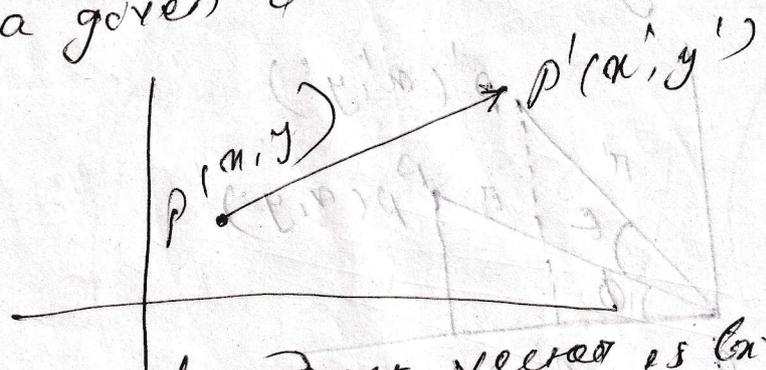
3 → Scaling

4 → Reflection

5 → Shearing

Translation

In translation, the object is displaced from a given distance with some direction.



If the displacement vector is $(tx)^i + (ty)^j$

$$P' = T_v \times P$$

where T_v is the translation matrix.

Each point (x, y) can be represented in a graphical display device in homogeneous form as:

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$x' = x + tx$$

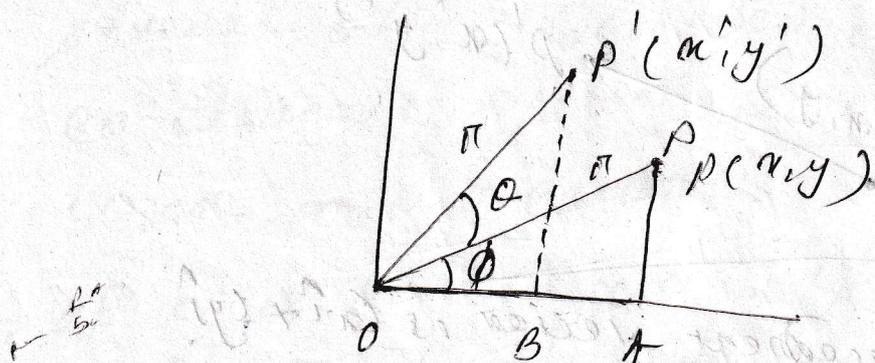
$$y' = y + ty$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$P' = T_v \cdot P$$

Rotation (cartesian)

It is the process of rotating the object with some angle θ° .



Let π' be the distance of point P' from origin

$$\text{In } \triangle OAP, \quad \left. \begin{aligned} OA &= \pi \cos \phi = x \\ PA &= \pi \sin \phi = y \end{aligned} \right\} \text{--- (1)}$$

$$\text{In } \triangle OBP', \quad \pi' = OB = \pi \cos(\phi + \theta)$$

$$\Rightarrow x' = \pi \cos \phi \cdot \cos \theta - \pi \sin \phi \sin \theta$$

$$\Rightarrow x' = x \cos \theta - y \sin \theta$$

$$\text{Similarly } y' = P'B = \pi \sin(\phi + \theta)$$

$$\Rightarrow y' = \pi \sin \phi \cos \theta + \pi \cos \phi \sin \theta$$

$$\Rightarrow y' = y \cos \theta + x \sin \theta$$

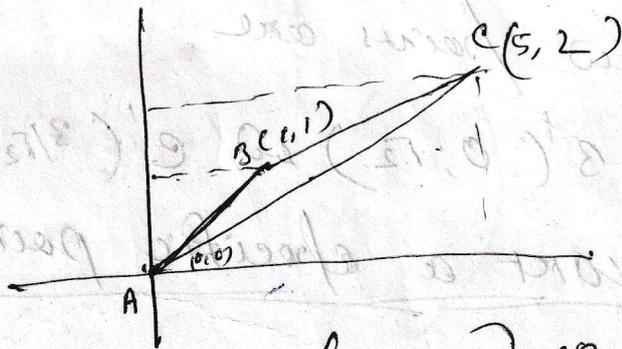
$$\boxed{\begin{aligned} x' &= x \cos \theta - y \sin \theta \\ y' &= x \sin \theta + y \cos \theta \end{aligned}}$$

$$\text{so } \boxed{p' = R_{\theta} \times p}$$

where $R_{\theta} \rightarrow$ Rotation matrix.

