SRINIX COLLEGE OF ENGINEERING, BALASORE

DEPARTMENT OF MECHANICAL ENGINEERING

NAME OF THE SUBJECT-MECHANISMS AND MACHINES

BRANCH-MECHANICAL ENGINEERING

SEMESTER-5^{TH}

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COURSE OUTLINES OF MODULE-I

1. Steering Gear Mechanism.
2. Hooke's Joint.
3. Double Hooke's Joint.
4. Cam Profile.

References-1. Theory of Machines- R S khurmi & J K Gupta- S Chand Publication.
2. Theory of Machines -S S Rattan-TMH Publication.

Steering Gear Mechanism

Davis steering gear
Ackermann steering gear

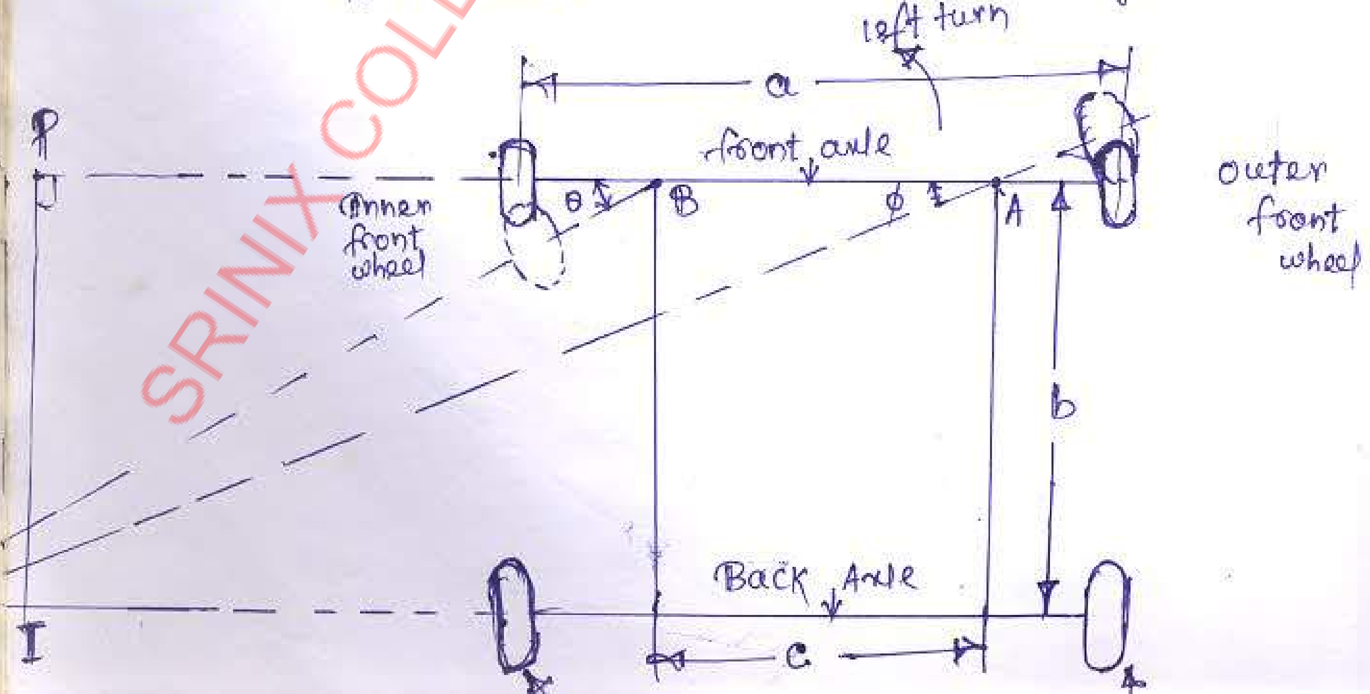
The steering gear mechanism is used for changing the dirn of two or more of the wheel axles with reference to the chassis so as to move the automobile in any desired path.

* Instantaneous centre :- It is a point at which the whole of the mechanism can be rotate at that point.

* How ~~to avoid cause~~ skidding ~~can occur~~..

Ans:- → Inorder to avoid skidding (i.e slipping of the wheels side ways), the two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheels.

→ If the instantaneous centre of the two front wheels do not coincide with the instantaneous centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear & tear of the tyres.



Correct Steering Mechanism

Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre.

Let 'A' and 'B' are the two pivot points on front axle as shown in fig.

The axis of inner wheel makes a larger turning angle θ than the angle ϕ subtended by the axis of outer wheel.

Let a = wheel track

b = wheel base

c = Distance betn the pivots A & B of the front axle.

Now from triangle IBP

$$\cot\theta = \frac{BP}{IP}$$

& from triangle IAP

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP}$$

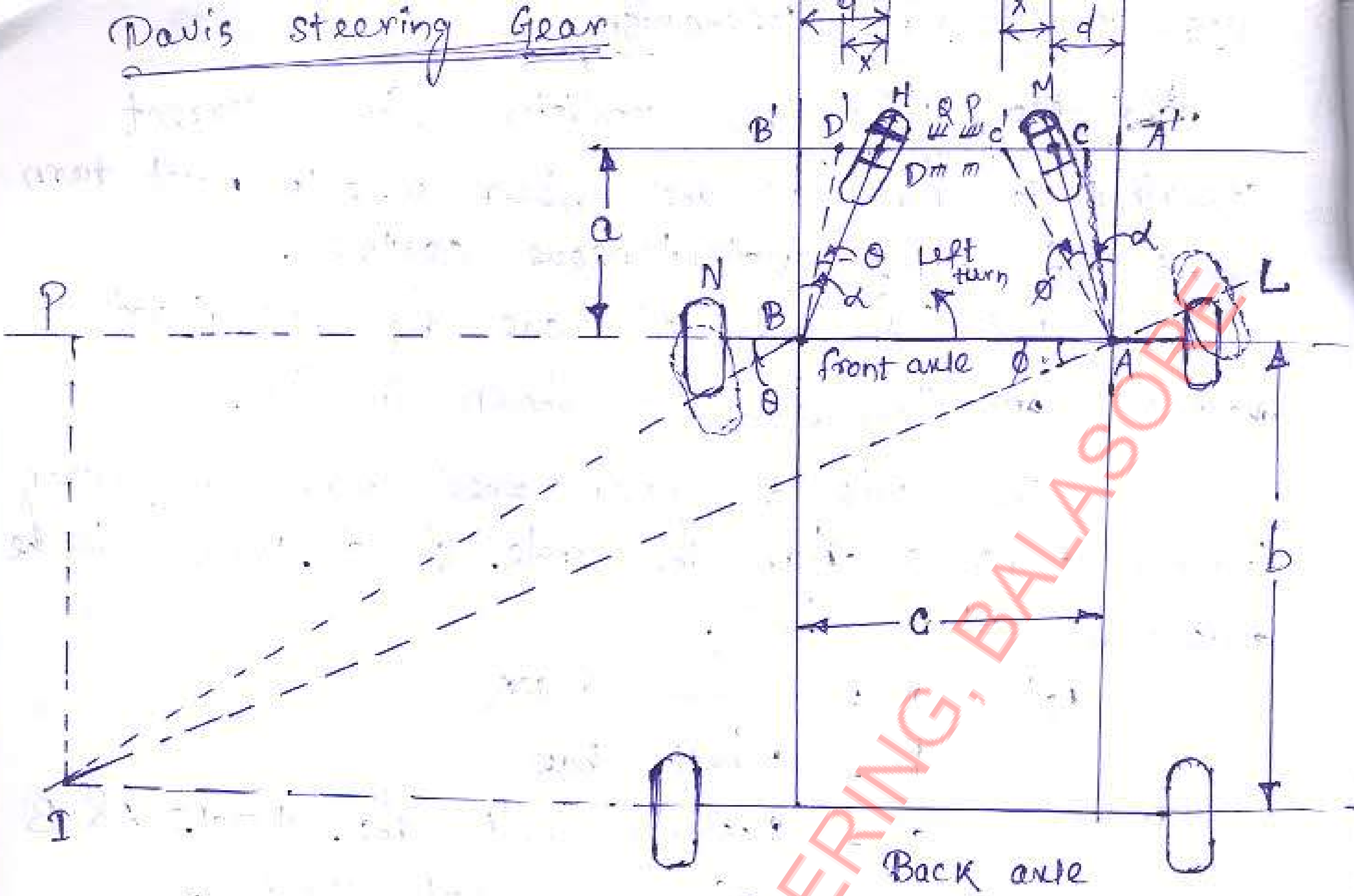
$$\Rightarrow \cot\phi = \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta$$

$$\Rightarrow \boxed{\cot\phi - \cot\theta = c/b}$$

This is the fundamental eqn. for correct steering.

If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

Davis steering Gear



The davis steering gear as shown in fig 07 is an exact steering gear mechanism.

The slotted links AM & BH are attached to the front wheel axle, which turn on pivots A & B respectively.

The rod CD is constrained to move in the dirn. of its length by the sliding member at P & Q.

~~These~~ constraints are connected to the slotted link AM & BH by a sliding & a turning pair at each end.

The steering is affected by moving CD to the right or left of its normal position. C'D' shows the position of CD for turning to the left.

Let a = vertical distance betn AB & CD.

b = wheel base

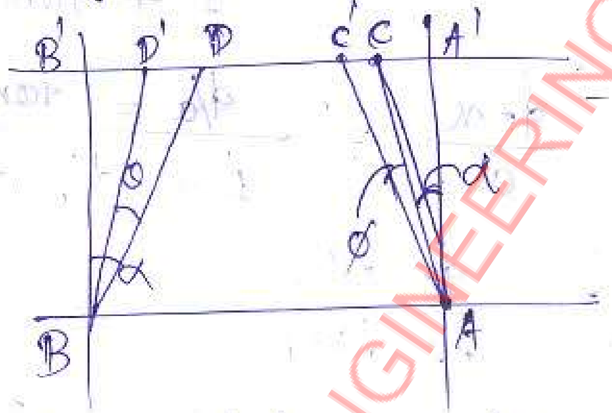
d = Horizontal distance betn AC & BD.

c = Distance betn the pivots A & B of the front axle.

x = Distance moved by AC to AC' = CC' = DD'

α = Angle of inclination of the links AC & BD to the vertical.

from triangle AA'C'.



$$\tan(\alpha+\phi) = \frac{A'C'}{AA'} = \frac{d+x}{a}$$

from triangle AA'C.

$$\tan\alpha = \frac{A'C}{AA'} = \frac{d}{a}$$

from triangle BB'D' $\tan(\alpha-\theta) = \frac{B'D'}{BB'} = \frac{d-x}{a}$

we know that $\tan(\alpha+\phi) = \frac{\tan\alpha + \tan\phi}{1-\tan\alpha \cdot \tan\phi}$

$$\Rightarrow \frac{d+x}{a} = \frac{d/a + \tan\phi}{1 - d/a \times \tan\phi}$$

$$\Rightarrow \frac{d+x}{a} = \frac{d+a\tan\phi}{a-d\tan\phi}$$

$$\Rightarrow (d+x)(a - d\tan\phi) = a(d + a\tan\phi)$$

$$\Rightarrow a\cdot d - d^2\tan\phi + ax - dx\tan\phi = ad + a^2\tan\phi$$

$$\Rightarrow a^2\tan\phi + d^2\tan\phi + dx\tan\phi = a\cancel{d} - a\cancel{d} + ax$$

$$\Rightarrow \tan\phi\,(a^2 + d^2 + dx) = ax$$

$$\Rightarrow \boxed{\tan\phi = \frac{ax}{a^2 + d^2 + dx}}$$

Similarly $\tan(\alpha - \theta) = \dfrac{\tan\alpha - \tan\theta}{1 + \tan\alpha\cdot\tan\theta}$

$$\Rightarrow \frac{d-x}{a} = \frac{d/a - \tan\theta}{1 + d/a\,\tan\theta}$$

$$\Rightarrow \frac{(d-x)}{a} = \frac{d - a\tan\theta}{a + d\tan\theta}$$

$$\Rightarrow (d-x)(a + d\tan\theta) = a \times d - a^2\tan\theta$$

$$\Rightarrow \cancel{ad} + d^2\tan\theta - ax - dx\tan\theta = \cancel{ad} - a^2\tan\theta$$

$$\Rightarrow \tan\theta\,(d^2 - dx + a^2) = ax$$

$$\Rightarrow \boxed{\tan\theta = \frac{ax}{a^2 - dx + d^2}}$$

We know that for correct steering

$$\cot\phi - \cot\theta = c/b$$

$$\Rightarrow \frac{1}{\tan\phi} - \frac{1}{\tan\theta} = c/b$$

$$\Rightarrow \frac{a^2 + dx + d^2}{ax} - \frac{a^2 - dx + d^2}{ax} = c/b$$

$$\Rightarrow \frac{2\,dx}{ax} = c/b \qquad \left(\frac{d}{a} = \tan\alpha\right)$$

$$\Rightarrow \frac{2d}{a} = c/b \quad \Rightarrow 2\tan\alpha = c/b$$

$$\Rightarrow \boxed{\tan\alpha = \frac{c}{2b}}$$

* In a Davis steering gear, the distance betn the pivots of the front axle is 1.2M & the wheel base is 2.7M. find the inclination of the track arm to the longitudinal axis of the car, when it is moving along a straight path.

