

Module - I

# DSP (Digital Signal Processing)

5th sem ETC

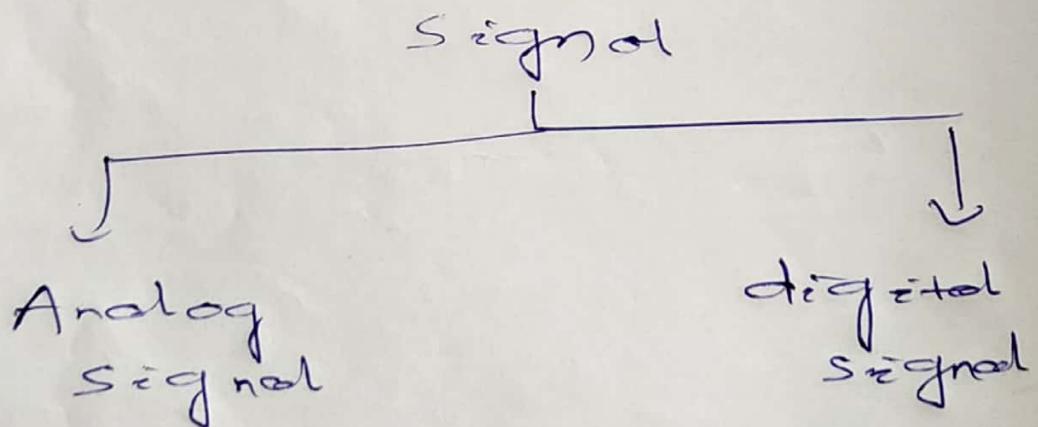
## Signal

- It is a physical quantity
- which varies with respect to time.
- It is the representation of messages, informations and data.

ECG, (Electrocardiogram)

EEG (ElectroEncephalogram)

These are two examples of signals.

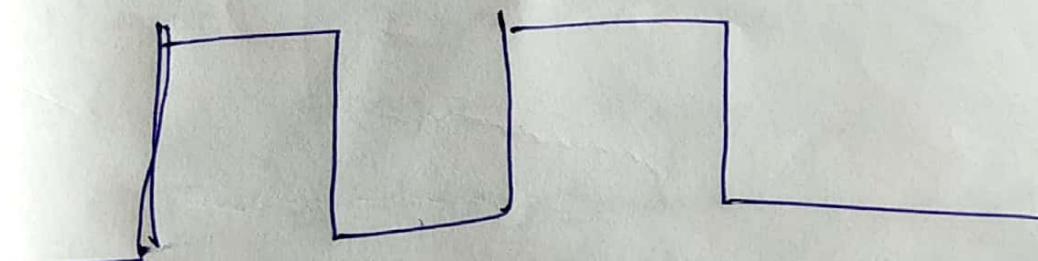
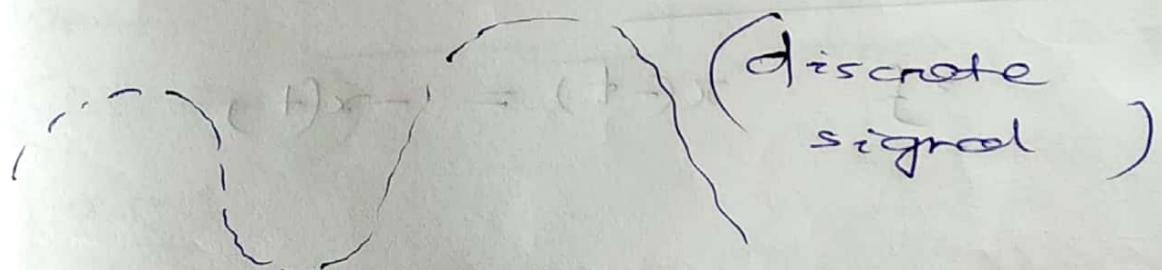
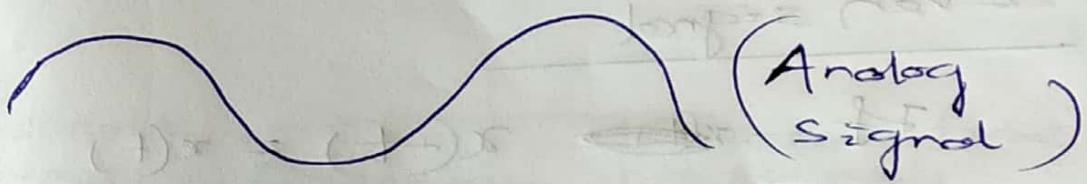


## Analog signal

It is the signal which is continuous in nature

## Digital signal

It is the signal which is discrete in time and quantized in amplitude



digital signal

## Sampling

It is a process of converting analog signal to discrete signal.