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{R_{\theta}} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Q) Find out the co-ordinates of a triangle $A(0,0), B(1,1), C(5,2)$ into angle 45°



Ans: The triangle represented in homogeneous form is

$$ABC = \begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$A's'c' = R_{\theta} \times ABC$$

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

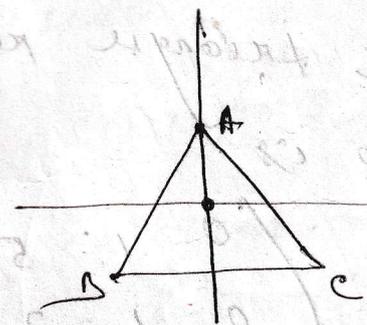
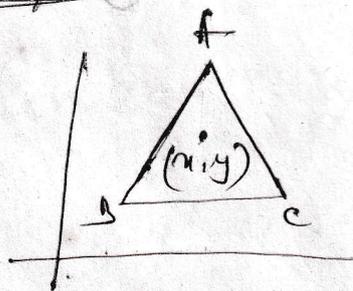
$$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} & \frac{5}{\sqrt{2}} - \frac{2}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} & \frac{5}{\sqrt{2}} + \frac{2}{\sqrt{2}} \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{3}{\sqrt{2}} \\ 0 & \frac{2}{\sqrt{2}} & \frac{7}{\sqrt{2}} \\ 1 & 1 & 1 \end{pmatrix}$$

So, the new points are

$$A'(0, 0), B'(0, \sqrt{2}) \text{ and } C'(\frac{3}{\sqrt{2}}, \frac{7}{\sqrt{2}})$$

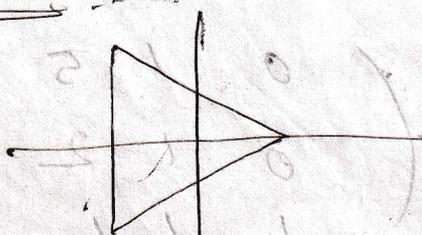
Rotation with a specific point:

Step-1



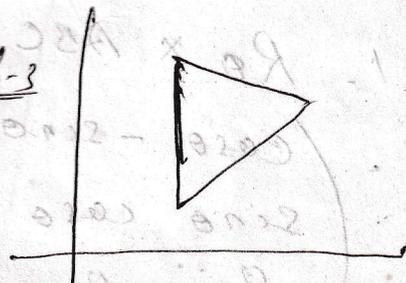
Translation

Step-2



Rotation

Step-3



Reverse translation

$$P' = (T_v \times R_\theta \times T_v^{-1}) P$$

Q) Write the general matrix form for rotating an object to any angle θ° w.r.t a point (h, k)

Ans: Here the rotation can be done as follows

Step-1: The point (h, k) should coincide the origin. \therefore For that

$$T_v = \begin{pmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}$$

Step-2:

It should be rotated an angle θ° , for that

$$R_\theta = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Step-3: The pivot is reverse transfer to original position. i.e.

$$T_{-V} = \begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

So the general for composite transformation is

$$C_T = (T_V \times R_\theta \times T_{-V})$$

$$= \begin{pmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos \theta & -\sin \theta & h(1-\cos \theta) + k \sin \theta \\ \sin \theta & \cos \theta & k(1-\cos \theta) - h \sin \theta \\ 0 & 0 & 1 \end{pmatrix}$$

Q) Rotate the rectangle ABC $A(0,0), B(1,1)$
 $C(5,2)$ to angle 45° by keeping the point
 C fixed.