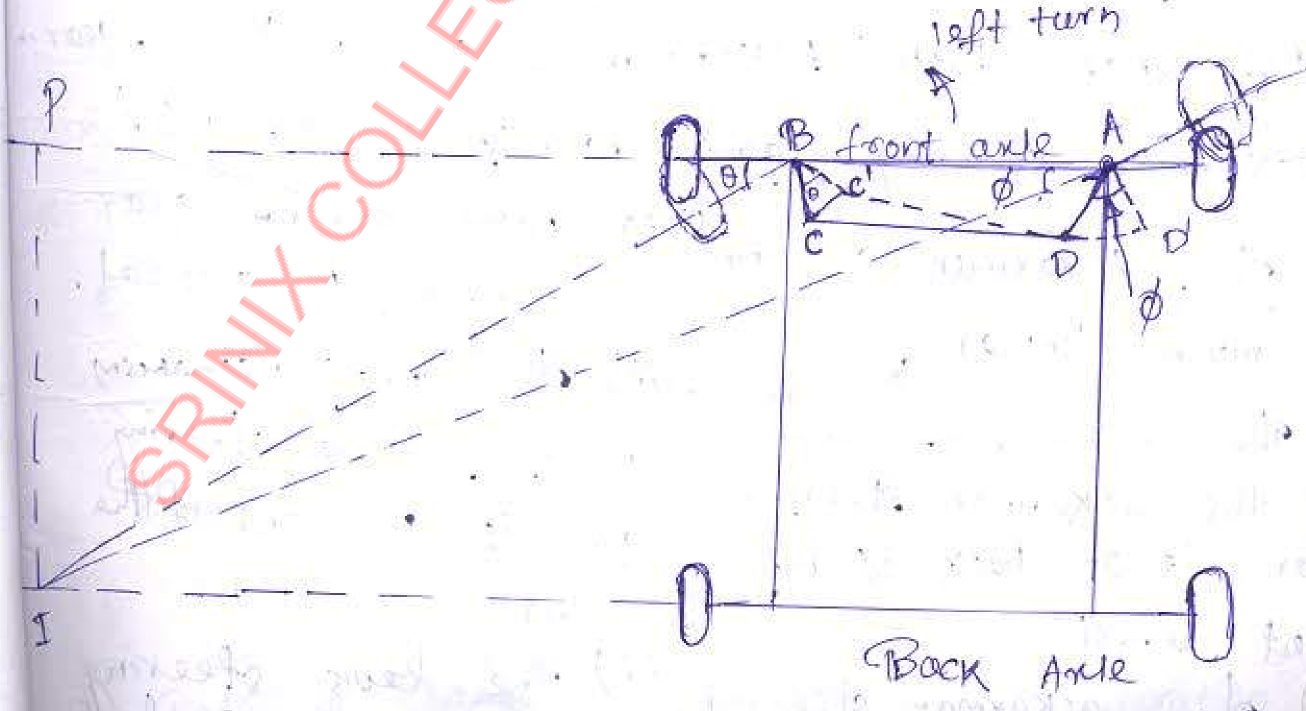
Ans:- Given data $c = 1.2\,m$, $b = 2.7\,M$

$\alpha = ?$

$$\tan\alpha = \frac{c}{2b}$$

$$\Rightarrow \tan\alpha = \frac{1.2}{2\times 2.7} \Rightarrow \boxed{\alpha = 12.5^\circ}$$

* Ackerman steering Gear :-



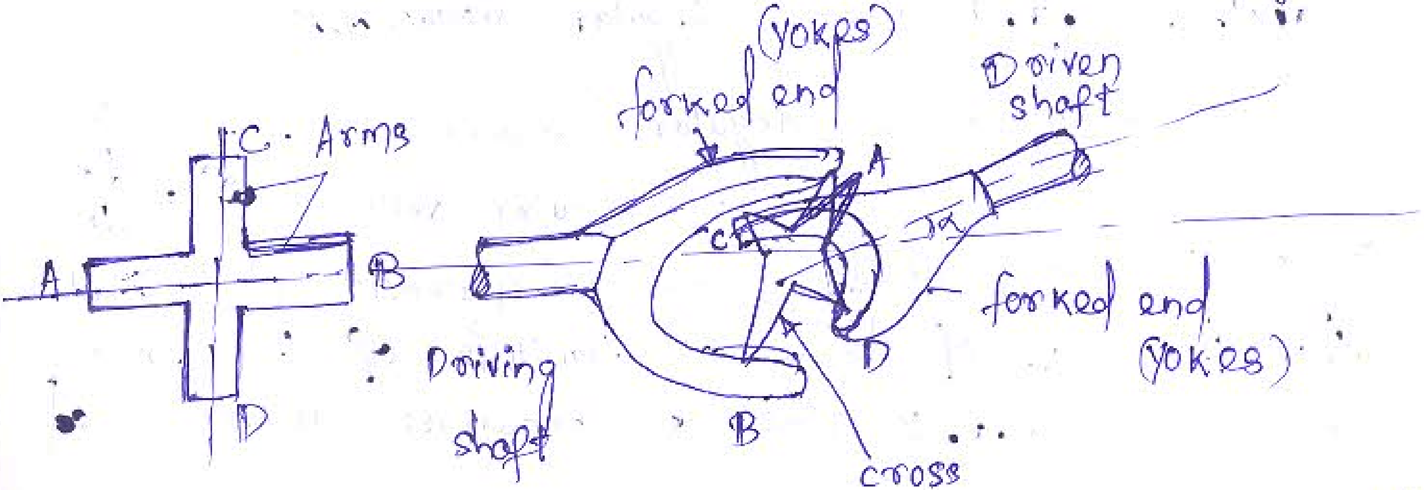
In Ackerman steering gear, the mechanism ABCD is a four bar crank chain as shown in fig. The shorter links BC and AD are of equal length & are connected by hinge joints with front wheel axles. The longer links, AB & CD are of unequal length. The following are the only three positions for correct steering.

1. When the vehicle moves along a straight path, the longer link AB & CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle.
2. When the vehicle is steering to the left, the position of the gear is shown by dotted lines. In this position, the lines of the front wheel axle intersect on the back wheel axle at I, for correct steering.
3. When the vehicle is steering to the right, the similar position may be obtained.

* Difference betn Ackerman & Davis steering gear:

Ackerman steering gear	Davis steering gear
(i) The Ackerman steering mechanism is much simpler	(i) Davis steering gear mechanism is critical.
(ii) The whole mechanism of the ackerman steering gear is on back of the front wheels	(ii) The whole mechanism of the davis steering gear is in front of the wheels.
(iii) The ackerman steering gear consists of turning pair	(iii) The davis steering gear consists of sliding

* Universal ~~or~~ Hooke's Joint :- Shaft with Angular misalignment



Universal or Hooke's Joint

- A Universal joint (universal coupling, U-joint, cardan joint, Hardy spicer joint) is a joint or coupling connecting rigid rods whose axes are inclined to each other and is commonly used in shafts that transmit rotary motion.
- It consists of a pair of hinges located close together, oriented at 90° to each other connected by a cross shaft.
- The universal joint is not a constant velocity joint.
- Universal joint is used to connect two non parallel but intersecting at a small angle i.e (5°-20°).
- Universal joint is situated in betn gear box (engine) and differential or back axle of an automobile.

* Relationship betn Angular velocity of driving & driven shaft:

Let ω_1 = Angular velocity of driving shaft in rad/sec

ω_2 = Angular velocity of driven shaft in rad/sec

$$\boxed{\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}} \qquad \boxed{\frac{N_2}{N_1} = \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}}$$

where N_1 = Angular speed of driving shaft in rpm

N_2 = Angular speed of driven shaft in rpm.

θ = Angle made by driving shaft

α = Angle betn driving & driven shaft

* Maximum and minimum speeds of driven shaft

$$(\omega_2)_{max} = \frac{\omega_1}{\cos\alpha} \qquad (N_2)_{max} = \frac{N_1}{\cos\alpha}$$

* Condition for equal speeds of the driving & driven shaft

$$\boxed{\tan\theta = \pm\sqrt{\cos\alpha}}$$

* Angular acceleration of the driven shaft

$$\Rightarrow \frac{d\omega_2}{dt} = \frac{-\omega_1^2\cos\alpha \cdot \sin 2\theta \cdot \sin^2\alpha}{(1-\cos^2\theta \cdot \sin^2\alpha)^2}$$

Maximum angular acceleration of the driven shaft

$$\boxed{\cos 2\theta = \frac{\sin^2\alpha(2-\cos^2 2\theta)}{2-\sin^2\alpha}}$$

If the value of $\alpha < 30^\circ$ then

$$\boxed{\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}}$$

* Max^m. fluctuation of speed

Max^m. speed of the driven shaft

$$(\omega_2)_{max} = \frac{\omega_1}{\cos\alpha}$$

min^n. speed of the driven shaft

$$(\omega_2)_{min} = \omega_1\cos\alpha$$

fluctuation of speed of the driven shaft

$$(q) = (\omega_2)_{max} - (\omega_2)_{min}$$

$$= \frac{\omega_1}{\cos\alpha} - \omega_1\cos\alpha$$

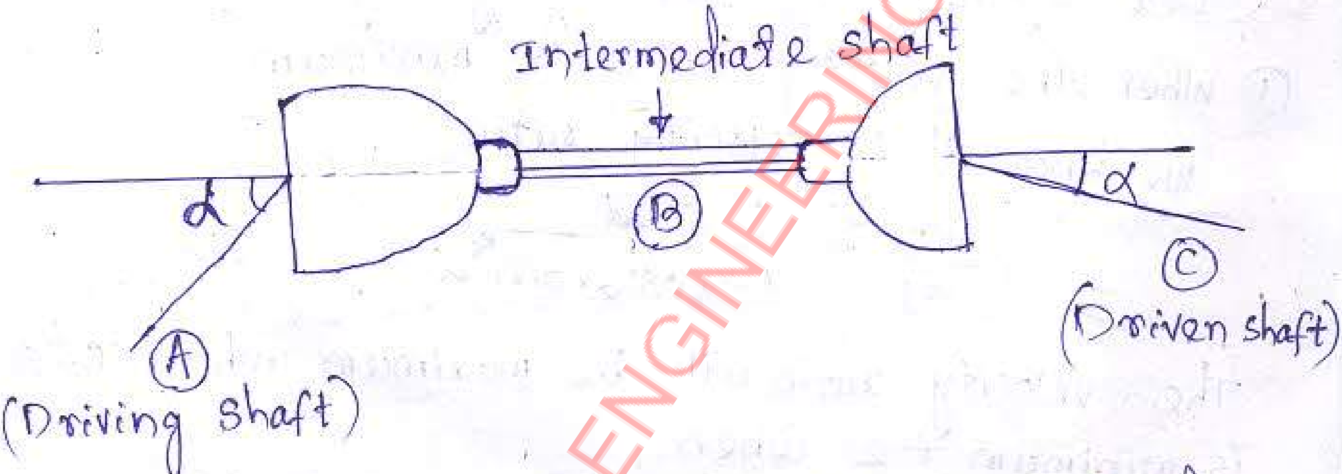
$$= \omega_1\left(\frac{1}{\cos\alpha} - \cos\alpha\right) = \omega_1\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$\boxed{q = \frac{\omega_1\sin^2\alpha}{\cos\alpha} = \omega_1\tan\alpha \cdot \sin\alpha}$$

Since α is a small angle, therefore
$\cos\alpha \simeq 1$ & $\sin\alpha \simeq \alpha$ radians

$$\boxed{\text{Max}^m \text{ fluctuation of speed } (q_{max}) = \omega_1 \alpha^2}$$

Double Hooke's Joint



- In a single Hooke's joint, the speed of the driven shaft is not uniform although the driving shaft rotates at a uniform speed.
- In order to have a constant velocity ratio, an intermediate shaft with a Hooke's joint at each end is used. This type of joint is known as double Hooke's Joint.
- This joint gives a constant velocity ratio, if
  1. → The axes of the driving & driven shafts are in the same plane.
  2. → The driving and driven shafts make equal angles with the intermediate shaft.