## Quantization

It is a process of converting discrete signal to digital signal.

### Even signal

$$\text{If } \cancel{x(-t)} \quad x(-t) = x(t)$$

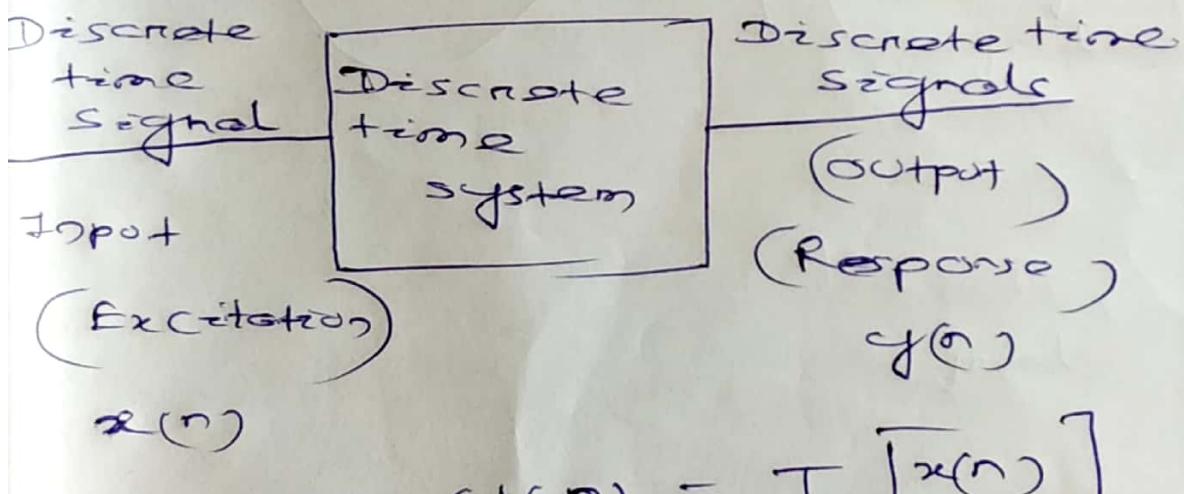
### Odd signal

$$\text{if } \cancel{x(-t)} \quad x(-t) = -x(t)$$



## Discrete time system

A discrete time system is a device or algorithm that operates on a discrete time signal called as input (Excitation) and processes. After processing, it produces another discrete time signal called as output or (response).



$$\text{Here } y(n) = T[x(n)]$$

Here  $x(n)$  = excitation

$y(n)$  = Response

$T$  = transformation called as operator.

# Classification of Discrete time signals.

- There are following types of discrete time signals:
  - Even signal
  - Odd signal
  - Periodic signal
  - Aperiodic signal
  - Energy signal
  - Power signal
  - Impulse signal / impulse function
  - Step signal / step sequence
  - Ramp signal.

Even signal

$$\text{If } x(-n) = x(n)$$

Odd signal

$$\text{If } x(-n) = -x(n)$$

Periodic signal

$$\text{If } x(n+kT) = x(n)$$

Aperiodic signal

$$\text{If } x(n+kT) \neq x(n)$$

## Energy signal

The Energy  $E$  of a signal  $x(n)$  is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

## Power signal

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

## Impulse signal / Impulse function

$$x(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{otherwise} \end{cases}$$