Ans here $h=5, k=2, \theta=45^\circ$

So the composite matrix is

$$\begin{pmatrix} \cos \theta & -\sin \theta & h(1-\cos \theta) + k \sin \theta \\ \sin \theta & \cos \theta & k(1-\cos \theta) - h \sin \theta \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 5(1-\frac{1}{\sqrt{2}}) + 2 \times \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 2(1-\frac{1}{\sqrt{2}}) - 5 \times \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{5(\sqrt{2}-1)}{\sqrt{2}} + \frac{2}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{2(\sqrt{2}-1)}{\sqrt{2}} - \frac{5}{\sqrt{2}} \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1}B^{-1}C^{-1} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{5(\sqrt{2}-1)}{\sqrt{2}} + \frac{2}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{2(\sqrt{2}-1)}{\sqrt{2}} - \frac{5}{\sqrt{2}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

So the point is (2, 1, 1)

6) Magnify the rectangle $A(0,0)$, $B(4,1)$, $C(4,4)$, $D(0,4)$
 to size 10×4 side

$A'D'C' = \text{Say } XABC$

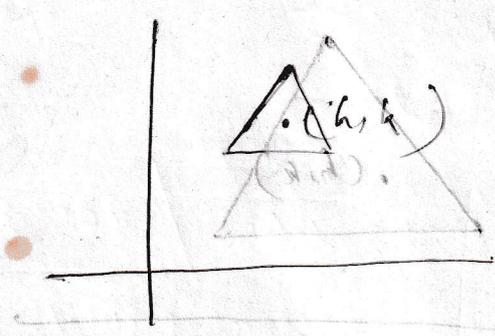
$$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 2 & 1 & 0 \\ 0 & 2 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= A'(0,0) \quad D'(2,2) \quad C(10,4)$$

2) Scaling w.r.t a specific point:

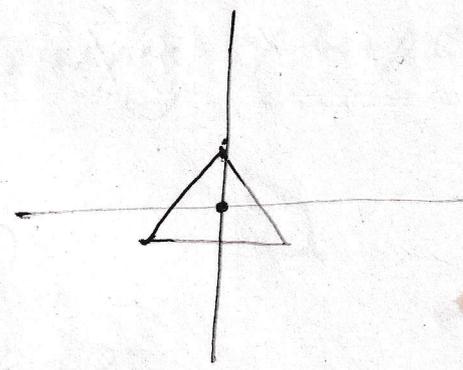
Scaling w.r.t a specific point involves.



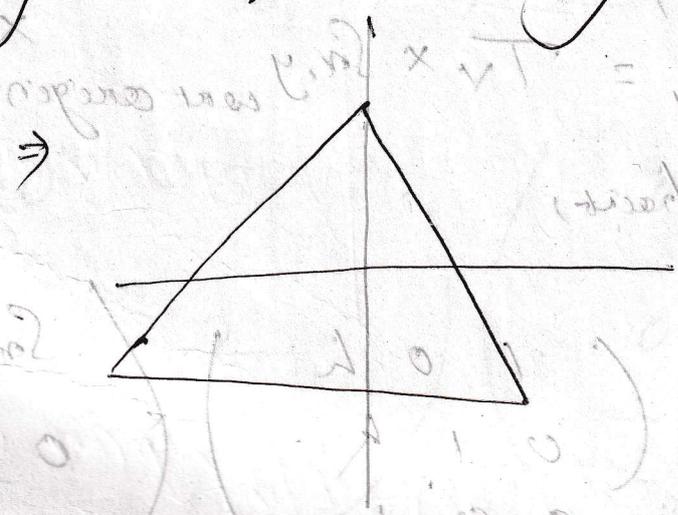
Step-1:

i) Translate point (h, k) to origin.