* The angle betn the axes of two shafts connected by Hooke's Joint is 18°. Determine the angle turned through by the driving shaft when the velocity ratio (VR) is maxm. and unity.

Ans :- Given data

Angle betn driving & driven shaft (α) = 18°

Let θ = Angle turned through by the driving shaft.

① When the velocity ratio is maximum

We know that velocity ratio

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

The velocity ratio will be maximum when $\cos^2\theta$ is minimum i.e when

$\cos^2\theta = 1$ or when $\theta = 0°$ or $180°$

② When the velocity ratio is unity

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

$$\Rightarrow 1 = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

$$\Rightarrow 1-\cos^2\theta \cdot \sin^2\alpha = \cos\alpha$$

$$\Rightarrow \cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha}$$

$$\Rightarrow \cos\theta = \pm\sqrt{\frac{1-\cos\alpha}{1-\cos^2\alpha}} = \pm\sqrt{\frac{\cancel{(1-\cos\alpha)}}{(1+\cos\alpha)\cancel{(1-\cos\alpha)}}}$$

$$\Rightarrow \cos\theta = \pm\sqrt{\frac{1}{1+\cos\alpha}} = \pm\sqrt{\frac{1}{1+\cos 18}}$$

$$\Rightarrow \cos\theta = \pm 0.7159$$

$$\Rightarrow \boxed{\theta = 44.3° \text{ or } 135.7°}$$

* Two shafts are connected by a Hooke's Joint. The driving shaft revolves uniformly at 500 rpm. If the total permissible variation in speed of the driven shaft is not to exceed $\pm 6\%$ of the mean speed. find the greatest permissible angle betn the centre lines of the shafts.

Ans:- Given data

speed of driving shaft (N_1) = 500 rpm.

Angular velocity of the driving shaft $(\omega_1) = \frac{2\pi N_1}{60}$

$(\omega_1) = \frac{2\pi \times 500}{60} = 52.4$ rad/s.

Let α = Greatest permissible angle betn the centre lines of the shafts.

Since the variation in speed the driven shaft is $\pm 6\%$ of the mean speed (i.e speed of driving shaft)

therefore total fluctuation of speed of the driven shaft

$\Rightarrow q = 12\%$ of Mean speed $(\omega_1) = 0.12\omega_1$

$\Rightarrow q = \omega_1 \left(\frac{1-\cos^2\alpha}{\cos\alpha}\right) \Rightarrow 0.12\omega_1 = \omega_1\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$

$\Rightarrow \cos^2\alpha + 0.12\cos\alpha - 1 = 0$

$\boxed{\cos\alpha = 0.19}$

$\Rightarrow \boxed{\alpha = 19.64^\circ}$

* Two shafts are connected by a universal joint. The driving shaft rotates at a uniform speed of 1200 rpm. Determine the greatest permissible angle betn the shaft axes so that the total fluctuation of speed does not exceed 100 rpm. Also calculate the maxm & minm speeds of the driven shaft.

Ans:- Given data $N_1 = 1200$ rpm, $q = 100$ rpm

Let α = Greatest permissible angle betn the shaft axes.

We know fluctuation of speed $(q) = N_1\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$

$\Rightarrow 100 = 1200\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right) \Rightarrow \frac{1-\cos^2\alpha}{\cos\alpha} = 0.083$

$\Rightarrow \cos^2\alpha + 0.083\cos\alpha - 1 = 0$

$\boxed{\therefore \alpha = 16.4^\circ}$

$(N_2)_{Max} = N_1/\cos\alpha = \frac{1200}{\cos 16.4^\circ} = 1251$ rpm

$(N_2)_{min} = N_1 \cos\alpha = 1200\cos 16.4^\circ = 1151$ rpm

CAM PROFILE

→ A cam is a rotating machine element which gives reciprocating or oscillating motion to another element known as follower.

→ The cam usually rotates at constant speed and drives the follower whose motion depends upon the shape of the cam.

→ cam acts as a driver whereas follower is the driven.

→ The cam and follower have a line contact and constitute a higher pair.

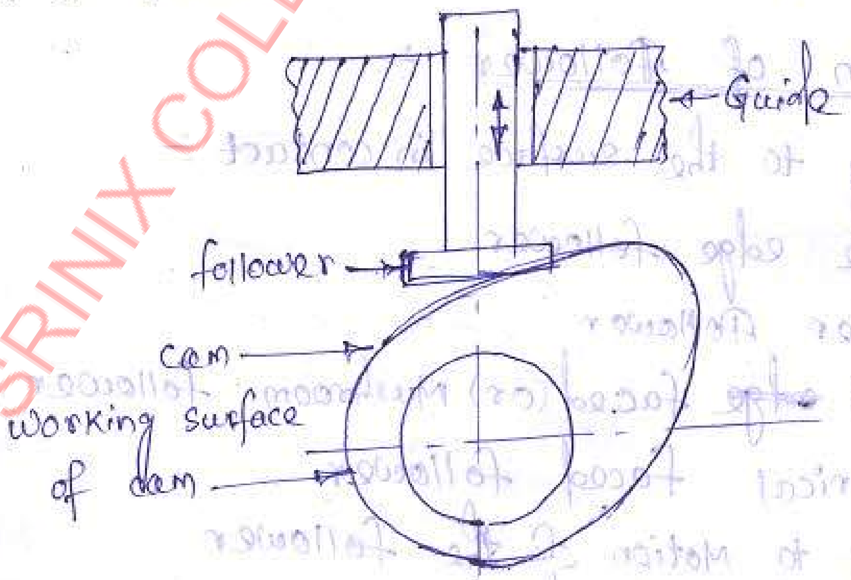
→ Ex :- Inlet and Exhaust valves of I.C. engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes etc.

→ In cam Mechanism three essential members are

(i) cam which has a curved or straight surface

(ii) follower

(iii) frame which supports and guide the follower & cam.



Classification of cams

Cams classified according to following types

* According to shape

(1) Wedge & Flat cams
(2) Radial or Disc cams
(3) Spiral cams
(4) Cylindrical cams
(5) Conjugate cams
(6) Spherical cams
(7) Globoidal cams

* According to follower movement

(1) Rise - Return - Rise (R-R-R)
(2) Dwell - Rise - Return - Dwell (D-R-R-D)
(3) Dwell - Rise - Dwell - Return - Dwell (D-R-D-R-D)

* According to manner of constraint of the follower

(1) Pre-loaded spring cam
(2) Positive-drive cam
(3) Gravity cam

Fundamental law of cam

The cam function must be continuous through the 1^{st} and 2^{nd} derivatives of displacement across the entire interval (360° rev. of the cam shaft)

S = Displacement - s

V = Velocity - $\frac{\partial s}{\partial t}$

A = Acceleration - $\frac{\partial^2 s}{\partial t^2}$

J = Jerk - $\frac{\partial^3 s}{\partial t^3}$

* **Classification of Follower**

① According to the surface in contact :-

(a) Knife edge follower

(b) Roller follower

(c) Flat ~~edge~~ faced (or) Mushroom follower.

(d) Spherical faced follower.

② According to Motion of the follower

(a) Reciprocating (or) translating follower

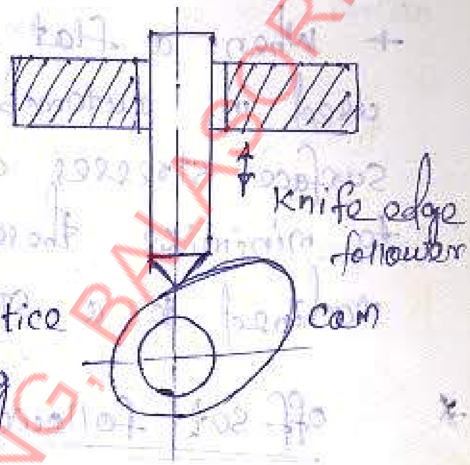
(b) oscillating (or) rotating follower

(3) According to the path of motion of the follower.
(a) Radial follower
(b) off-set follower

* Knife edge follower :-

→ When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower.

→ It is seldom (limited) used in practice because the small area of contacting surface results in excessive wear.



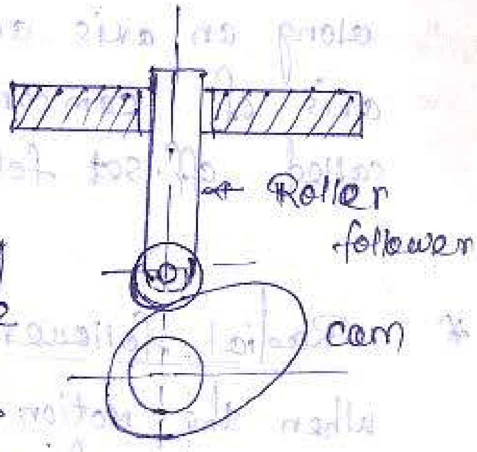
* Roller follower :-

→ When the contacting end of the follower is a roller, it is called a roller follower.

→ The roller follower are extensively used where more space is available

→ Ex :- stationary oil & gas engines & aircraft engine

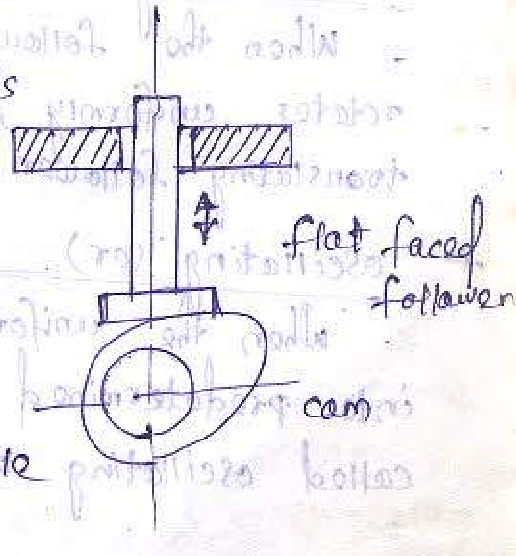
→ Wear & tear will be less



* Flat faced or Mushroom follower :-

→ When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower.

→ The flat faced follower are generally used where space is limited Ex :- cams which operate the valves of automobile engines.