## Step signal / Step sequence

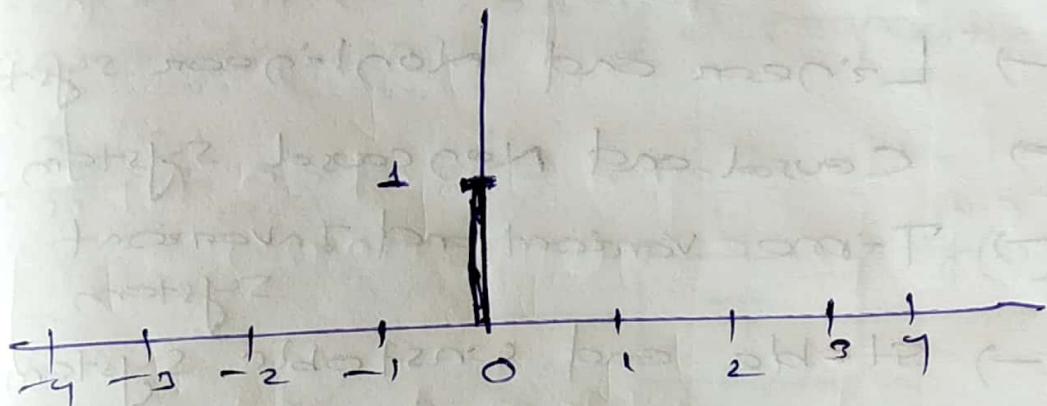
$$x(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

## Ramp signal

$$x(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

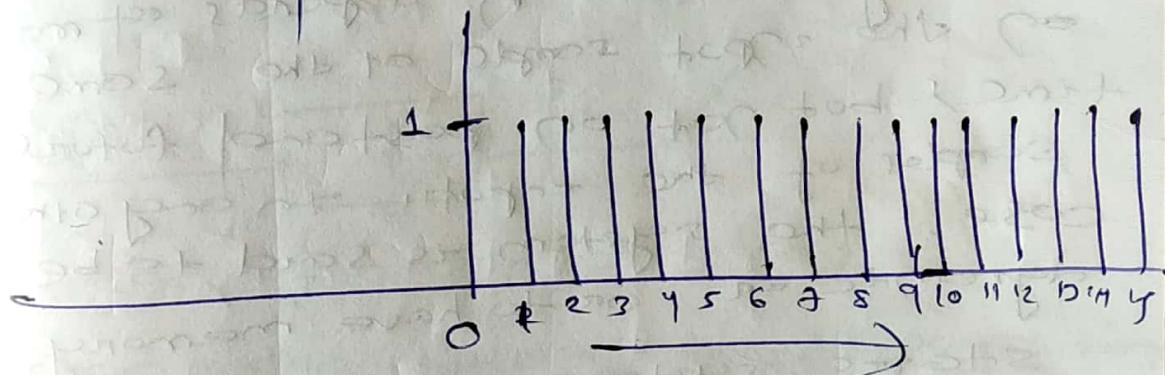
## Impulse signal / Impulse function

$$x(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{otherwise} \end{cases}$$



## Step signal / step sequence

$$x(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



# Types of discrete time systems

- There are following classifications of discrete systems
- static and dynamic system
- Linear and Nonlinear system
- Causal and Noncausal system
- Time variant and Invariant system
- stable and unstable system.

## ① static and dynamic system

A discrete time system is called static or memoryless if its output at any instant  $n$  depends at most on the input sample at the same time, but not on past and future samples of the input. In any other case, the system is said to be dynamic or to have memory.