$$T_1 = \begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

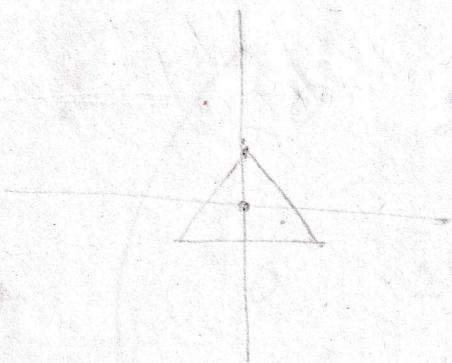
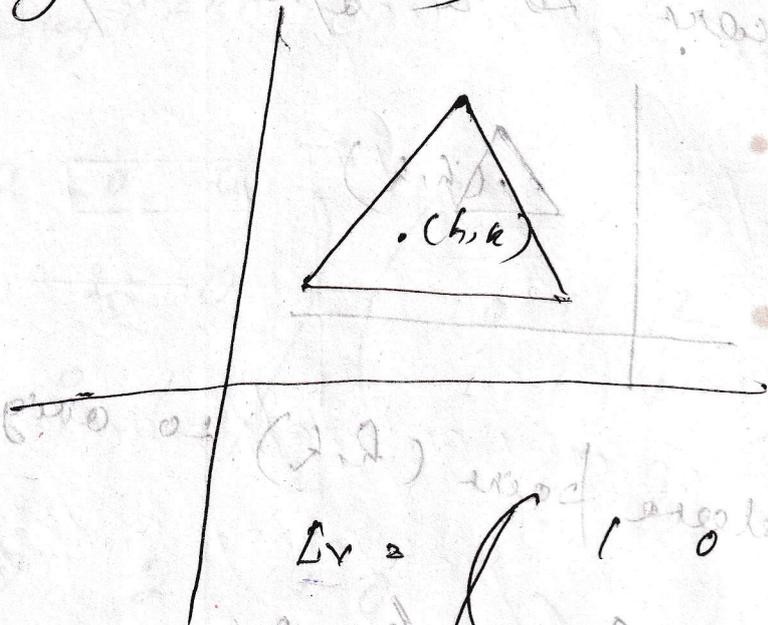


Step-2: Apply the required scaling



$$S_{x,y} = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Step-3: Apply translation the point from origin to (h, k)



$$T_v = \begin{pmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

So composition operation

$$S_{a,y} = T_v \times S_{a,y} \text{ cent origin} \times T_v^{-1}$$

(work a part)

$$S_{a,y} = \begin{pmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} S_a & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} S_a & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} S_x & 0 & h(1-S_x) \\ 0 & S_y & k(1-S_y) \\ 0 & 0 & 1 \end{pmatrix}$$

6) Magnify the triangle $A(0,0)$, $B(1,1)$, $C(5,2)$ to twice by keeping the pt. $C(5,2)$ fixed.

Ans:

$$2 \sim 0, \quad h=5, k=2$$

$$S_x=2, S_y=2$$

$$\begin{pmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$2 \times 0 + 0 + 5 \quad 2 - 5$$

$$\begin{pmatrix} -5 & 0 & 5 \\ -2 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

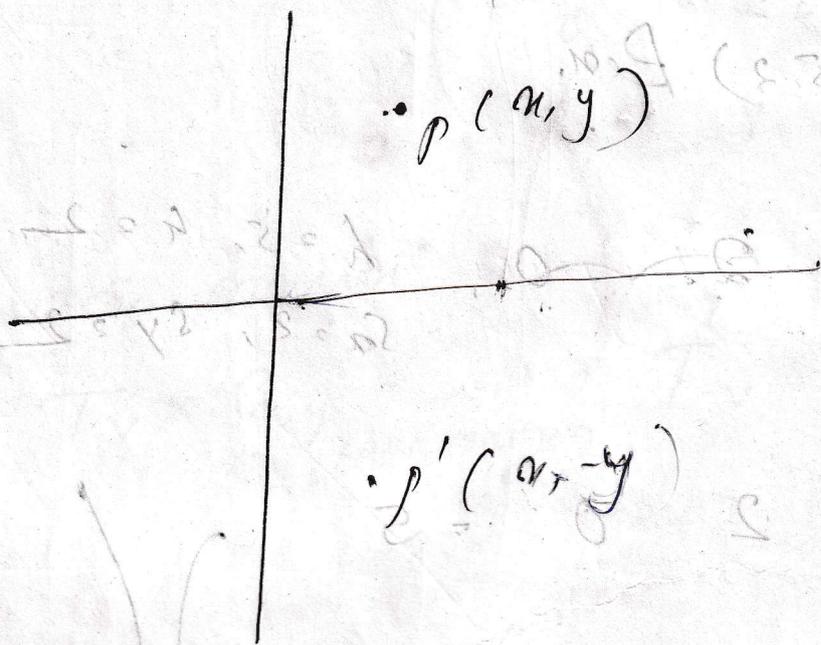
So let $A' = (-5, -2)$

$$A' = (-5, -2)$$

$$C' = (5, 2)$$

*) Reflection: It is the process of finding out the mirror image of a point.