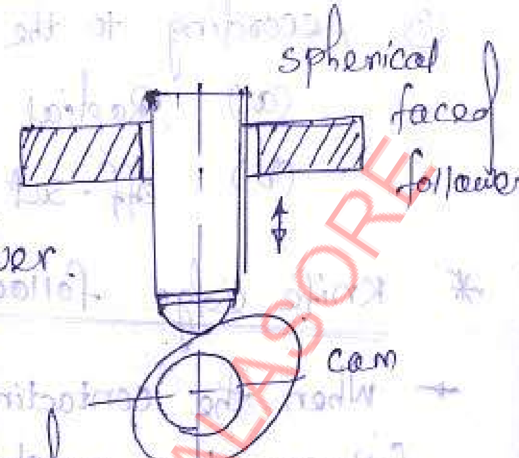
→ Wear & tear is less.

circular it is then

* Spherical faced follower :-

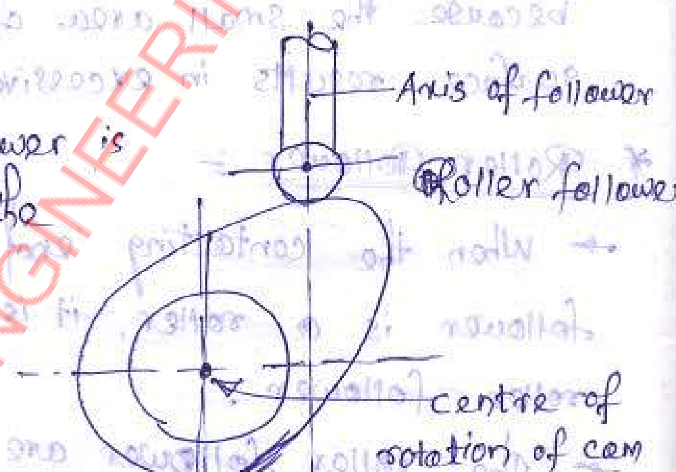
→ When the contacting end of the follower is of spherical shape, it is called a spherical faced follower.

→ When a flat faced follower is used in automobile engines, high surface stresses are produced. Inorder to minimize these stresses, the flat end follower is machined to a spherical shape.



* Off-set follower :-

When the motion of the follower is along an axis away from the axis of cam centre, it is called off-set follower.



* Radial follower :-

When the motion of the follower is along an axis passing through the centre of cam, it is known as radial follower.

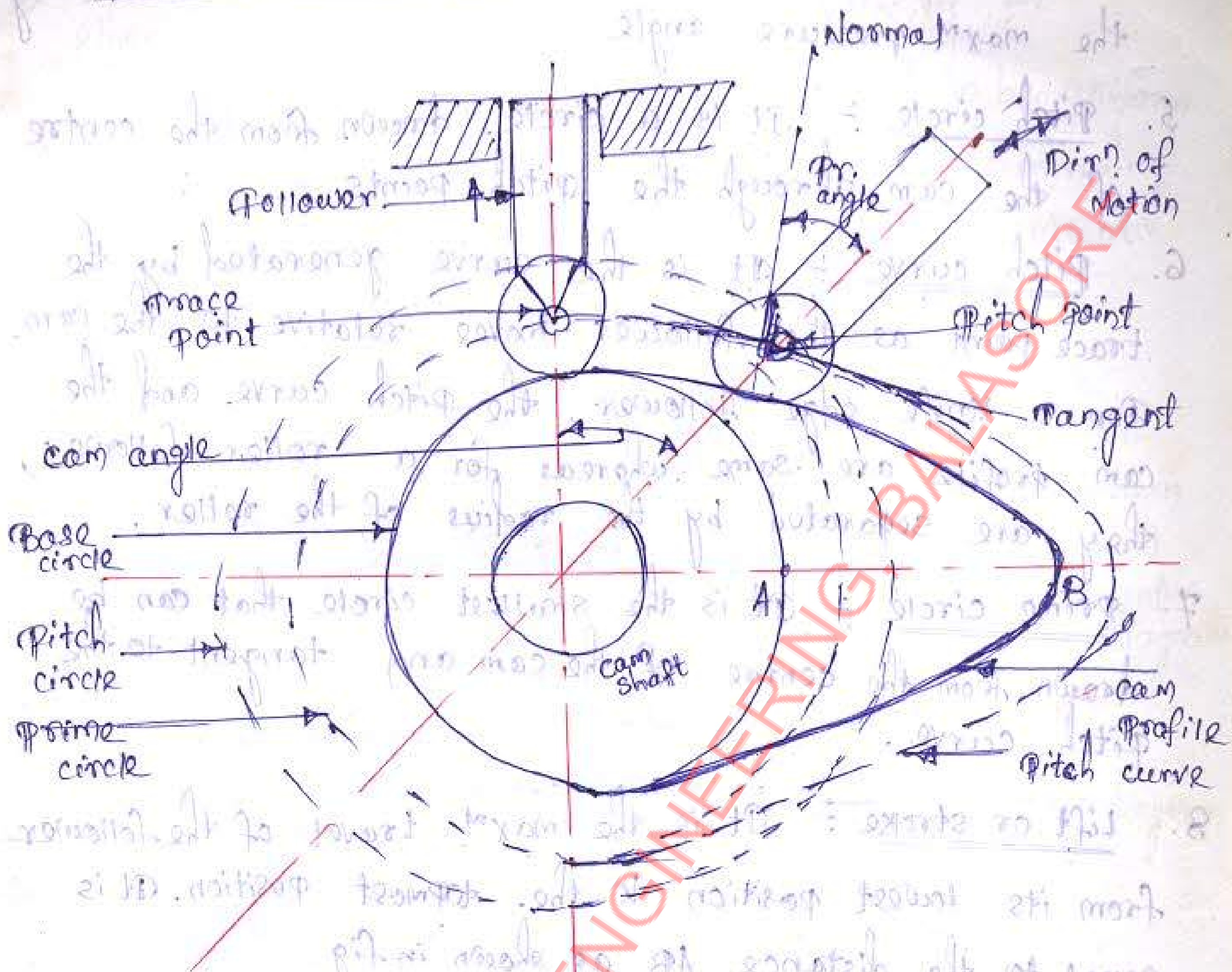
* Reciprocating (or) translating follower :-

When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower.

* Oscillating (or) rotating follower :-

When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower.

* Nomenclatures for cam profiles :-



1. Base circle :- It is the smallest circle that can be drawn to the cam profile.

2. Trace point :- It is a reference point on the follower and is used to generate the pitch curve. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In roller follower, centre represent trace point.

3. Pressure angle :- It is the angle betn the dirn of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pr. angle is too large, a reciprocating follower will jam in its bearings.

4. Pitch point :- It is a point on the pitch curve having the max^m pressure angle.

5. Pitch circle :- It is a circle drawn from the centre of the cam through the pitch points.

6. Pitch curve :- It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.

7. Prime circle :- It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve.

8. Lift or stroke :- It is the max^m travel of the follower from its lowest position to the topmost position. It is equal to the distance AB as shown in fig.

9. Period of dwell :- It is the period during which the follower remains stationary during some finite rotation of the cam.

10. Cam angle :- It is an angle of rotation of the cam for a definite displacement of the follower.

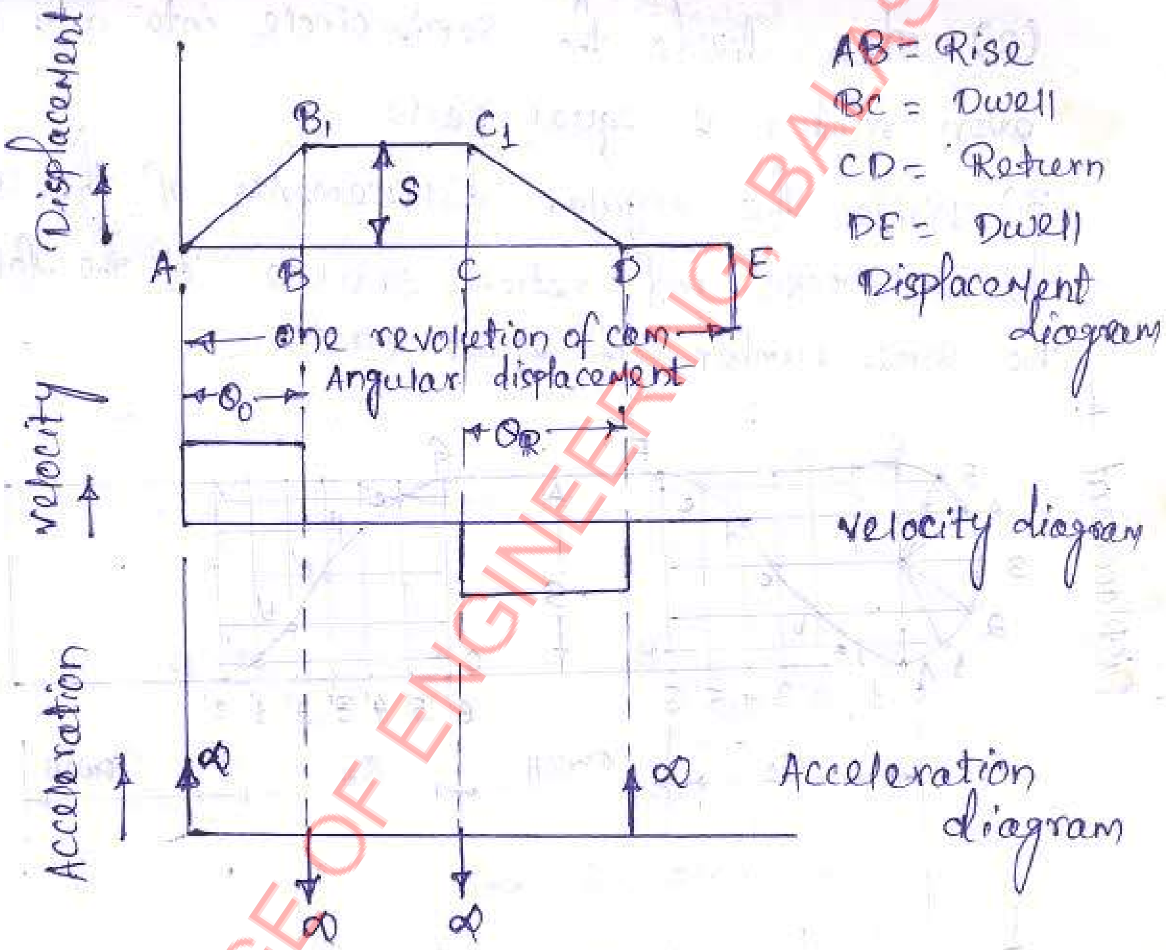
11. Cam profile :- The surface in contact with the follower is known as cam profile. This is the actual working curve of the cam.

* Motion of the follower

(1) Uniform velocity (2) Simple harmonic Motion (SHM)
(3) Uniform acceleration & retardation
(4) cycloidal Motion

* Displacement, velocity and Acceleration Diagrams when follower moves with uniform velocity :-

The motion of a follower is said to be uniform when it travels the same distance in each succeeding time interval. Hence its velocity will be constant.

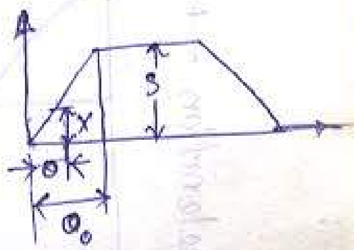


S = stroke of the follower

θ_0 and θ_R = Angular displacement of the cam during out stroke & return stroke of the follower respectively in radians

ω = Angular velocity of the cam in rad/s.

from fig: $\theta = \omega t$ [θ = Angle turned by cam in time (t)]



The displacement of follower during out stroke in time (t)

$$\boxed{x = \frac{S}{\theta_0} \times \theta} \qquad \left(\frac{x}{\theta} = \frac{S}{\theta_0}\right)$$