### static systems

$$\rightarrow y(n) = x(n)$$

$$y(n) = \alpha x(n)$$

$$y(n) = a x(n) + b x^2(n)$$

### Dynamic systems

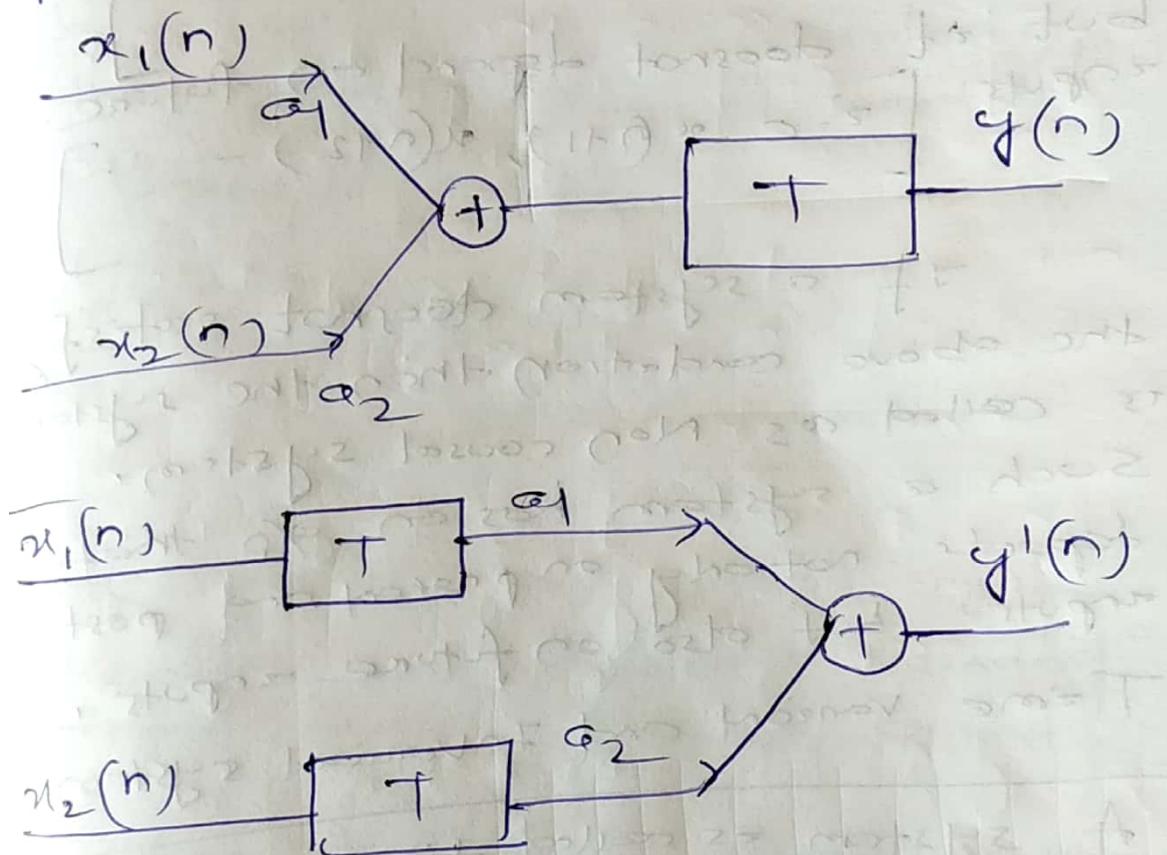
$$y(n) = x(n) + 3x(n-1)$$

$$y(n) = x(n) + (n-1)x(n+1)$$

## Linear and Non linear systems

A system is said to be linear if it obeys superposition principle

A system is said to be non linear if it does not obey superposition principle.



Here if  $y(n) = y_1(n)$

then the system obeys superposition principle. It is called as Linear

if  $y(n) \neq y_1(n)$

then the system does not obey superposition principle. It is called as Non linear.

## Causal and Non Causal system

A system is said to be causal if the output of the system at any time  $n$  depends only on present and past inputs [i.e.,  $x(n)$ ,  $u(n-1)$ ,  $u(n-2) \dots$ ]

but it does not depend on future inputs [i.e.  $x(n+1)$ ,  $u(n+2) \dots$ ]

If a system does not satisfy the above condition then the system is called as Non causal system. Such a system has an output that depends not only on present and past inputs but also on future inputs.

## Time variant and Invariant system

A system is called time invariant if its output input characteristics does not change with time i.e  $y(n) = x(n) + x(n-1)$

A system is called time variant system if its output input characteristics changes with time i.e

$$y(n) = n x(n)$$

## Stable and Unstable system

A system is bounded input / bounded output (BIBO), then the system is called as stable.

If a system does not provide bounded input and bounded o/p then type of system is called as unstable system.

example ①  $y(t) = t x(t)$

$$\textcircled{1} \quad y(t) = \frac{d x(t)}{dt}$$

$y(t) = t x(t)$  is unstable system

because it is providing bounded input and unbounded output.

$y(t) = \frac{d x(t)}{dt}$  is stable system

because it is bounded input and bounded output.

# Analysis and response of discrete time linear LTI system

 LTI system means the system which is both linear and time invariant.

Linear means the system which obey superposition principle.

Time invariant system means the system's output which is not varied by changing of the input of the system.