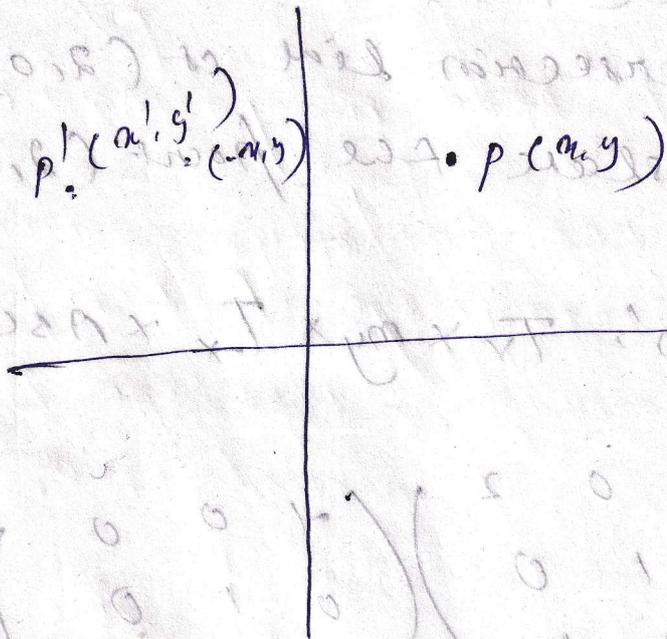
Case-1: when mirror is on the x-axis



$$P'(x', y') = M_x \times P(x, y)$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Case-2



$$P'(a', y') = My^x P(a, y)$$

$$\begin{pmatrix} a' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ y \\ 1 \end{pmatrix}$$

Q) Find the mirror image of a diamond shaped polygon $A(-1, 0), B(0, -2), C(0, 2), D(0, 2)$ about the mirror line

(a) $x = 2$

(b) $y = 2$

(c) $y = x + 2$

Ad: Since the line is $x=2$, so the
 intersection line is $(2,0)$, so
 translate the point $(2,0)$ to origin

So $A^{-1}B^{-1}C^{-1}D^{-1}T_v \times my \times T_{-x} \times A^{-1}B^{-1}C^{-1}D^{-1}$