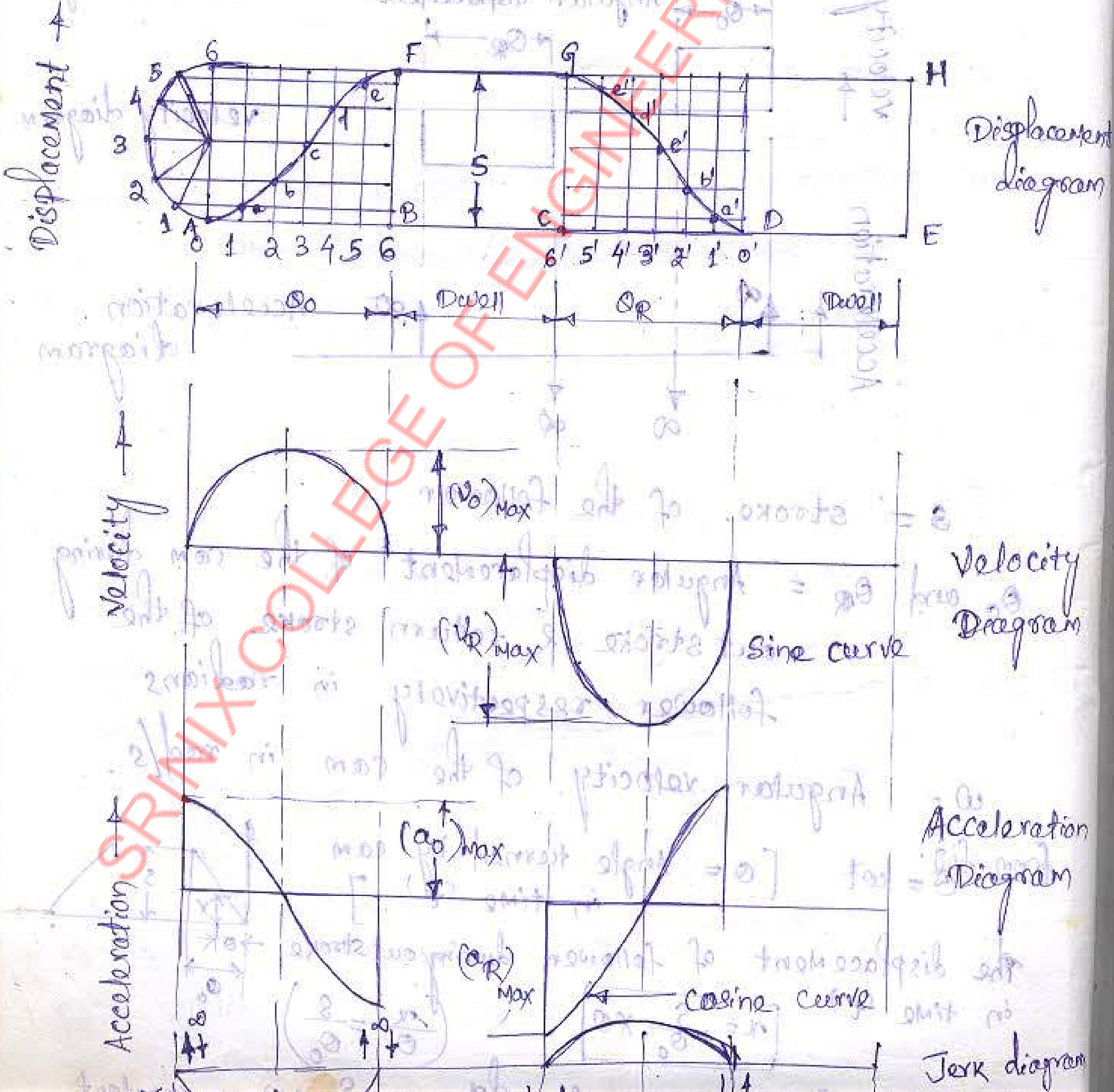
* Displacement, velocity and Acceleration Diagrams when the follower with SHM :-

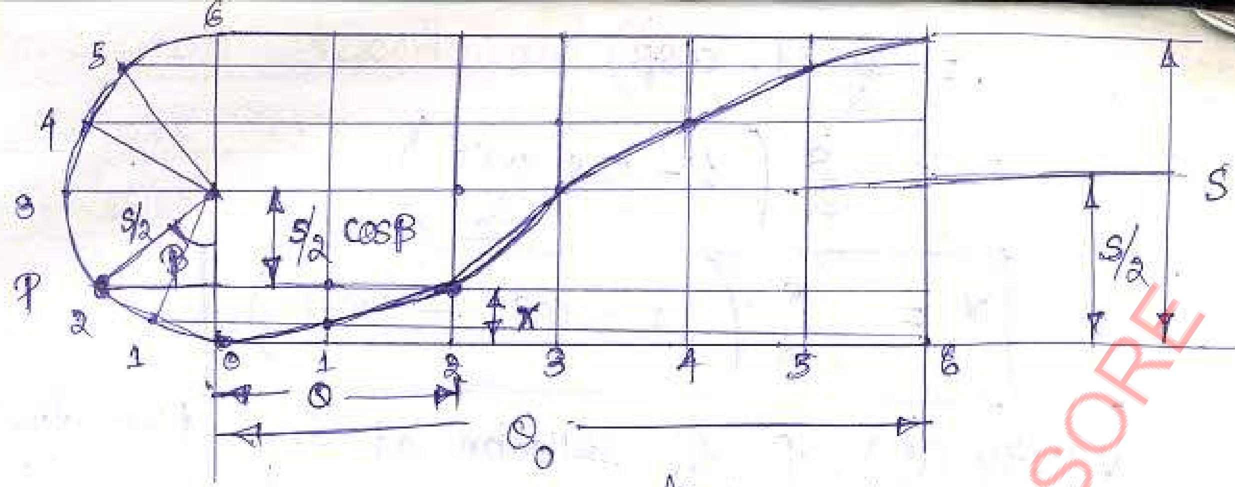
Displacement Diagram

(1) First draw a semi-circle on the follower stroke as diameter.

(2) Then divide the semi-circle into any convenient even number of equal parts

(3) Divide the angular displacements of the cam during the outstroke and return strokes of the follower into the same number of equal parts.





Angular displacement

Let x = Displacement of the follower in time 't'. This displacement is also known as the lift of the follower.

β = Angle turned by a point (P) moving along the circumference of the semi-circle in time 't'

θ = Corresponding angle turned by cam in time 't'

θ_0 = Angle turned by cam during outstroke.

ω = Angular velocity of cam.

Then $\theta = \omega \times t$

When the angle turned by cam is 'θ_0', the corresponding angle turned by point (P) will be π. When the angle turned by cam is 'θ', the angle turned by point 'P' will be $\left(\frac{\pi}{\theta_0} \times \theta\right)$

But when the angle turned by cam is 'θ', the corresponding angle turned by point 'P' is β

$\Rightarrow \theta_0 - \pi$

$\Rightarrow 1 - \pi/\theta_0$

$\boxed{\beta = \frac{\pi}{\theta_0} \times \theta}$

$\Rightarrow \theta - \frac{\pi}{\theta_0} \times \theta = \beta$

Now the displacement (x) of the follower in time 't' is given by $x = \frac{S}{2} - \frac{S}{2}\cos\beta$

$$= \frac{S}{2}\left(1 - \cos\frac{\pi \times \theta}{\theta_0}\right)$$

$$\boxed{x = \frac{S}{2}\left(1 - \cos\frac{\pi}{\theta_0} \times \omega \times t\right)}$$

Velocity (v_0) of the follower at any time during outstroke

$$(v_0) = \frac{dx}{dt} = \frac{d}{dt}\left[\frac{S}{2}\left(1 - \cos\frac{\pi}{\theta_0} \times \omega \times t\right)\right]$$

$$= \frac{S}{2}\left[0 - \left(-\sin\frac{\pi\omega t}{\theta_0}\right) \times \frac{\pi\omega}{\theta_0}\right] = \frac{S\pi\omega}{2\theta_0} \times \sin\frac{\pi\omega t}{\theta_0}$$

$$\Rightarrow (v_0) = \frac{S}{2} \times \frac{\pi\omega}{\theta_0} \times \sin\left(\frac{\pi\theta}{\theta_0}\right) \quad [\because \omega t = \theta]$$

The velocity will max^m when $\theta = 90°$

$$\Rightarrow \sin\left(\frac{\pi\theta}{\theta_0}\right) = \sin\frac{\pi}{2}$$

$$(v_0)_{max} = \frac{S}{2} \times \frac{\pi\omega}{\theta_0} \times \sin\left(\frac{\pi}{\theta_0} \times \frac{\theta_0}{2}\right)$$

$$\Rightarrow \frac{\pi\theta}{\theta_0} = \frac{\pi}{2}$$

$$\boxed{(v_0)_{max} = \frac{S}{2} \times \frac{\pi\omega}{\theta_0}}$$

$$\Rightarrow \boxed{\theta_0 = 2\theta} \Rightarrow \boxed{\theta = \frac{\theta_0}{2}}$$

Acceleration (a_0)

$$\Rightarrow \frac{dv_0}{dt} = \frac{d}{dt}\left(\frac{S}{2} \times \frac{\pi\omega}{\theta_0} \times \sin\frac{\pi\omega t}{\theta_0}\right)$$

$$= \frac{S}{2} \times \frac{\pi\omega}{\theta_0} \times \cos\frac{\pi\omega t}{\theta_0} \times \frac{\pi\omega}{\theta_0}$$

$$\boxed{a_0 = \frac{S}{2} \times \left(\frac{\pi\omega}{\theta_0}\right)^2 \times \cos\frac{\pi\theta}{\theta_0}} \quad (\omega t = \theta)$$

The acceleration will be max^m when ($\theta = 0°$)

$$\boxed{(a_0)_{max} = \frac{\pi^2\omega^2 S}{2\theta_0^2}}$$

$$\boxed{(v_0)_{max} = \frac{\pi\omega S}{2\theta_0}}$$

$$\boxed{(v_R)_{max} = \frac{\pi\omega S}{2\theta_R}}$$

$$\boxed{(a_0)_{max} = \frac{\pi^2\omega^2 S}{2\theta_0^2}}$$

$$\boxed{(a_R)_{max} = \frac{\pi^2\omega^2 S}{2\theta_R^2}}$$

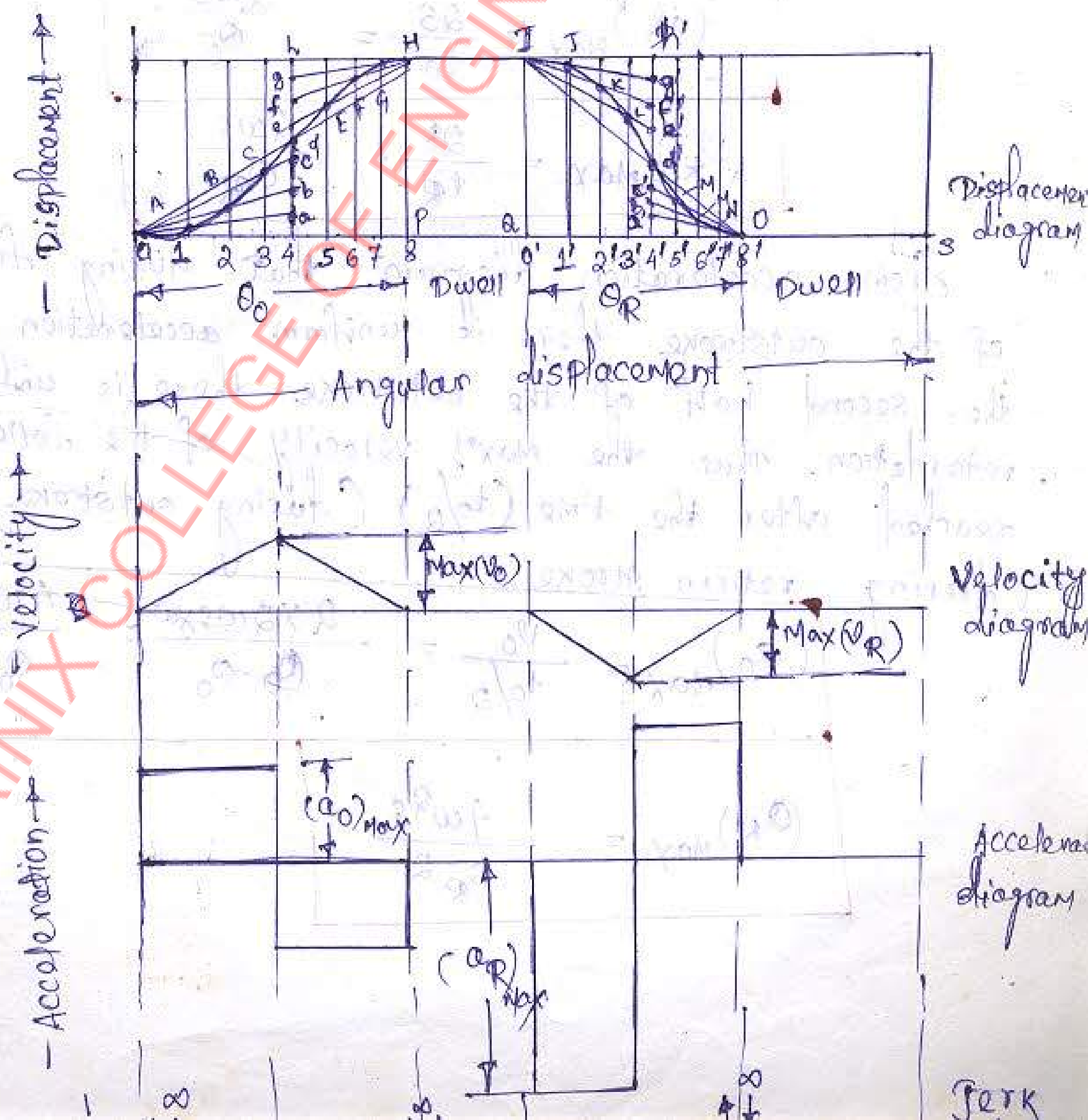
* Displacement, velocity and Acceleration Diagrams when the follower moves with uniform Acceleration and Retardation :-

① Divide the angular displacement of the cam during outstroke (θ_o) into any even number of equal parts and draw vertical lines through these points.