## Convolution

The process of finding out the output of any system if input and impulse responses are given

$$y(n) = x(n) * h(n)$$

Here   $\rightarrow$  It represents the symbol of convolution

$$y(n) = \sum_{n=-\infty}^{\infty} h(k) x(n-k)$$

$$y(n) = \sum_{n=-\infty}^{\infty} h(k) x(n-k)$$

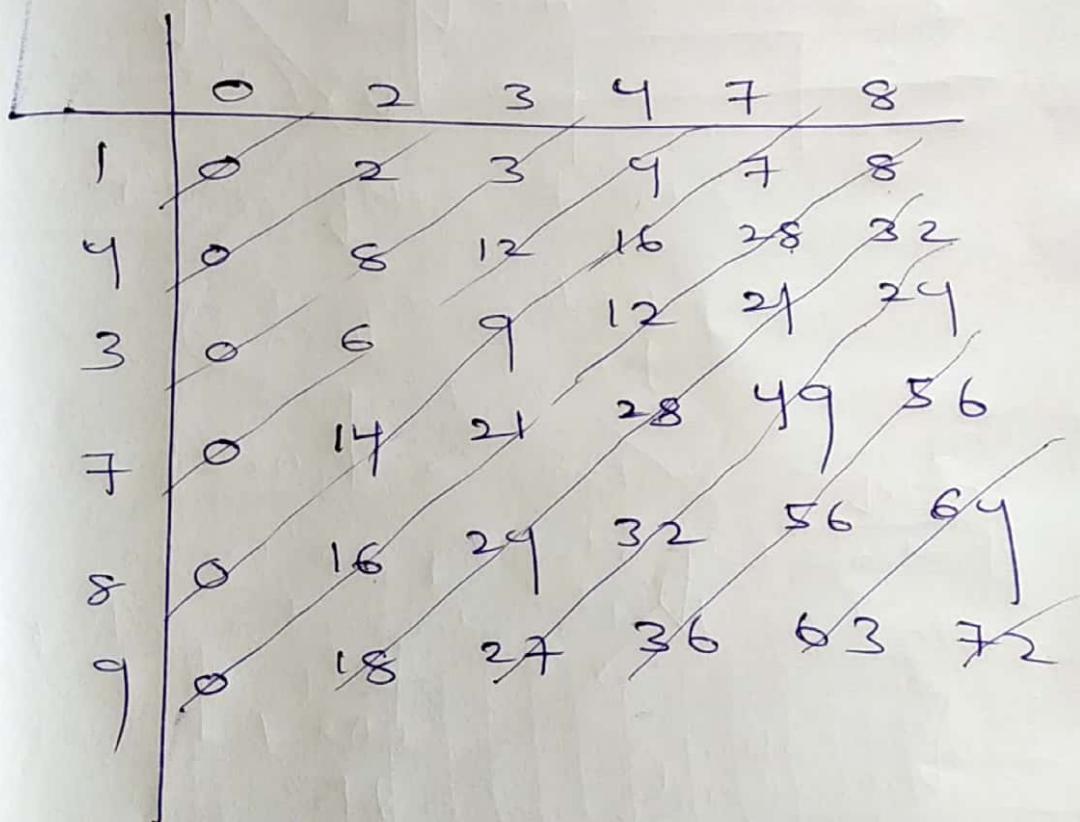
There is another method, by which we can calculate the convolution

example

Given

$$x(n) = \{ 0, 2, 3, 4, 7, 8 \}$$

$$h(n) = \{ 1, 4, 3, 7, 8, 9 \}$$



Here,  $y(n) = \{ 0, 2, 11, 22, 46, 85, 95, 123, 132, 148, 127, 72 \}$

two signals. The general formula for correlation is

$$\int_{-\infty}^{\infty} x_1(t)x_2(t - \tau)dt$$

There are two types of correlation:

- Auto correlation
- Cros correlation

### Auto Correlation Function

It is defined as correlation of a signal with itself. Auto correlation function is a measure of similarity between a signal & its time delayed version. It is represented with  $R(\tau)$ .

Consider a signals  $x(t)$ . The auto correlation function of  $x(t)$  with its time delayed version is given by

$$R_{11}(\tau) = R(\tau) = \int_{-\infty}^{\infty} x(t)x(t - \tau)dt$$

[+ve shift]

Cross correlation is the measure of similarity between two different signals.

Consider two signals  $x_1(t)$  and  $x_2(t)$ . The cross correlation of these two signals

$R_{12}(\tau)$  is given by

$$R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t)x_2(t - \tau) dt$$

[+ve shift]

$$= \int_{-\infty}^{\infty} x_1(t + \tau)x_2(t) dt$$

[-ve shift]

If signals are complex then

$$R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t)x_2^*(t - \tau) dt$$

[+ve shift]

$$= \int_{-\infty}^{\infty} x_1(t + \tau)x_2^*(t) dt$$

[-ve shift]