$$= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

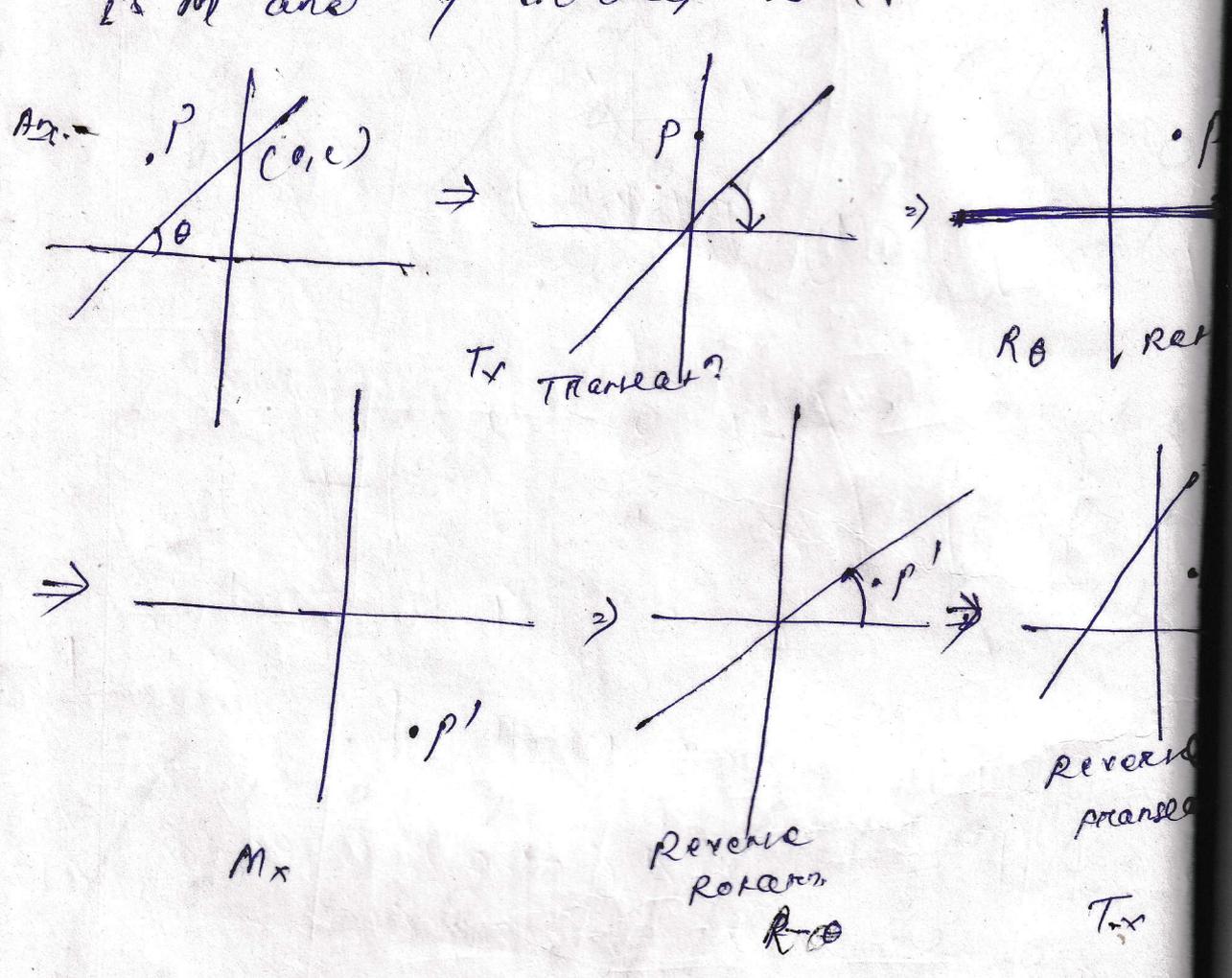
$$\begin{pmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 4 & 3 & 4 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Case 3: When mirror is any arbitrary line

Let the line is $y = mx + c$

Q) Derive the composition matrix for find the mirror image of the point, when the mirror is on the line L whose eqn is M and Y intercept is C .



$$\underline{\text{matrix}} = (T_x \times R_\theta \times M_L \times R_{-\theta} \times T_{-x})$$

$$\begin{pmatrix} \frac{1-m^2}{1+m^2} & \frac{-2m}{1+m^2} & \frac{-2cm}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & \frac{2c}{1+m^2} \\ 0 & 0 & 1 \end{pmatrix}$$

or prove that the reflection about the line $y = a$ is obtained by reversing the co-ordinates i.e. $m(x, y) = (y, x)$

Ans Reflection about $y = a$ of (x, y) is

$$(T_x \times R_\theta \times M_a \times R_{-\theta} \times T_{-y}) (x, y)$$

$$m = 1, c = 0$$

$$\begin{pmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & \frac{-2cm}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & \frac{2c}{1+m^2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} y \\ x \\ 1 \end{pmatrix} = (y, x)$$

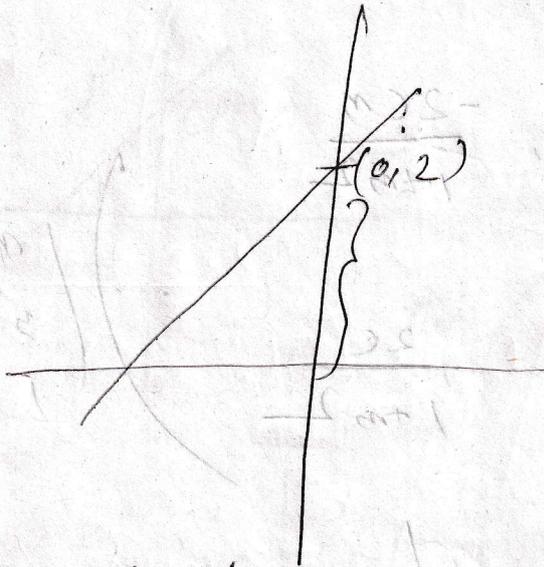
Q. 12.09.11

a) Find the reflection of a triangle.