② Divide the stroke of the follower (S) into the same number of equal even parts.

③ Then join these points to obtain the parabolic curve for the outstroke of the follower.

④ In the similar way, the displacement diagram for the follower during return stroke may be drawn.



We know that time required for the follower during outstroke $t_0 = \theta_0/\omega$

time required for the follower during return stroke

$$t_R = \theta_R/\omega$$

Mean velocity of the follower during outstroke

$$= S/t_0$$

Mean velocity of the follower during return stroke

$$= S/t_R$$

Since the max^m. velocity of the follower is equal to twice the mean velocity, therefore max^m velocity of the follower during outstroke

$$\boxed{(V_0)_{max} = \frac{2S}{t_0} = \frac{2\omega S}{\theta_0}}$$

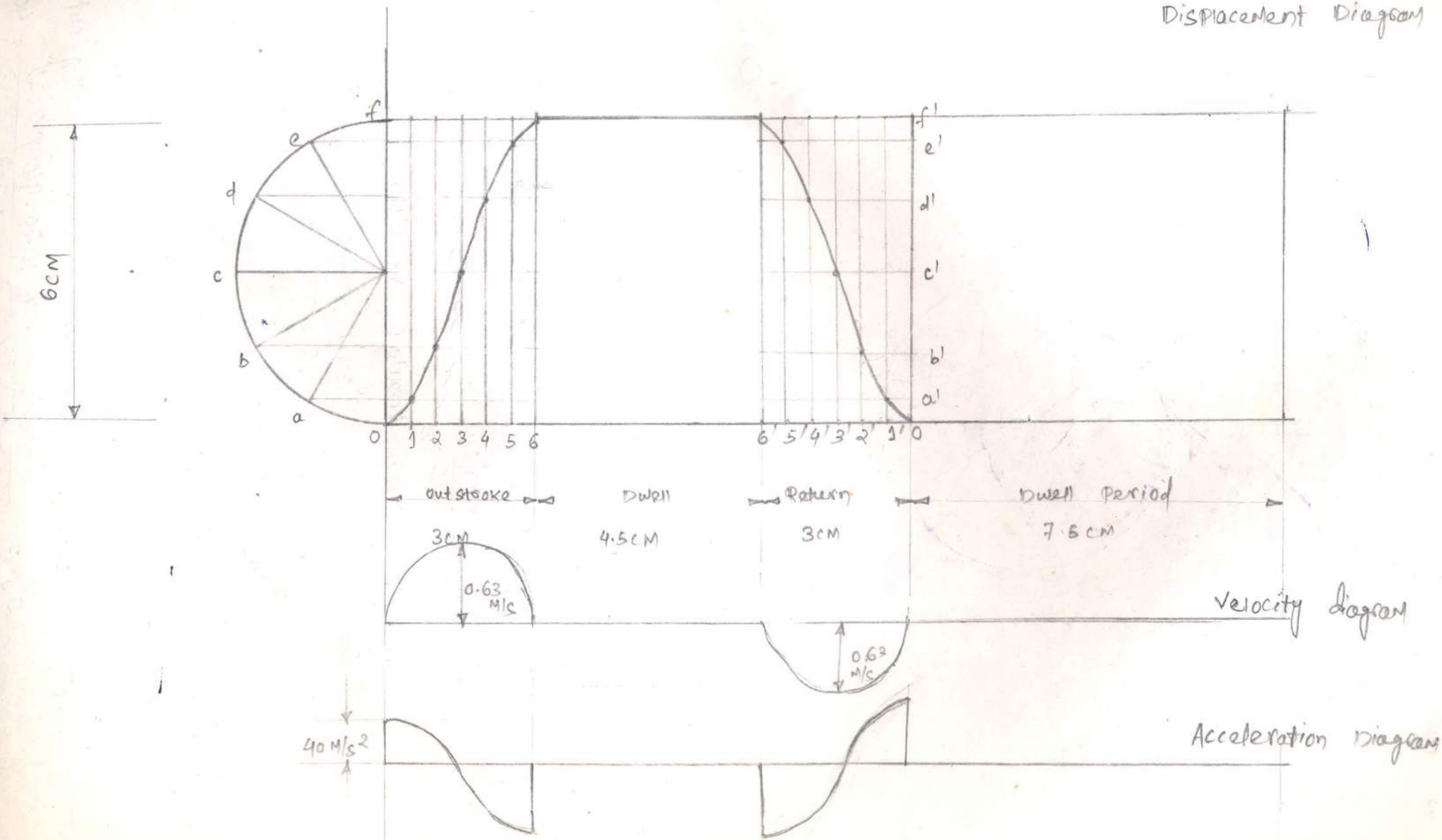
$$\boxed{(V_R)_{max} = \frac{2S}{t_R} = \frac{2\omega S}{\theta_R}}$$

from acceleration diagram, that during first half of the outstroke there is uniform acceleration & during the second half of the outstroke there is uniform retardation. Thus, the max^m velocity of the follower is reached after the time $(t_0/2)$ (during outstroke) & $t_R/2$ (during return stroke).

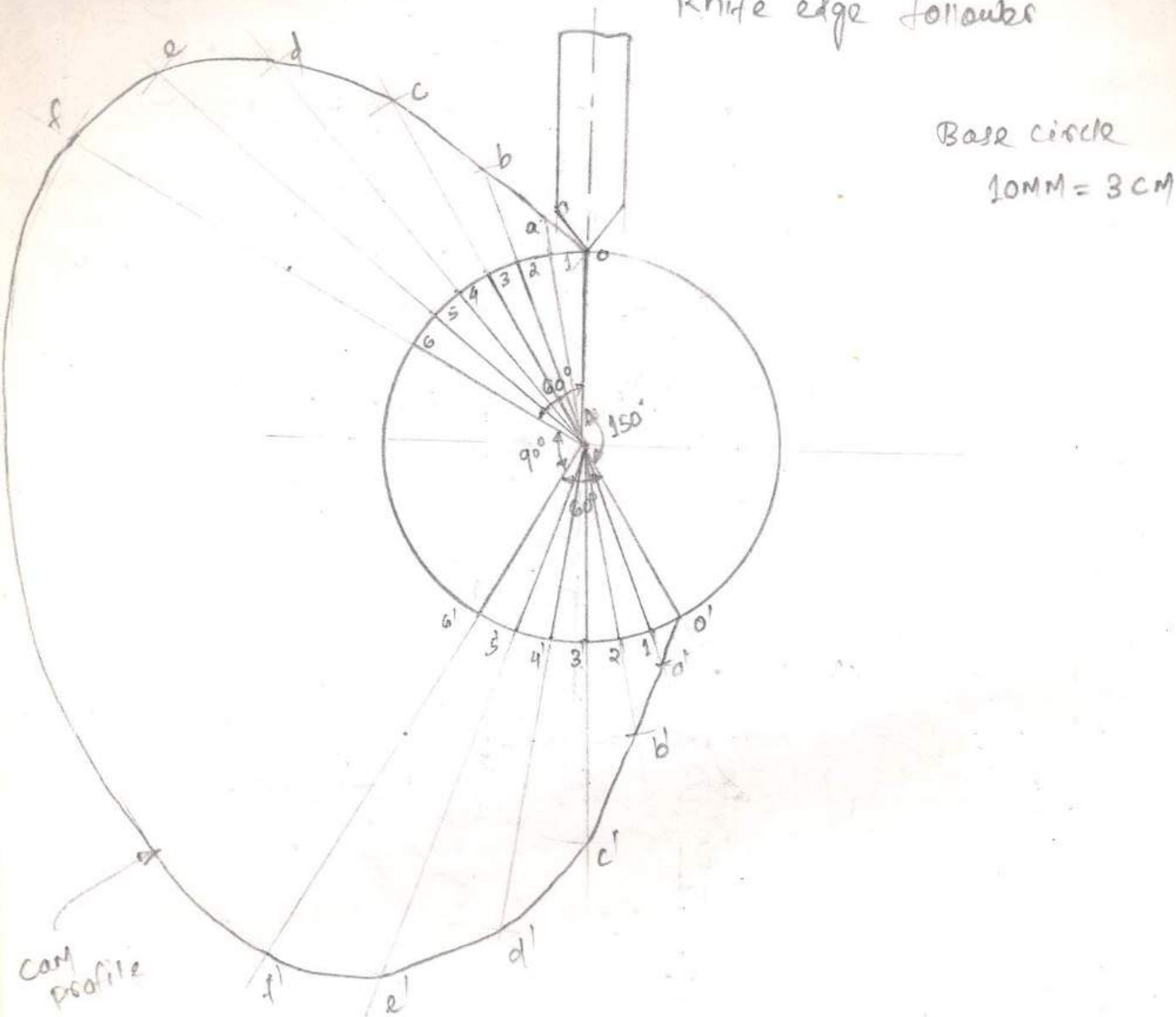
$$\boxed{(a_0)_{max} = \frac{V_0}{t_0/2} = \frac{2 \times 2\omega S \times \omega}{\theta_0 \cdot \theta_0} = \frac{4\omega^2 S}{\theta_0^2}}$$

$$\boxed{(a_R)_{max} = \frac{4\omega^2 S}{\theta_R^2}}$$

* Draw the cam profile for a knife-edge follower executing SHM with a stroke of 20MM with the following parameters given below. The outstroke or rise stroke is 60°, dwell period is 90°. the return stroke is 60°, rest part is dwell period. The base circle of the cam profile is 10 MM radius. Also calculate the Max^m^. velocity and Max^m^. acceleration executed by follower when cam rotates 200 rpm.


Displacement Diagram
6CM
f
e
d
c
b
a
0 1 2 3 4 5 6
6' 5' 4' 3' 2' 1' 0
f'
e'
d'
c'
b'
a'
out stroke
Dwell
Return
Dwell Period
3CM
4.5CM
3CM
7.5CM
0.63 M/s
0.63 M/s
Velocity diagram
40 M/s²
Acceleration Diagram

⇒ Axis of follower passing through the centre of cam.



Maximum velocity of the follower during out stroke

$$\Rightarrow V_{O\,max} = \frac{\pi s \omega}{2\theta_0}$$

we know ω = Angular velocity of the cam

$$= \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s}$$

$$V_{O\,max} = \frac{\pi \times 0.02 \times 20.94}{2 \times 1.05} \qquad \theta_0 = 60° \times \frac{\pi}{180°} = 1.05 \text{ radian}$$

$$= 0.63 \text{ M/s}$$

Max^{m}. acceleration of the follower during out stroke

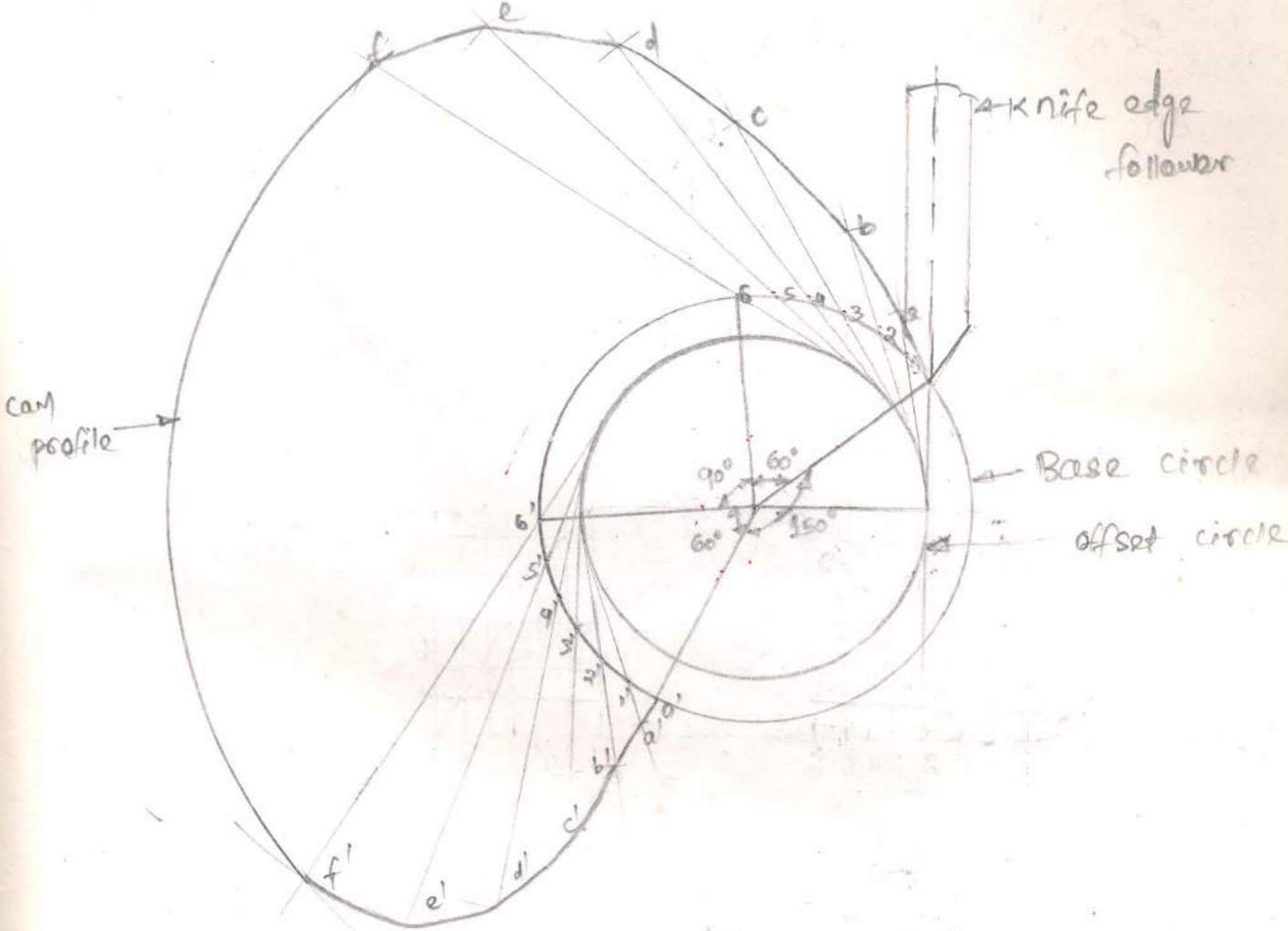
$$a_0 = \frac{\pi^2 \omega^2 s}{2(\theta_0)^2} = \frac{\pi^2 \times 20.94^2 \times 0.02}{2 \times 1.05^2} = 40 \text{ M/s}^2$$

⇒ Axis of follower is at a distance of 8MM from the centre of cam.

$10\,MM = 3\,CM$ on paper

$1\,MM = \frac{3}{10}\,CM$

$8\,MM = \frac{3}{10} \times 8 = 2.4\,CM$



* A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower at the end of a valve rod, motion.

1. To raise the valve through 50 mm during 120° rotation of the cam.
2. To keep the valve raised through next 30°.
3. To lower the valve during next 60° and.
4. To keep the valve closed during rest of the revolution i.e.

The dia. of the roller is 20 mm and dia. of the cam shaft is 25 mm. Draw the profile of the cam when (a) line of stroke of the valve rod passes through the axis of the cam sha (b) the line of the stroke is offset 15 mm from the axis of cam shaft

Scale $10^\circ = 0.5$ CM on paper

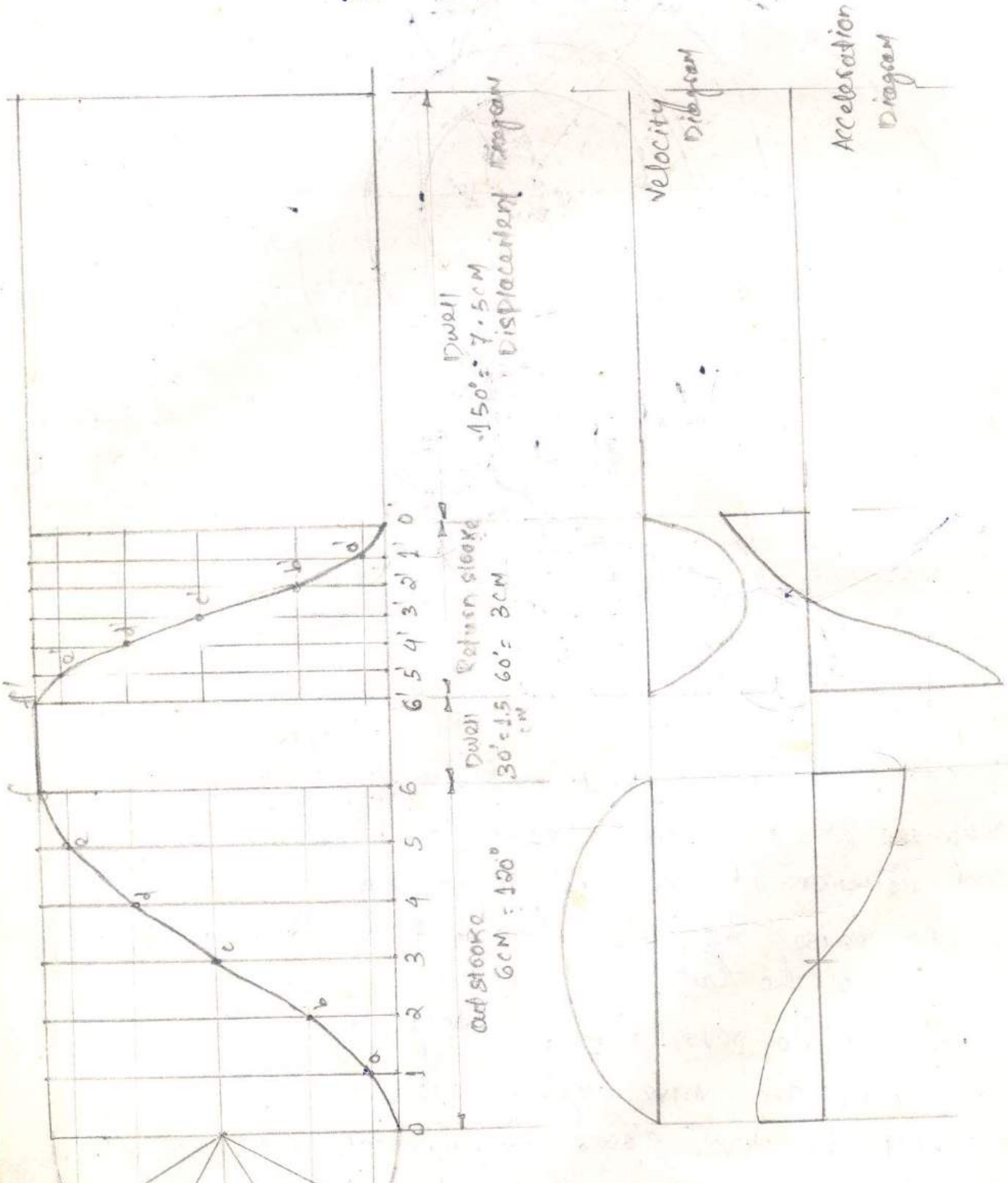
$120^\circ = 6$ CM on paper

$30^\circ = 1.5$ CM on paper

$60^\circ = 3$ CM on paper

$150^\circ = 7.5$ CM on paper

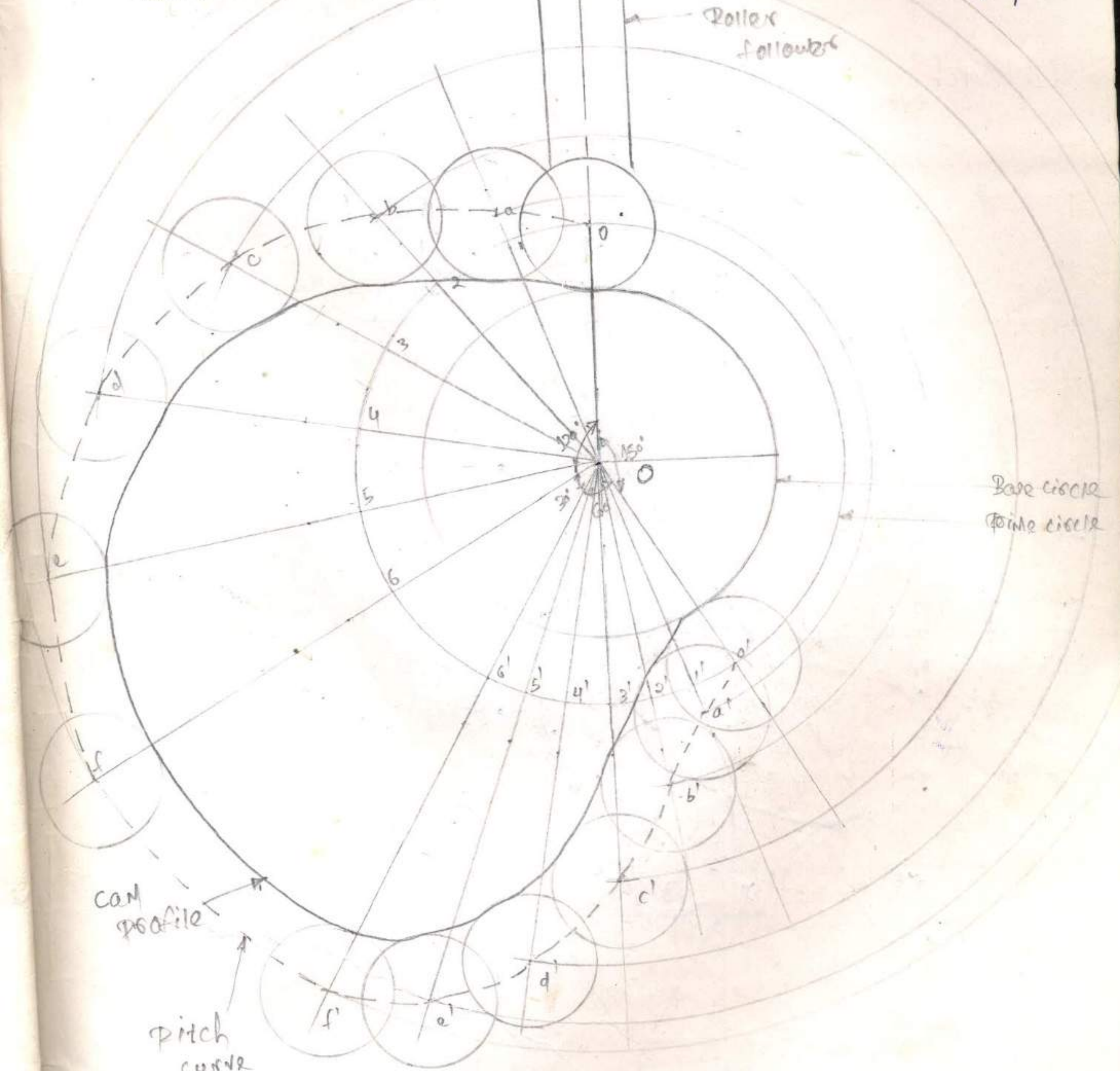
50MM = 6CM on paper



(a) profile of the cam when the line of ~~stroke~~ stroke of the valve rod passes through the axis of the cam shaft.