PER $\left| \begin{array}{ccc} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{array} \right|$

after reflection

about a line $x - 2y = -4$



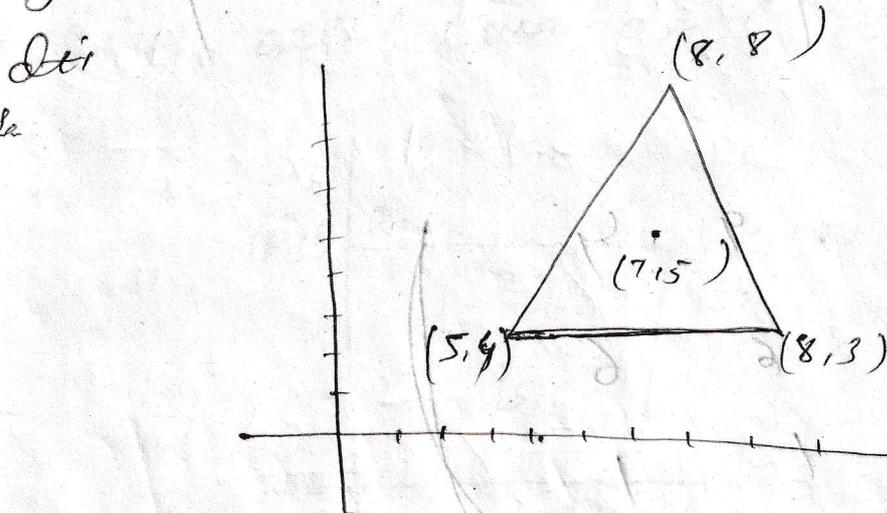
$y = \frac{1}{2}x + 4$

$mx + c = 2x - 1$
 $mx + c = 0$
 $c = 2$

$m = \frac{1}{2}$

$P'Q'R' = \left| \begin{array}{ccc|c} \frac{1-m}{1+m} & \frac{2m}{1+m} & \frac{-2cm}{1+m} & 2 \\ \frac{2m}{1+m} & \frac{m^2-1}{1+m} & \frac{2c}{1+m} & 4 \\ 0 & 0 & 1 & 1 \end{array} \right|$

Q7) Locate the new position of ΔABC after its rotation by 90° about its centroid, clockwise



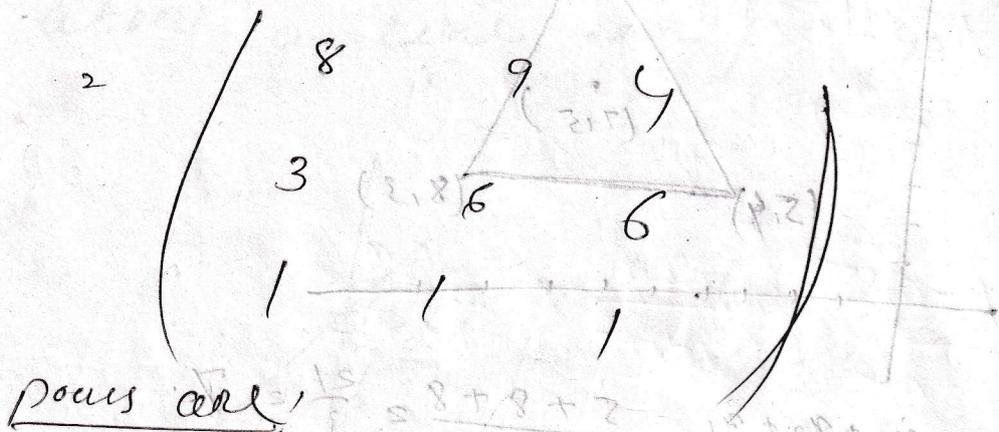
Centroid $x = \frac{x_1 + x_2 + x_3}{3} = \frac{5 + 8 + 8}{3} = \frac{21}{3} = 7$
 $y = \frac{y_1 + y_2 + y_3}{3} = \frac{4 + 3 + 8}{3} = \frac{15}{3} = 5$