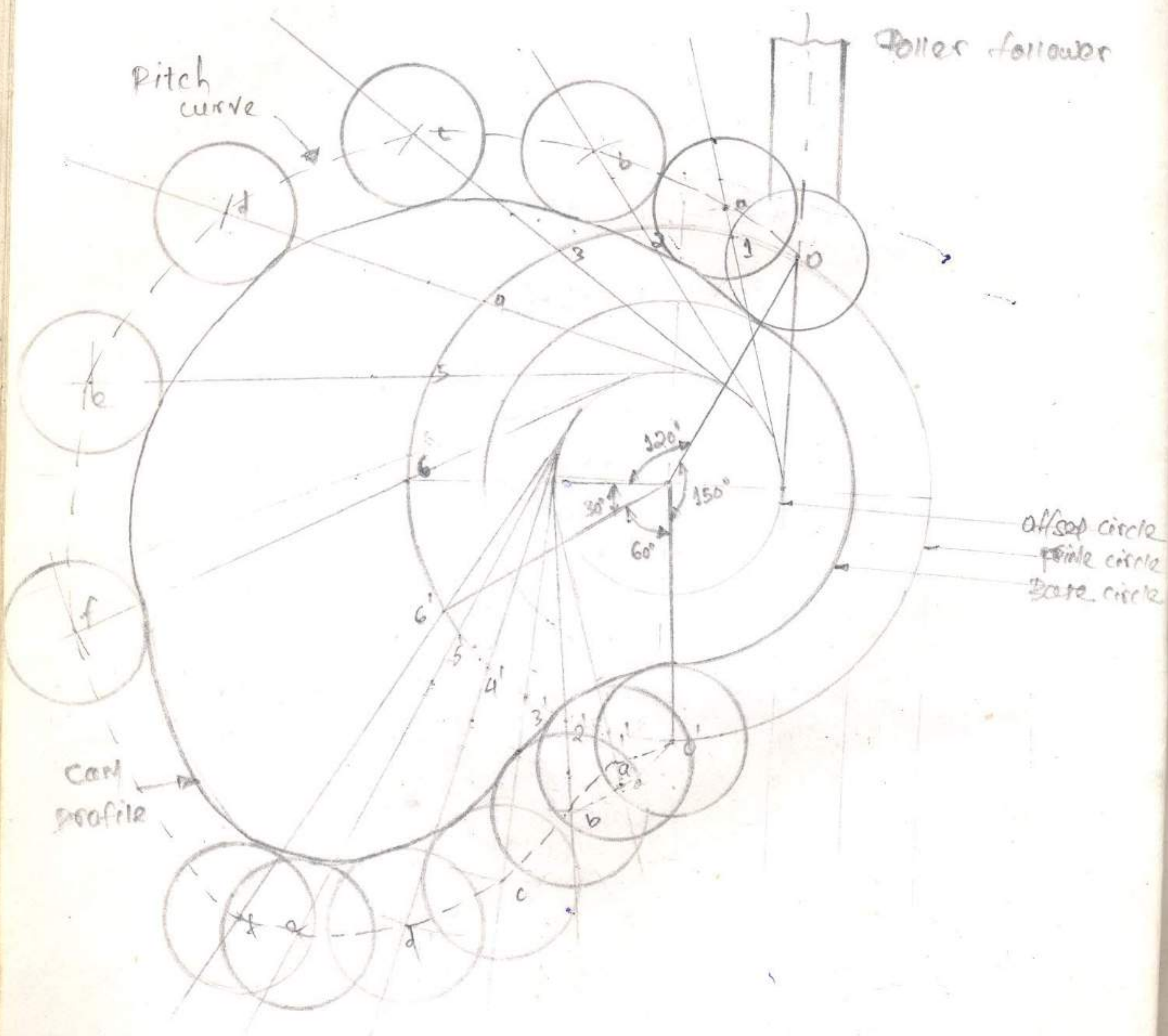
Radius of ~~pitch~~ Base circle = 25MM = 3CM on paper

Radius of prime circle = 25 + 10 = 35MM = 4.2CM on paper



(b) the line of the stroke is offset 15 mm from the axis of the cam shaft.

offset circle
15mm = 1.8 cm
on paper



* A cam drives a flat reciprocating follower in the following manner :-

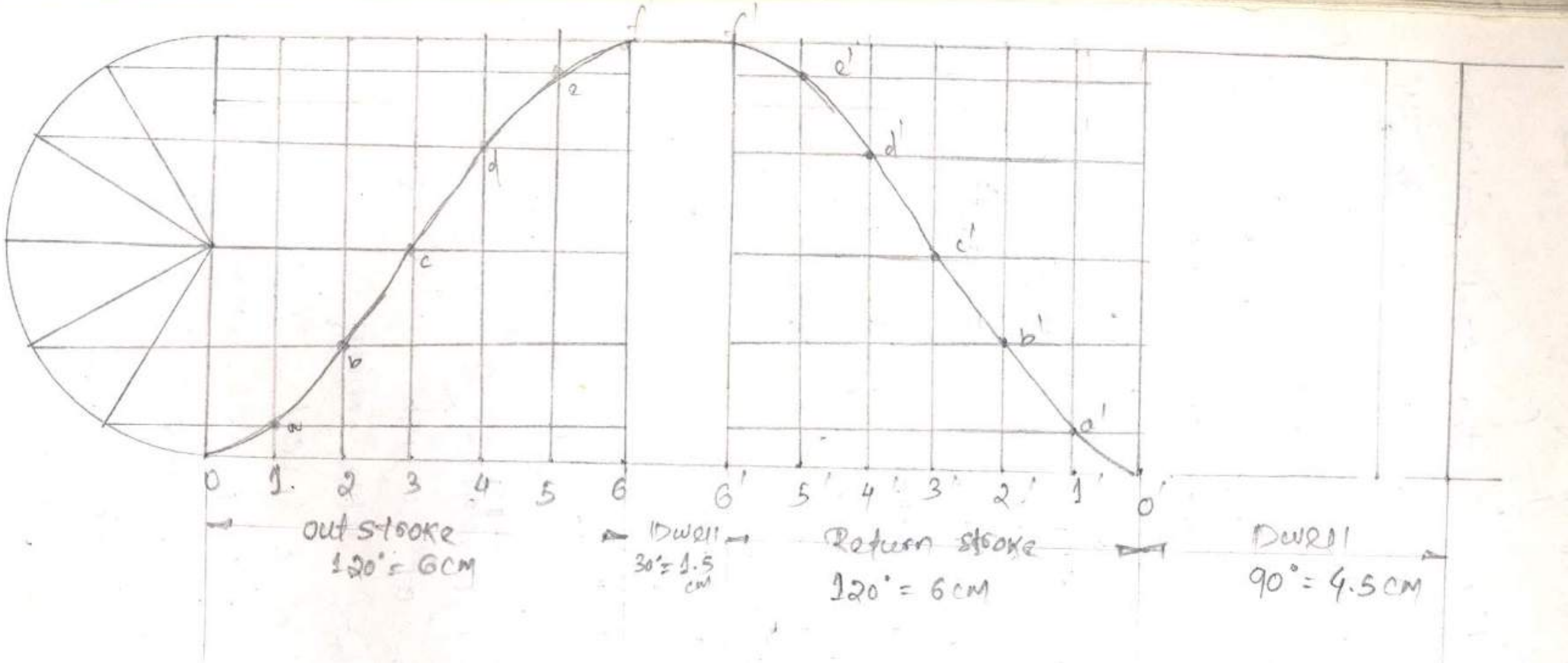
During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with SHM. The follower dwells during next 30° of cam rotation. During next 120° of cam rotation, the follower moves inwards with SHM. The follower dwells for the next 90° of cam rotation. The minm. radius of the cam is 25 mm. Draw the profile of the cam.

Ans:-

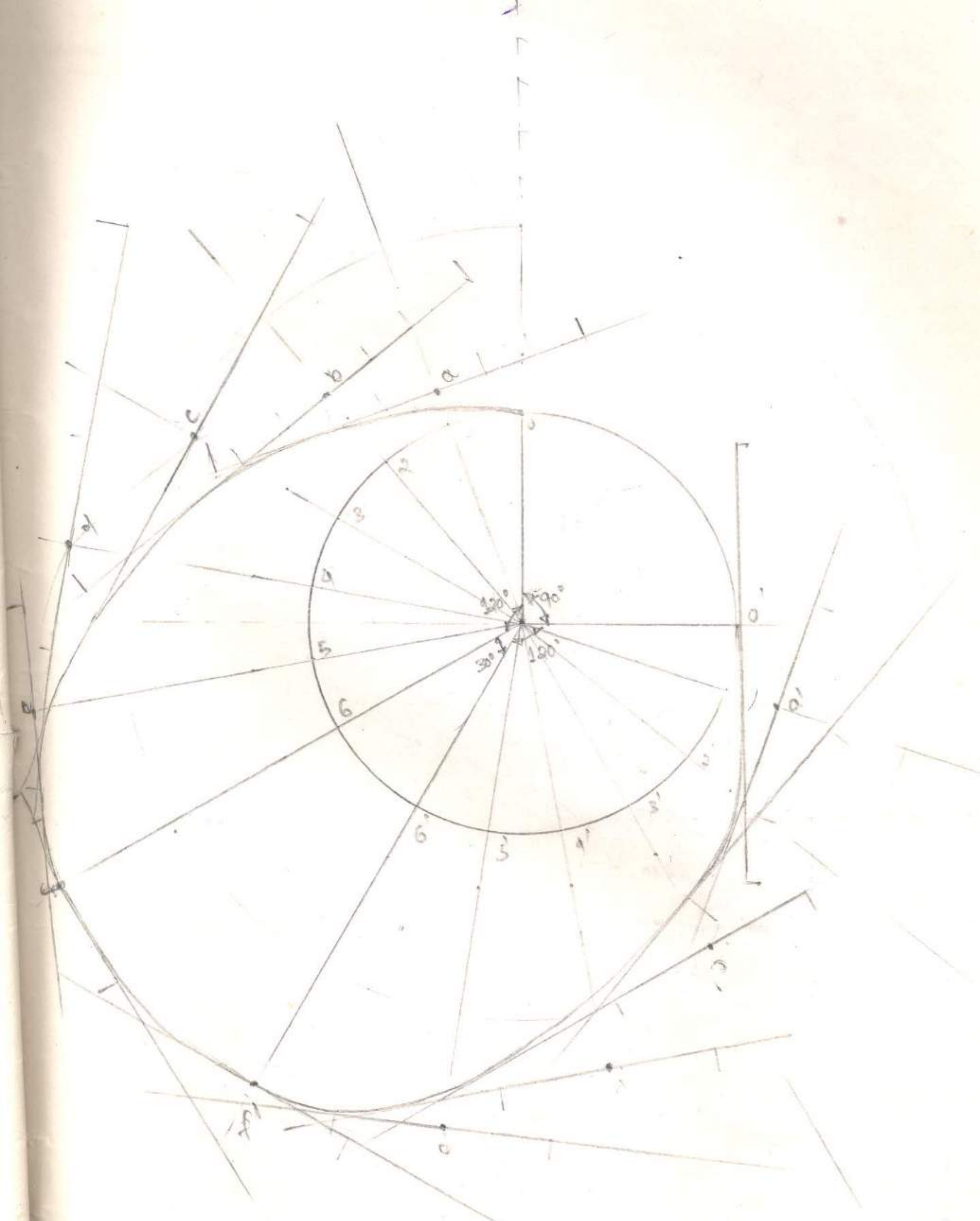
10° = 0.5 cm on paper
120° = 6 cm on paper
30° = 1.5 cm on paper
120° = 6 cm on paper
90° = 4.5 cm on paper

Radius of pitch circle = 25 mm
= 4 cm on paper

20 mm = 6 cm on paper



0 1 2 3 4 5 6
6' 5' 4' 3' 2' 1' 0'
a b c d e f
f' e' d' c' b' a'
out stroke
120° = 6CM
Dwell
30° = 1.5 CM
Return stroke
120° = 6 CM
Dwell
90° = 4.5 CM


a
b
c
d
0
2
3
4
5
6
0'
120°
90°
120°
30°