$$\begin{pmatrix} -\cos\theta & -\sin\theta & k(1-\cos\theta) + k\sin\theta \\ \sin\theta & \cos\theta & k(1-\cos\theta) - k\sin\theta \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 7(1-0) + 5 \times 1 \\ 1 & 0 & 8(1-0) - 7 \times 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 12 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 0 & -1 & 1 & 2 & 5 & 8 & 8 \\ 1 & 0 & -2 & & 4 & 3 & 8 \\ 0 & 0 & 1 & & 1 & 1 & 1 \end{array} \right)$$



points are

$$(8, 3), (9, 6) \text{ and } (4, 6)$$

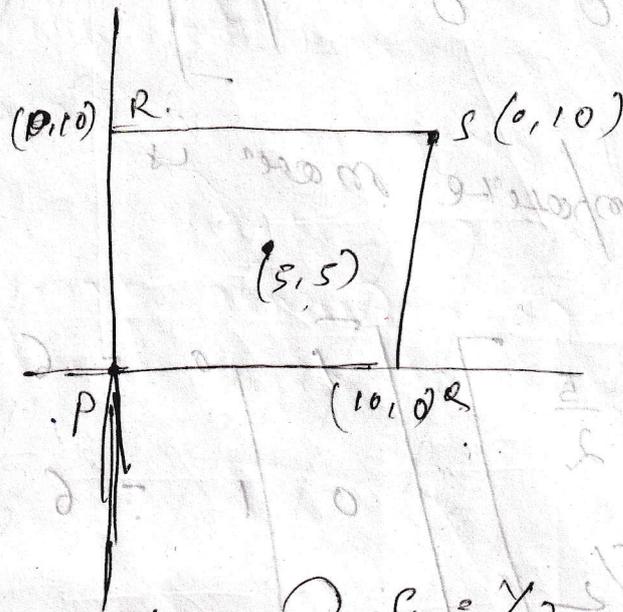
6) A mirror is placed such that it passes through $(3, 0)$ and $(0, 2)$ and the reflected view of the triangle whose vertices are $(3, 4)$, $(5, 5)$ and $(4, 6)$ on the mirror.

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right)$$

c) Transfer the square $p(0,0)$, $q(10,0)$, $r(0,10)$, $s(10,10)$ into a master pixel co-ordinate system with half of original size with corner at $(-1,-1)$

Ans,



Here $S_x = 1/2$ and $S_y = 1/2$

In the 1st step we have to apply scaling with the point $(5,5)$,

The composition of matrix is

$$\begin{pmatrix} S_x & 0 & h(1-S_x) \\ 0 & S_y & k(1-S_y) \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & 5/2 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 1 \end{pmatrix}$$

~~Also the co-ordinates of the~~

Step-2: Translate the point from $(5,5)$ to $(-1,-1)$

So translate on moment

$$T_V = \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

So the composite matrix is

$$\left[\begin{array}{cc|c} \frac{1}{2} & 0 & \frac{5}{2} \\ 0 & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 1 \end{array} \right] \left[\begin{array}{cc|c} 1 & 0 & -6 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cc|c} 3/2 & 0 & -7/2 \\ 0 & 1/2 & -7/2 \\ 0 & 0 & 1 \end{array} \right]$$

Now p' & r/s'

$$\left[\begin{array}{cc|c} 1/2 & 0 & -1/2 \\ 0 & 1/2 & -1/2 \\ 0 & 0 & 1 \end{array} \right] \left[\begin{array}{ccc|ccc} 0 & -10 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 10 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array} \right]$$

$$\begin{bmatrix} 2.5 & 4.5 & 4.5 & -6.5 \\ -0.5 & -4.5 & 4.5 & 4.5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

The composite matrix is

$$T_V \times S_m$$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 0 & -3.5 \\ 0 & 1/2 & -3.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -3.5 & 1.5 & 1.5 & -3.5 \\ -3.5 & -3.5 & 1.5 & 1.5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$